THE ELEMENTS OF LOGIC

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THE ELEMENTS OF LOGIC

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PREFACE

THE aim of this book is to meet the requirements of students who take logic as one of their subjects for an Ordinary or Pass Degree at the Universities, and at the same time to provide an introduction to logic for those who intend to pursue the study of the subject to a more advanced stage.

The book was begun by Professor Latta some years before the war. When he had completed the manuscript (except the exercises) up to p. 204 of the work as it now appears, he was compelled to discontinue writing, owing to the pressure of administrative and other work which he undertook during the war. The heavy demands of this work so impaired his health that he was unable to do anything further towards completing the book. When I was asked to complete it, I at first thought it would be possible to do so by using material from Professor Latta's lecture notes. But, partly because Professor Latta had, in the part which he completed, departed in places from the doctrine of the lecture notes and more especially because he was not in the habit of lecturing on induction to his Ordinary Class, this proved impossible. I have, however, used material from his lecture notes in Chapter XV, in the criticism of Mill's doctrine that inference is from particular to particular in Chapter XVIII, and in the account of Aristotle's view of induction in Chapter XIX. But for the form in which this material now appears, as well as for the remainder of the text and the exercises, I am responsible. My main aim has been to follow the lines which seemed necessary to complete the book consistently with the position taken in the earlier part, which remains substantially in the form in which Professor Latta put it.

In a work of this kind it is difficult to acknowledge one's obligations to other writers adequately. Such acknowledgement had been attempted in the footnotes. But the position taken in the book has been so influenced by the logical writings of Bradley, Bosanquet and Mr. Joseph that special mention of them is necessary. I should also mention my own deep indebtedness to Professor Latta, under whose guidance I began the study of logic, and to whose teaching and encouragement I owe the greater part of what I know about the subject.

Of the exercises a few were taken from Professor Latta's papers, and others have appeared in examination papers in Glasgow and Belfast. Those of the test questions at the end of the book marked (O) have been taken from the First Public Examination papers of the University of Oxford. Where exercises have been taken from other books on logic, the name of the author is given in brackets after the exercise.

It is a pleasure to acknowledge the assistance which I have received in the preparation of the book. Mr. W. D. Ross of Oriel College, Oxford, read Professor Latta's original manuscript and gave most helpful advice as to how it should be completed. Later, he read the manuscript of almost the whole of the work and made many valuable criticisms and suggestions. The proofs were read by Miss M. J. Levett and Mr. George Brown, Lecturers in Logic and Metaphysics in the University of Glasgow, and by my colleague Mr. O. de Selincourt, all of whom made many suggestions for the improvement of the book. If I had taken more of the advice thus given me, probably the book would be better than it is. For the errors which remain I alone am responsible.

A. MACBEATH.

March, 1929.

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CHAPTER I

A GENERAL DESCRIPTION OF LOGIC

In beginning the study of any science, we must have some general idea of its subject-matter and of the special way in which the science is concerned with its subject-matter. We cannot begin with a rigorous definition, for such a definition cannot be understood without considerable knowledge of the subject. And in the case of many sciences, including those which deal with the thought, conduct and feelings of man, there is the further difficulty that no one definition can claim general assent. But some sort of preliminary description of the nature and purpose of the science is necessary at the outset, and it is the object of this chapter to give such a description of Logic.

Etymologically, logic is the science of λόγος, *i.e.* of reasoned discourse or language, discourse which expresses thought. In all intelligible discourse, whether written or spoken, there is continuity; we pass more or less naturally and inevitably from sentence to sentence. But the special quality of this continuity is different in different kinds of discourse. Consider, for instance, such different types of discourse as the oratorical, the poetic and the scientific. Each of these has a certain predominating quality which pervades it, gives it continuity and enables us to distinguish it from the others. The object of oratory is to persuade its hearers to do, or to refrain from doing, something. It is charged with feeling, often with passion. It appeals not merely to the intelligence of its hearers, but to their prejudices, their imagination, their humour, their feelings and

desires. It ought, no doubt, to be reasoned, to have intellectual continuity; but excellent oratory may contain very bad reasoning. The continuity of poetry, again, is mainly a continuity of imagination and emotion. The unity of a poem is an artistic unity; it includes such elements as harmonious rhythm, rhyme and assonance, as well as fine imagery and subtle suggestion. There may be an element of reasoning; but it is not essential, and it is always subordinate. On the other hand, the predominant feature of a scientific book or statement is its intellectual or rational continuity, a continuity of thought or reasoning, as distinct, say, from imagination or feeling. We do not look for eloquence, emotion, or pictorial imagery in Euclid or in Newton's *Principia*; but we do expect a very rigid connection of statement with statement, of thought with thought, so that each step may be shown to follow, with an iron necessity, that which precedes it.

A simple example of each of these kinds of discourse will make the difference obvious. As an instance of an oratorical statement we may take the well-known sentence of Burke: "Because half-a-dozen grasshoppers under a fern make the field ring with their importunate chink, while thousands of great cattle, reposed beneath the shadow of the British oak, chew the cud and are silent, pray do not imagine that those who make the noise are the only inhabitants of the field, that they are many in number or that they are other than the little, shrivelled, meagre, hopping, though loud and troublesome, insects of the hour." Coleridge's lines will serve as an example of the poetic:

> "In Xanadu did Kubla Khan
> A stately pleasure-dome decree:
> Where Alph, the sacred river, ran
> Through caverns measureless to man
> Down to a sunless sea."

And from these we may pass to the bare scientific argument: "A magnitude required for the solution of a problem must satisfy a particular equation, and as the magnitude x alone

satisfies this equation, it is therefore the magnitude required."

If we were to express the quotation from Burke in a form similar to that of the scientific argument, we should state it somewhat in this way : " Noisy people are not necessarily numerous or worthy of much consideration." Similarly Coleridge's lines would be reduced to a statement about the building of a palace by Kubla Khan at Xanadu near the river Alph. It is obvious that we should thus completely fail to express what Burke and Coleridge intended ; and yet we might fairly claim to have stated the purely intellectual element in their sentences. Accordingly, while in all discourse there is an intellectual element, language may be used to express much more than pure thought or reasoning, and we may therefore discriminate in it various kinds of continuity.

Now we may turn our attention to any one of the different forms of continuity in discourse (including others as well as those which have been mentioned), and we may try to discover what is meant by rhetorical continuity or poetic continuity or intellectual continuity—what exactly is their nature and what are the laws which they must obey, whatever may be the special things which are rhetorically, poetically or intellectually connected together. And according as we thus deal with the rhetorical, the poetic or the intellectual quality of human discourse, we are constructing the science of rhetoric, that of poetics (literary criticism), or that of logic.

As the subject-matter of logic is the intellectual element in discourse, we may describe logic generally as the science of thought. There has been much discussion of the question whether there can be thought without language ; but that is a problem with which we need not specially concern ourselves. Practically logic deals with thought as it is expressed in language, and if there be any other thought, it cannot differ essentially from the thought which is thus expressed. But language, as we have seen, expresses more than thought,

and it is therefore important to remember that logic is concerned with thought itself rather than with the verbal forms in which it is expressed. The significance of this will be more evident later, when we come to consider the distinctions between logical terms and names, and between logical propositions and sentences. Now, when we say that logic is the *science* of thought, we mean that logic investigates, or endeavours to make explicit, the principles of thought, the principles on which thinking depends. Every science tries to discover principles. The business of a science is not merely to observe facts or events and to arrive at general statements about them, but to bring to light the fundamental conditions, laws or principles, which are present in the events and which govern their appearance. The statement, for instance, that 'All bodies fall to the ground' is a general statement, based on observation of facts, but it is not a scientific statement, because it does not express a scientific principle. On the other hand, the law of gravitation is a scientific principle, because it is a statement of the fundamental conditions on which depend the motions of the planets, as well as countless other events, observed and unobserved, which are continually occurring throughout the whole material universe. Similarly we can distinguish between the principles which the science of meteorology or the science of ethics aims at discovering and the weather saws or the proverbial maxims of conduct, which are rough generalizations from experience. Every science presupposes that facts, events or actions are the expressions of principles, and it endeavours to attain knowledge of these principles, to make us clearly conscious of them.

Now in all our human activities thinking is involved. The thinking may be good or bad, clear or confused, narrow and one-sided or broad and comprehensive ; but we can do nothing without thinking of some sort. We are often unaware of our thinking ; we imagine that we are merely observing or listening or acting without any thought ; and we hardly ever make our thinking an object of study. But

in all our action thinking is present, though it varies greatly in degree and in quality; and the scientific study of thinking is as important as any other branch of science. The main difference between science and common knowledge or opinion is that scientific thinking is deeper, more comprehensive and more accurate than that of ordinary knowledge. All science is an expression of thinking, and it is an expression of thinking at its best, thinking of the highest type. But the scientist, though he thinks well, does not necessarily think about thinking itself. He takes it for granted; he uses it without examining it or inquiring into its nature and laws. Thus he speaks of methods of observation and experiment, induction and deduction, analysis and synthesis, hypothesis, theory, fact and law, proof, demonstration and probability, cause and effect, axioms and postulates, description and explanation, definition and classification, and many other forms and methods of thinking, which he does not scientifically investigate. These are his tools or instruments, and he does not necessarily inquire into their nature, any more than the gardener studies the chemistry of his spade or the dynamics of digging. But each of these tools of science is itself a nest of problems, which it is the business of logic, as the science of thought, to investigate. In this sense logic may be described by the old phrase, *scientia scientiarum*, the science of the sciences, since all the other sciences are part of its subject-matter, in so far as they are expressions of clear and accurate thinking.[1] Logic, of course, investigates thought in *all* the forms in which it is expressed; but it is specially concerned with the sciences, because in them thought has its clearest and most adequate expression.

[1] This must be carefully distinguished from the erroneous view of Jevons (*Elementary Lessons in Logic*. p. 6) that the termination 'logy' in geology, biology, etc., means 'logic,' and that therefore 'each science is a special logic,' *e.g.* 'logic applied to explain the formation of the earth's crust,' etc. The termination 'logy' is really 'logia,' not 'logic,' and it means reasoned discourse or scientific statement about a subject. The special sciences are not 'logics'; they are objects of logical study, just as the earth's crust is an object of geological study.

Thought always has an object ; that is to say, all thinking is thinking about something. There is no such thing as mere thinking *in vacuo*. Thought cannot be separated from its object, although the two may be distinguished from one another. In physics we think about matter and energy, in biology we think about life, in psychology we think about mental processes. While our thinking is essentially the same in each case, it nevertheless varies in each, according to the nature of its subject-matter. This will be evident when we consider that the methods of physics differ from those of biology, and the methods of both are different from those of psychology. These methods are simply ways of thinking. There is thus a very close relation between thought and its object. Now in logic thinking is its own object, for logic, as the science of thought, is thinking about thought. On this account some logicians have held that logic investigates 'the form of thought to the exclusion of the matter.' "A form," says Jevons [1], "is something which may remain uniform and unaltered, while the matter thrown into that form may be varied. Medals struck from the same dies have exactly the same form, but they may be of various matter, as bronze, copper, gold or silver. A building of exactly the same form might be constructed either of stone or bricks," etc. Similarly the form of thought is the way in which we think of things, the matter of thought is the various particular objects we think of. And it is contended that logic deals solely with the form of thought. Thus Sir W. Hamilton [2] argues : "If it be maintained that Logic takes not merely the form but the matter of thought into account (the matter, you will recollect, is a collective expression for the several objects about which thought is conversant), in that case Logic must either consider all those objects without distinction or make a selection of some alone. Now the former of these alternatives is impossible ; for if it were required that Logic should comprise a full discussion of all cogitable objects, in other words, if Logic must draw within its sphere all other sciences, and

[1] *Op. cit.* pp. 4-5. [2] *Lectures*, vol. iii. p. 16.

thus constitute itself in fact the one universal science, every one at once perceives the absurdity of the requisition and the impossibility of its fulfilment. But is the second alternative more reasonable? Can it be proposed to Logic to take cognisance of certain objects of thought to the exclusion of others? On this supposition, it must be shown why Logic should consider this particular object and not also that; but as none but an arbitrary answer, that is no answer at all, can be given to this interrogation, the absurdity of this alternative is no less manifest than that of the other. The particular objects, or the matter of thought, being thus excluded, the form of human thought alone remains as the object-matter of our science."

Now, while the object of logical study is undoubtedly the form of thought, it does not follow that logic takes no account of the matter of thought. The form is inseparable from the matter, although we may distinguish between them. A medal may be the same in form, although the material of which it is made may vary. But it can never be pure form; it must always be made of *some* material. And we can study the form of the medal, not apart from the material, but as it appears in this or that or any material. The form and the matter mutually affect one another. You may get a better impression of the die in one material than in another, and the die itself has material as well as form. 'Form without matter' is quite a different thing from 'form in any matter which is capable of that form.' There is no such thing as 'form without matter,' except in *Alice in Wonderland*, where the grin remained after the face of the Cheshire cat had vanished. Not merely logic, but all the other sciences, investigate the form of their phenomena, that which remains the same in a great variety of instances. But they study the form *in* the phenomena, and not *apart from* the phenomena. Physics, for instance, studies the forms, the universal laws or principles, of such phenomena as electricity, light and gravity, which appear in countless instances. But it would be manifestly absurd to say that it studies these forms or principles without taking

into account the phenomena, the matter, the instances in which the laws appear. And it would be equally absurd to say that physics, because it takes into account the matter, the phenomena, as well as the form, must either study *all* the phenomena, *e.g.* every thunderstorm, every flash of light, every fall of an object towards the ground, or must choose its phenomena arbitrarily. In this respect, logic is in the same position as any other science. Just as physics studies the form of electricity as it appears in various instances, so logic studies the form of thought as it appears in various instances. Neither science studies the form alone, neither is compelled to study every instance, and neither chooses its instances in a purely arbitrary way. On the other hand, just as physics is interested in particular phenomena not merely in themselves but for the sake of the laws or principles which they exhibit, so logic is concerned with the matter of thought, not on account of its intrinsic interest, but solely because of the forms of thinking which appear in various objects of thought. The form of thought is thus the primary interest of logic.

Logic has sometimes been called (*e.g.* by the Port-Royal logicians) 'the art of thinking.' It does not, however, teach us how to think, nor is it (as the Stoics suggested) an instrument for the discovery of truth. We can think quite well without studying logic, and we discover truth, not by logic, but by observation, experiment and reasoning, which are themselves part of the subject-matter of logic. Just as aesthetics is not the art of painting or sculpture or poetry or music, so logic is not the art of thinking. But though logic is not an art in this sense, it is not without practical use. Much of our thinking is careless and slipshod, and logic endeavours to set forth ideals of thinking, in the light of which we may criticise our reasonings. We sometimes feel convinced that an argument is unsound, without being able to see exactly what its weakness is, and logic may give us considerable aid in locating the fallacy and thus making it possible for us to correct it. By compelling us to

make clear to ourselves the nature of evidence in different spheres of thought, it disciplines our thinking and develops a habit of scrutinising and checking our inferences. But while logic sets up ideals and standards of accurate thinking, it does not enable us to conform to these ideals. We may conform to them without any knowledge of logic, and we may know them without conforming to them.

As logic discovers standards or ideals of thinking, and as it lays down rules to which reasonings must conform if they are to be valid, it is sometimes described as a normative science, in distinction from such positive sciences as physics, physiology, psychology, etc. Positive sciences are sciences of fact ; they endeavour to ascertain the laws in accordance with which events actually happen in the outer world or in the world of mind. Logic, ethics and aesthetics, on the other hand, are often called normative sciences, because their business is to discover certain norms or standards to which our thoughts, actions and feelings ought to conform and by means of which these thoughts, actions and feelings may be tested and pronounced to be correct or incorrect, right or wrong. The laws of a positive science, being universal statements of what actually happens in certain circumstances or under certain conditions, are laws which cannot be broken ; while the laws of a normative science, resembling the laws of a country or of a game, can be broken. A positive science tells us what is ; a normative science tells us what ought to be. This is a useful distinction, but it may be misleading if it is pressed too far. For it suggests that there is something arbitrary about the laws of logic. But the standards which logic discovers are not mere rules ; they are ideals which express the true nature of thinking, just as mathematical points, lines, surfaces and solids (which are ideal and never exist as perceived facts) express the true nature of spatial relations. The science of logic is essentially an inquiry into the nature of thought, and its rules or laws depend entirely upon this.

As thought cannot be studied without taking account of the objects of thought, and as thought and its objects

constitute the whole of knowledge, including everything that we can know, logic in its higher developments passes into theory of knowledge and metaphysics, and every exposition of logic is based on certain metaphysical presuppositions. This should be frankly recognised at the outset, as it explains the differences in the treatment of elementary logic by logicians of various schools. But, in an elementary treatise, it is unnecessary to discuss to any great extent the metaphysical implications of logic.

EXERCISE I

1. Explain and illustrate the statement that language may be used to express more than pure thought. Distinguish carefully between the thought element and the other elements expressed in your illustrations.

2. (*a*) How does science differ from ordinary knowledge? (*b*) What is meant by calling logic the *science of thought*?

3. What do you understand by (*a*) the form, and (*b*) the matter of thought? In what sense, if any, is logic concerned with the matter of thought?

4. "A man who could not think without training could never be trained to think" If this is so, what is the value of logic?

5. Explain and exemplify the distinction between positive and normative sciences. Is there any sense in which logic can be regarded as a positive science?

6. In what sense are all sciences *formal*? Is there any important sense in which logic is more formal than other sciences?

CHAPTER II

THE ELEMENTS OF REASONING

We have seen that logic is the science of reasoned discourse, or of discourse in so far as it expresses thought, and we must now endeavour to get a general idea of the nature of thought and its elements. If we look at any scientific book (*e.g.* a book on geometry) we shall find that it consists of a series of statements, not disconnected, like a book of sayings or aphorisms, but linked together by such words as 'if,' 'because,' 'therefore,' 'consequently,' or such phrases as 'it follows,' 'it is evident,' 'it may be inferred,' etc. In books on geometry the words 'because' and 'therefore' occur so frequently that they are usually indicated by signs (∵ and ∴). The connection which is thus expressed is what we mean by a 'thought' connection, as distinguished from the other connections we recognise in discourse (*e.g.* the poetical, the oratorical, etc.). All statements which are connected in this way are reasonings, and we may therefore say that, in this sense, reasoning is the subject-matter of logic. Accordingly we must try to get some general idea of what reasoning is and of the elements of which it consists.

A reasoned or inferred statement is one which is made not as a result of direct observation, but on the ground of some other statement or statements. It states not that the fact is so, but that it must be so if the given statements are true. Every reasoning then contains these three elements: (1) a given element consisting of one or more statements, ideas or facts, called the grounds, reasons or premises; (2) another statement, idea, or fact which depends on the

former or follows from them—called the consequent or conclusion ; and (3) the connection in virtue of which (2) follows from (1). But we have seen that in discourse statements are connected in various ways, and we must therefore inquire : what exactly is the nature of the connection which is an element in reasoning, the connection between ground and consequent, or premise and conclusion ?

Let us take a simple instance. When we say that 'Heat, being motion, can be converted into mechanical force,' we are stating a reasoning. Its explicit ground or premise is the statement that 'Heat is motion,' and its consequent or conclusion is the statement that 'Heat can be converted into mechanical force.' But it also implies the further premise that 'Motion can be converted into mechanical force.' Now what is the connecting link which constitutes the reasoning and enables us to draw the conclusion from the premises ? It evidently is to be found in the nature of motion. The conclusion follows from the premises, or the reasoning is valid, only on condition that the same thing, motion, has these two characteristics, (1) that heat is a form of it, and(2) that it (motion) can be converted into mechanical force. The connection which makes the reasoning possible is thus the sameness or identity of motion as having these two characteristics. If the motion of which heat is a form were not the same as the motion which can be converted into mechanical force, the conclusion would not follow from the premises and the reasoning would be invalid. It is the identity of motion in the two premises that brings heat and mechanical force together in such a way that we can relate them in the conclusion.

But while the two premises have a common point, while heat and mechanical force have an identical element, they are not the same in all respects. If they were, there would be no reasoning. It is therefore insufficient to say that the connection which constitutes reasoning is an element of sameness or identity. We must say that it is an identity between different things, *i.e.* an identity in difference.

Heat and mechanical force are identical in so far as they are both manifestations of motion, but they are different manifestations of it. The identity of motion in its different manifestations constitutes the connection between them but this does not abolish the difference between them in other respects. We must say, then, that the connection which constitutes reasoning is an identity in certain respects along with a difference in others.

This is true not merely of reasoning but of all our knowledge. To know is to find or make connections between things. This is evident when we consider any of our statements. To take a specially obvious example, if I say, 'The line AB is equal to the line CD,' I am asserting a connection between the two lines : I am indeed declaring that there is an identity between them in respect of length. If again it is said that in a competition of any kind 'John Jones and Thomas Smith arc equal,' the assertion is that these men are identical in merit, but not neccssarily in height or weight or in any other respect. It is certainly implied that they are not identical in all respects, *e.g.* in name or in personality. If they were identical in all respects, we should be able to say only that 'John Jones is John Jones,' which is equivalent to saying nothing. In such statements, then, we assert a connection between two things, which are in some respects different from one another. Again, to take a less obvious instance, if I say, 'That roof is red,' I assert a connection of identity between the roof and redness. They are identical in respect of colour, but not necessarily in other respects. The roof is not mere colour : it differs from the colour red in many other ways. The statement thus expresses an identity in difference. All our knowing has this characteristic : it is a double-edged process, in which we recognise things as being the same in some respects and different in others. It is one and the same process by which we identify or connect things and discriminate or distinguish between them. If all things were the same in all respects, or if they were all different without any sameness (if they had nothing in common) we could never know

them. We could make no statements about them and we could not reason about them. It may be urged that in negative statements, *e.g.* 'Man is not a fly,' we are merely discriminating and not connecting or identifying. But even in such statements we do not mean to say that the two things (*e.g.* man and fly) are different in *all* respects. If the statement has any meaning, it implies that the things spoken of have something, however little, in common. This point, however, we shall have to consider more fully in a later chapter (p. 66).

Now, wherever we find identity in difference, we have what is called a *universal.* Anything that is one and the same in various forms, instances, occasions or relations is a universal, *e.g.* a term that is applicable in the same sense to each of several objects or a term that is applicable to one object at various times or under varying conditions. The term ' motion,' for instance, is a universal, in so far as it is applicable in the same sense to the various kinds of motion, such as heat, light, mechanical motion, etc. All adjectives, *e.g.* red, hard, swift, round, are universals in that they may be applied in the same sense to each of many different objects. Similarly terms which refer to one object only, *e.g.* the sun, the House of Commons, Cleopatra, that roof, are universals, in so far as they are applicable to one and the same thing or person at various times, under various conditions or in various relations. A law of nature is a universal, inasmuch as it applies in the same sense to a great variety of events or phenomena. A medical remedy is a universal, because it applies to all cases of a particular kind of illness. A reasoning is a universal, in so far as it is valid for various people or under various conditions. In each of these and endless other cases, we have something which is the same in different instances, at different times or places, or under different conditions.

Logic, as the science of thought or reasoning, is thus concerned primarily with universals. If there were no universals there would be no reasoning, for, as we have seen, reasoning implies a connection of identity between different

things. But when we consider our actual reasonings, we find the universal element appearing in different ways. In some of our reasonings a general principle or law is applied to an individual case or cases ; in others we try to establish a general principle or law from various individual facts or instances ; and in others we reason from one individual case or fact to another. If I infer that I must die because all men are mortal, I am reasoning from a general law to an individual case. If I infer from many astronomical observations that the planets move in elliptical orbits, I am reasoning from various individual facts to a general law. If I infer that the king is at Windsor, because the Royal Standard is flying, I am reasoning from one fact (that the standard is flying) to another fact (that the king is at Windsor). In the first of these reasonings, the universal, the common element, on which the inference depends, is explicit or at least relatively explicit. It is expressly stated in the general law that all men are mortal. In the second the universal is implicit in the ground or premises and is made explicit in the conclusion. The planets move in orbits which are different, but which are found to have something in common. By careful astronomical observation we are enabled to state precisely what they have in common, and we do this in the conclusion, which makes the universal explicit in the general law that all the planets move in elliptical orbits. In the third reasoning the universal is implicit, *i.e.* it is implied but not definitely expressed. Why am I entitled to infer that the king is at Windsor, because the Royal Standard is flying? Only because I know or believe it to be a general rule that when the Royal Standard is flying at any of the Royal palaces, the king is there. That general rule is the universal on which the inference proceeds ; but although it is implied in the reasoning it is not explicitly stated.

Now on the basis of this distinction reasonings have been divided into two main classes, deductive and inductive. A deductive reasoning is one in which we reason from a general principle or law (an explicit universal) to an indi-

vidual case or cases. An inductive reasoning is one in which we reason from various individual facts or instances to a general principle or law (the universal becoming explicit in the conclusion). Reasonings from one fact to another (the universal being explicit neither in the premises nor in the conclusion) are usually classed as inductive, but some of them seem obviously deductive. For sometimes the universal is not stated only because it is considered too familiar to need statement. When I infer that I must die because I am a man, I am reasoning from one fact to another ; but my reasoning differs only in verbal form from that in which I infer that I must die because all men are mortal ; and the latter is deductive. In other reasonings from one fact to another we do not grasp the implicit universal sufficiently clearly to be able to express it ; as I may reason that it will rain to-day because the appearance of the sky is like what it was on another day when it did rain. Such reasonings are not deductive. The fact of the matter is that reasonings from one fact to another do not naturally fall into either division of the classification into deductive and inductive ; and as we shall see later (p. 234) there are other reasonings in the same position. For the present we state the distinction in the traditional way, leaving its adequacy to be considered later.

We shall follow the usual course and begin with the study of the reasonings termed deductive. Every deductive reasoning expressed in language consists of statements, one of which is the conclusion of the reasoning, while the others are the premises on which the conclusion depends. These statements are called propositions, and we may at present provisionally describe a proposition as a premise or conclusion of a reasoning. For instance, in the reasoning :

All men are fallible,
All kings are men,
∴ All kings are fallible,

there are evidently three propositions, the conclusion and two premises. Now in every proposition something is said

about something else, either affirmatively or negatively, or (in the technical language of logic) something is predicated of something else, either affirmatively or negatively. In the proposition 'All men are fallible,' 'fallible' is predicated affirmatively of 'all men.' In the proposition 'No planets are self-luminous,' 'self-luminous' is predicated negatively of 'all planets.' That of which something else is predicated in the proposition is called the subject of the proposition, while that which is predicated is called the predicate. Thus in the first of these examples, 'all men' is the subject and 'fallible' is the predicate of the proposition; in the second example 'all planets' is the subject and 'self-luminous' is the predicate.[1] The subject and predicate of a proposition are called its terms, and in general a term in logic may be defined as a subject or predicate of a proposition.

For many centuries the doctrines of Deductive Logic have been expounded under three main heads: (1) the doctrine of Terms, (2) the doctrine of Propositions, and (3) the doctrine of Reasonings. But many unnecessary difficulties and misunderstandings have been introduced into the discussion of logical questions by treating terms as if they were independent of propositions, and propositions as if they were independent of reasonings. Thus it has been said that, corresponding to terms, propositions and reasonings, there must be three different kinds of thought or operations of the mind. Terms (or the ideas which terms express) are supposed to be given to us by simple apprehension, propositions are supposed to result from a different action of the mind, called judgment, in which two terms are compared so as to ascertain whether they agree or differ; and reasoning is supposed to be a third act of the mind, in which we pass from one or more given propositions to a proposition different from those given. This has led people to imagine that terms are in some way independent of propositions, and that propositions are independent of reasonings,

[1] The logical subject and predicate are evidently different from the grammatical subject and predicate, *i.e.* the subject and predicate of a sentence. The distinction between them is explained in ch. v. p. 51 ff.

or that we can first get terms, then go on to combine them into propositions, and finally combine propositions so as to produce reasonings. But all this is quite artificial and misleading: it is not an accurate account of our actual thinking. We never think in separate terms or ideas. All thinking involves connection and discrimination, and therefore we always think in judgments, statements or propositions. And our judgments or propositions are never separate and isolated, any more than our terms or ideas. All our judgments or propositions have grounds or reasons of some sort, whether these grounds be good or bad, explicit or implicit. In other words our judgments or propositions have always some rational connection with other judgments or propositions. Our actual thought is always reasoning of some kind, and accordingly propositions by themselves are parts of reasonings (premises or conclusions), considered apart from their context, and similarly terms by themselves are parts of propositions (subjects or predicates), considered separately for convenience of study. We may study separately the root or the flower of a plant; but the root and the flower presuppose the plant, and we cannot make the plant by sticking together the flower, the root, the stalk, the leaves, etc. In the same way we may study terms and propositions separately, but terms presuppose propositions and propositions presuppose reasonings, and we do not make reasonings by combining propositions nor propositions by combining terms. Accordingly when we study the logical problems connected with terms, we must keep in view the fact that they are integral parts of propositions, and we must in the same way study propositions as parts of reasonings.

EXERCISE II

1. Explain and exemplify the nature of universals and indicate the part which they play in reasoning.

2. Discuss the relation between terms, propositions, and reasonings. Which of them do you consider the unit of concrete thinking. and why?

3. Explain and illustrate the following: premise, universal, conclusion, proposition, subject, predicate.

4. In the following reasonings state *(a)* which are inductive and which deductive, *(b)* which statements are premises and which conclusions, *(c)* point out the universals or connections on which the reasonings depend :

(1) The removal of the thyroid gland dulls the intelligence, for this happened in the case of Mr. Jones, Mr Robinson, etc.

(2) The train is coming, for the signal is down.

(3) Socrates must have been a happy man, for all wise men are so.

5. " Every proposition is an implicit reasoning." Explain, illustrate and discuss.

CHAPTER III

TERMS

As we have seen, a term is the subject or predicate of a proposition: it stands for what is spoken about in a proposition (the subject) or for what is predicated of it (the predicate). Terms must be carefully distinguished from words. Words are oral or written signs by means of which we communicate with one another. In them we express not merely thoughts and reasonings, but feelings or emotions, questions, wishes, commands, etc. By means of terms, on the other hand, we do not express feelings, questions, wishes, etc. but only information, thought or reasoning. Terms are also distinct from words in two other respects. (1) A term may consist of one word or of an indefinite number of words, *e.g.* the proposition 'The man I saw in Glasgow to-day is the man I saw in London yesterday' contains two terms, each consisting of seven words. (2) There are some words which cannot stand by themselves as subject or predicate of a proposition, *e.g.* such words as verbs, adverbs, prepositions and conjunctions, *e.g.* 'is,' 'but,' 'how,' 'loudly,' etc., and such combinations of words as 'is quite clearly,' 'hardly to be,' etc. Words which can stand by themselves as terms have been called categorematic words, while those which cannot so stand have been called syncategorematic.

Some logicians regard names as equivalent to terms; but it is necessary to distinguish between a name and a term as well as between a word and a term. A name is a special kind of word or combination of words. Those who identify

names with terms (*e.g.* Jevons and Mill) usually quote with approval Hobbes's definition of a name : "A name is a word taken at pleasure to serve for a mark, which may raise in our mind a thought like to some thought which we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had before in his mind." This definition suggests that names are words which can be used by themselves, apart from their position as subject or predicate in a proposition. In so far as names can be used in this way, they are not identical with logical terms. For example, a dictionary consists of a list of names to which meanings are assigned. But these names, as they appear in the dictionary, are not terms. They are words or signs, having a certain significance or meaning, and the purpose of the dictionary is to inform us what meaning or meanings these signs have at present or have had in the past. Many names in the dictionary have several meanings assigned to them, and these various meanings have often no apparent connection with each other, *e.g.* such names as pound, play, bull, scale, etc. A pound, for instance, may mean a pound in money, or a pound in weight, or a place in which stray cattle are put. Play may mean the gambolling of young animals, or a drama, or an individual's skill in a game, or a play on words, or play of fancy, and so on. These meanings are indications of the various ways in which the names have been used or may be used as terms. And evidently the same *name* may stand for several different *terms.* When we consult a dictionary for the meaning of a name, we are not necessarily seeking for all its possible meanings. We read the name in a statement or proposition, and we try to find its meaning as used in that statement or proposition, *i.e.* its meaning as a term (or part of a term) in that proposition. If we find various meanings assigned to the name in the dictionary, we choose the meaning which best fits the context, *i.e.* the meaning which makes the proposition (and the whole context) intelligible. A name, considered by itself, is accordingly a name unused ; but a term is always a name or names used as subject or predicate

of a proposition, and as such it can have (or at least ought to have) only one meaning. So far as a name has more than one meaning, it stands for more than one term. We must, therefore, be careful to avoid identifying names with terms.

In opposition to the view that terms are mere names or words, some logicians prefer to speak of terms as notions or concepts. In doing so they rightly insist on the meaning of the name as logically important rather than the name itself, the verbal expression of the meaning. The concept or notion of anything is the thing as an object of thought. A concept is a universal, and it is generally regarded as the idea of certain common qualities, characteristics or elements which thought finds in a variety of things or kinds of things. For example, the concept 'animal' is not something which we perceive by our senses or picture in our imagination, but an idea of the common characteristics which belong to all the various things we call animals. Concepts, of course, do not refer merely to physical things. We can have a concept of imagination or of reasoning or of concepts themselves in their various kinds, the concept in each case being the idea of the common elements in the various instances. All terms are universals in this sense: each of them has a meaning which is the same in different thoughts, reasonings or propositions. But it is generally held that there are no concepts of individual things. There is, for instance, a concept of *a* blade of grass, because it is possible for us to recognise certain elements which all the various blades of grass have in common and which differentiate blades of grass from other things. But there is no concept of *this* blade of grass, because we can never discover all the elements or characteristics which make it *this* blade of grass and which differentiate it from every other blade of grass. We can never completely know an individual, because the characteristics of an individual are infinite; but it is possible to understand a concept completely, although our actual knowledge of many concepts may be very imperfect. But while there is no concept of an individual, terms which apply to individuals

are frequently used in propositions and reasonings, *e.g.* 'This blade of grass is broken'; 'James Smith is the only son of John Smith.' We cannot therefore use the expression 'concept' as equivalent to 'term,' for evidently some terms are not concepts. If we could extend the meaning of 'concept' so as to make it stand for any term regarded as a universal, there would not be so much objection to identifying terms and concepts. But such an extension of the meaning of 'concept' is not usual, and it might result in ambiguity and confusion.

When we speak of concepts, we are rightly insisting on the meaning of a term, *i.e.* its thought-aspect, as more important than the verbal expression, its word-aspect. But, on the other hand, we are apt to suggest that the term is nothing more than a thought or idea, and also that it is independent of a proposition, *e.g.* that a proposition is made by putting together two thoughts or ideas, two concepts. Such a suggestion as this is quite erroneous, and it is therefore better to use the expression 'term.' But we must remember that a term is not *merely* a name or a word or combination of words, nor *merely* a thought or idea. A term is the subject or predicate of a proposition, and it is an object of thought or part of an object of thought (either a physical or a mental object) expressed in language, which may be written, spoken, or gesture language (*e.g.* the dumb alphabet) or language represented mentally in imagination.

Logicians have distinguished terms into various kinds. Among the most important of these distinctions is that between general, collective and singular terms. A *general* term (sometimes called a *common* term) is a term which is used as applying equally and in the same sense to each of a number of things, having certain qualities in common. For example, in the propositions 'All birds are feathered'; 'Some men are wise'; 'No Athenians were Helots,' each of the terms is a general term. The term 'all birds' applies to each of the various kinds of birds and to each individual bird, because each must have the common qualities which

constitute the meaning of 'bird'; the term 'some men' applies to each of an indefinite number of men, having the common qualities which constitute the meaning of 'man,' and the same thing is true of the other terms 'feathered,' 'wise,' etc. When a term is used in this way, it is said to be used distributively, as opposed to collectively. A term is *collective*, when it is used as referring to a sum or group of individuals, and is applied not to each of the individuals by itself but only to the sum or group, the totality which they form: it is a term used as referring to a number of things taken as one undivided whole. In the proposition 'The British Army is a small army,' the subject is a collective term, because it clearly applies to the army as a whole and not to each of the individual soldiers, regiments, battalions, etc. The predicate, on the other hand, is a general term, because it applies to each of an indefinite number of armies, which have the common characteristic of being small. A general term can always be predicated affirmatively of each of the individuals to which it is applicable; but a collective term cannot be so predicated of each of the individuals to which, as a group, the term refers. If 'nation' is used as a general term, we can say that 'England is a nation,' 'France is a nation,' etc., England, France, etc. being the individuals to which the term is applicable. But if 'nation' is used as a collective term, meaning the various citizens considered as one whole or group, we cannot say that any individual citizen is a nation.

The last example suggests that the same *name* may not merely stand for different terms (as 'capital' may stand either for money and goods or for the chief town of a country), but may also be used as a term in different ways, distributively or collectively. We cannot, in fact, tell whether a term is general or collective until we know the proposition in which it occurs. In the proposition 'The crew were successful in saving all the passengers,' the subject is collective, for the proposition means not that each member of the crew saved all the passengers, but that the crew as a whole did so. On the other hand, in the proposi-

tion 'The crew were all foreigners,' the subject is distributive, because it is evidently meant that each member of the crew was a foreigner. Accordingly in every case in which it is necessary to distinguish between a general and a collective term, we must consider not merely the meaning of the name but the context in which it occurs as a term. This distinction between the distributive and the collective use of a term is of great importance in the testing of reasonings, as fallacies frequently arise from ignoring it. For instance, a term may be used distributively in one premise of a reasoning and collectively in the other, and on account of this a fallacious conclusion may be drawn. The reasoning appears to be valid, because we seem to be using exactly the same term in both premises ; but in reality it is invalid, because of the different ways in which the term is used. *E.g.* 'The people of the country are suffering from famine ; you are one of the people of the country ; therefore you are suffering from famine.' The term, 'the people of the country' occurs in both premises ; but in the first it is used collectively and in the second distributively and hence the conclusion does not follow.

As a general term is applicable to each of a number of things and a collective term is applicable to a number of things, not severally, but as one whole or group, a *singular* term is a term used as applying to one thing only, *e.g.* this house, the North Pole, Julius Caesar, the highest cornerstone in the east gable of the tower. A distinction is sometimes drawn between singular terms and proper names ; but there is considerable difference of opinion as to whether this distinction is valid. It is significant that we never speak of proper *terms*, but only of proper *names*, and this seems to suggest that the lines of distinction should be (*a*) between singular *terms* and general *terms*, and (*b*) between proper *names* and common *names*, rather than (*c*) between singular terms and proper names. This is confirmed by the fact that those who insist on the distinction between singular terms and proper names generally regard proper names as mere marks, used to point out individual

things or persons and having no meaning such as could be expressed in a dictionary, while common names are descriptive and have a dictionary meaning, in virtue of which they are applicable to one or more individuals. The proper names Baker or Merry do not mean that the bearers of these names are bakers or merry people or that they have anything in common; the proper name Caesar may belong to a man or a dog. But common names do imply that the things to which they are applicable have certain common qualities, expressed in the meanings of the names. So far it is possible to draw a valid distinction; but this is a distinction between names, not a distinction between terms. If a proper name is used as a term in an intelligible proposition, it always has a meaning. Kinchinjunga, as a proper name, taken by itself, might mean anything, a person or a place. But as a term in the proposition 'Kinchinjunga is difficult to climb,' it has a definite meaning. Its meaning, of course, may not be fully known to the person to whom the proposition is addressed, and I may therefore prefer to say 'The second highest mountain in the Himalayas is difficult to climb.' But there is no logical difference between the two terms, any more than there is a logical difference between a term expressed in English and the same term expressed in French. A proper name, when used in a proposition as applying only to one thing or person, should thus be regarded as a singular term.

The distinctions between the singular, the collective and the general term may be illustrated by the following propositions: (1) 'This thing is heavy'; (2) 'This group or collection of things is heavy'; (3) 'All these things are heavy'; (4) 'All things of a certain kind are heavy.' The subject of (1) is singular, the subject of (2) is collective, the subjects of (3) and (4) are general. But the subject of (2) may be regarded as singular, as well as collective, inasmuch as it refers to the group as one whole and not severally to the individuals which make up the group. And, as we shall afterwards see, it is necessary for certain purposes in reasoning to regard such terms as singular. We can also

discern a certain transition of character in these propositions as we pass from (1) to (4). In the subject of (1) the emphasis is on the unity, the oneness, of the individual thing. In the subject of (2) the unity is a unity of different things; but the emphasis is still on the unity. It is not the different things that are heavy, but the things taken together as one. In the subject of (3) we are still speaking of a definite number of things—all *these* things—and there is therefore still the suggestion of a group or collection; but the emphasis is no longer on their unity as a collection, but on some quality which is common to them all. *Each* of them is heavy, and though they may be different in other respects, they have a certain unity of character, which is more important than their unity as members of a group. It is in virtue of this unity of character in the things referred to that the subject of (3) is a general term. In the subject of (4) the unity of the things as members of a definite group entirely disappears, and emphasis is laid solely on their unity of character. The things are not a definite number of things, but all things of a certain kind, *i.e.* all things having certain common characteristics. The 'singular' and the 'collective' aspects have entirely disappeared, and the term is general in the fullest sense. Terms like the subject of (3) which refer to a definite number of things, of each of which something is predicated, are sometimes distinguished as enumerative general terms. For example, in the proposition 'Ceres, Pallas, Juno and Vesta are minor planets,' the subject is an enumerative general term. This distinction, however, will be further considered when we deal with propositions (p. 93).

Another distinction between terms, which has been recognised by logicians, is that of *abstract* and *concrete* terms. J. S. Mill describes the distinction in this way: "A concrete name[1] is a name which stands for a thing; an abstract name is a name which stands for an attribute of a thing." He gives the following examples: "*John, the sea, this table,*

[1] As pointed out above, Mill uses 'name' as identical with 'term.'

are names of things. *White,* also, is the name of a thing, or rather of things. *Whiteness,* again, is the name of a quality or attribute of these things. *Man* is a name of many things; *humanity* is a name of an attribute of these things. *Old* is a name of things; *old age* is a name of one of their attributes."[1] In these instances the names described as concrete are not merely names of things. They also imply attributes or qualities, in so far as they have a meaning. *Man, white, old,* etc. are names of things having certain attributes. The abstract names, on the other hand, are evidently names of *attributes* (or groups of attributes) *considered by themselves* and apart from any reference to the things which have these attributes. *White, e.g.* applies to things having the quality of whiteness; but *whiteness* stands for the quality itself, without reference to white things. Adjectives, which appear to stand for attributes or qualities, *e.g.* white, old, clever, punctual, are thus regarded by Mill as concrete names, because of their reference to things; while the corresponding abstract names, whiteness, old age, cleverness, punctuality, stand for the attributes or qualities considered by themselves.

Superficially this distinction between concrete and abstract names seems fairly clear; but it is exceedingly difficult to apply it in particular instances. We cannot readily determine what is to be called a 'thing.' Some logicians (*e.g.* Jevons) suggest, though they do not definitely say, that a 'thing' is to be regarded as a physical thing. In that case 'mind' would be an abstract name, and yet we have the abstract name 'mentality,' which suggests that 'mind' is concrete. If, on the other hand, things include mental as well as physical things, it would seem to be necessary to define a thing as a substance in the widest sense, *i.e.* as anything to which attributes belong. This view is adopted by several logicians, including Dr. Keynes, who states the distinction thus: "A concrete name is the name of anything which is regarded as possessing attributes, *i.e. as a subject of attributes*; an abstract name is the name of

[1] Mill, *System of Logic,* bk. i. ch. ii., sect. 4.

anything which is regarded as an attribute of something else, *i.e. as an attribute of subjects.*"[1] But the distinction, interpreted in this way, ceases to be a distinction between names and becomes a distinction between terms. Many names which are apparently abstract are names of things which can have attributes assigned to them, *e.g.* ambition, reverence, consciousness, solemnity. Such names as these can be used as the subjects of propositions in which attributes are predicated of them, and when so used as terms, these names must apparently be concrete. Indeed the subject of any proposition can be regarded as a subject of attributes, and accordingly, if an abstract *name* is used as an abstract and not a concrete *term*, it must be used not as a subject, but as a predicate. But abstract names are hardly ever, if at all, used by themselves as predicates, unless we include among abstract names the names of adjectives. Accordingly, Dr. Keynes points out that, "as logicians we have very little to do with the abstract use of names."[2] In every proposition that is not merely verbal the subject and the predicate are always concrete.

The distinction is thus a distinction between names, rather than a distinction between terms, and it is therefore of slight logical importance. As a distinction between names it arises owing to an inevitable tendency, in the progress of knowledge, to give independent recognition to qualities or attributes. We recognise qualities in things and we name them at first by means of adjectives or participles of verbs, which we use always as predicates. But presently we find it convenient to speak about the qualities we have recognised, and for that purpose we form, out of the adjectives or participles, nouns, which can be used as the subjects of propositions and can thus be spoken about. These are the names which are called abstract. Every abstract name has a corresponding concrete name, generally an adjective or participle, *e.g.* reverence, reverent; humanity, human; miserliness, misery; consciousness, conscious; tiredness, tired; endurance, endured, etc.

[1] *Formal Logic* (4th ed.), p 16. [2] *Ibid*, p. 19.

Another generally recognised distinction is that between *positive* and *negative* terms. If the distinction is regarded as a distinction between *names*, apart from their use in propositions, it may be said that a positive name is one which implies the presence of a quality or set of qualities, *e.g.* black, man, crafty, while a negative name is one which implies the absence of a quality or qualities, *e.g.* non-elector, immaculate, colourless. The distinction in this form is very difficult to draw, for there are no purely negative names, *i.e.* names which imply merely the absence of qualities. Every actual name has some positive meaning and thus implies however vaguely the presence of some quality, although the main function of the name may be to emphasise the absence of qualities. In any case, a distinction between mere names is not important for reasoning. But the distinction has been applied to terms in this way, that every term necessarily implies the possibility of another term which negates it and which is called its contradictory, *e.g.* *blue* has *not-blue* as its contradictory, *possible* has *impossible*, *man* has *not-man*, *the man who left his umbrella in the train* has *something other than the man who left his umbrella in the train.* In general, the contradictory of a term A is not-A or non-A or other than A. This distinction, as we shall afterwards see (p. 129 ff.) is convenient for certain processes of inference; but it is really a distinction which belongs to propositions and has been artificially transferred to terms. Contradiction is a matter of affirmation or denial, and affirmation or denial is the function of propositions alone. A term does not affirm or deny; it merely suggests something which may be affirmed or denied in a proposition. Consequently terms cannot, strictly speaking, be contradictory of one another. But when we desire to express an affirmative proposition in a negative form or a negative proposition in an affirmative form, it is convenient to use terms of the form not-A, non-A, etc., and these are called negative terms, *e.g.* instead of saying 'All birds are bipeds' it may be convenient to say 'No birds are non-bipeds.'

Aristotle objected to the use of such terms on the ground

that they are indeterminate or boundless, too vague to be intelligible either as subjects of attributes or as predicates of a subject. The name not-A stands for everything in the universe except A ; A and not-A taken together make up the whole universe. The name not-mountain, *e.g.* stands for 'shoes and ships and scaling-wax and cabbages and kings' and everything else in the world except mountains. The various things to which it refers seem to have no common qualities, and it seems therefore impossible to use it in any intelligible sense as a subject or predicate. Aristotle's contention is sound with reference to negatives of the form not-A, when they are regarded as names apart from their actual use in propositions. Two contradictory names, like A and not-A must be absolutely opposed to one another, *i.e.* there can be no middle term between them. Thus 'more' and 'less' are not contradictories, for there is a possible middle term, 'equal': the contradictories should be 'more' and 'not-more,' or 'less' and 'not-less.' "Credulous' and 'not credulous' are contradictories; but 'credulous' and 'incredulous' are not contradictories, because the one implies too great readiness to believe and the other too little readiness to believe, leaving a middle condition possible. If, then, we have two contradictory *names*, the negative name must be regarded as standing for everything (actual, possible or conceivable) to which the positive name does not apply. But this is not necessarily the case when the names are used as terms in a proposition or reasoning. The context generally limits very considerably the field of application of the negative term. The *name* 'non-electors,' *e.g.* may by itself stand for beetles and tadpoles, cameras and carpet-bags, suns and systems of metaphysics; but if we use it as a *term* and say 'Non-electors are invited to the meeting' or 'Non-electors are not likely to be canvassed,' we limit enormously the range of its application, because we clearly do not mean anything so absurd as that 'Beetles, etc. are invited to the meeting.' Similarly pairs of names which in themselves are not contradictory (*i.e.* not of the form A and not-A) may, when used

as terms in a particular context, be really contradictory, *i.e.* they may, in that context, be entirely exclusive of one another, with no middle term between them. For instance, the names 'moral' and 'immoral' are not contradictory as names, for there are many things, *e.g.* trees, triangles, electricity, etc. which are neither moral nor immoral but non-moral. But if in a proposition or reasoning we are speaking only of *persons* as moral or immoral, these may actually be contradictory terms, because all *persons* must be either moral or immoral. The context (or, as it is sometimes said, the universe of discourse) in this case may limit the application of the term 'non-moral' so that it becomes indistinguishable from 'immoral.' In the same way two names, neither of which appears to be negative in form, *e.g.* 'British subject' and 'foreigner,' may in certain contexts be contradictory terms. The *name* 'foreigner' is not the contradictory of the name 'British subject,' because to an American or a Frenchman a British subject is a foreigner. But in a particular proposition or reasoning the name 'foreigner' is frequently used as a term in such a way that it clearly means the opposite of 'British subject,' and in that case the two terms are contradictory. In general, if we wish to determine whether any two terms are contradictory of one another, we must ask whether, in the context in which they occur, they are used in a sense in which they are entirely exclusive of one another and exhaustive of a whole class. If they are so used they are contradictory terms.

Some logicians also recognise *privative* names, as distinct from positive and negative names. J. S. Mill[1] defines a privative name as "the name of something which has once had a particular attribute, or for some other reason might have been expected to have it, but which has it not," *e.g.* blind (unable to see), deaf (unable to hear), dead (having ceased to live). The distinction is not of much logical importance, and if we follow Mill's definition, there is no sure ground on which we can distinguish between a privative and a negative term. Expectations about the attributes

[1] *Op. cit.*, bk. i. ch. ii., sect. 6.

of things vary endlessly in different people and in the same person at different times, and therefore it seems hardly possible to discriminate between a negative term, as implying the absence of an attribute, and a privative term, as implying the absence of an attribute which 'might have been expected' to be present. Jevons [1] says that "we apply a privative term to anything which has not a quality which it was capable of having; we apply a negative term to anything which has not and could not have the quality." But we never actually deny any quality of a thing, unless there was some possibility of that quality belonging to the thing. If we try to invent a proposition which denies to its subject a quality which could not possibly belong to that subject, we shall be forced to make up absurdities like 'No steam-engine is acquainted with the differential calculus.'

[1] *Op. cit.*, p. 25.

EXERCISE III

1. Distinguish carefully between the following, giving examples of each: (*a*) words, (*b*) names, (*c*) terms, (*d*) concepts.

2. Explain the difference between the distributive and the collective use of a name. State whether the subject of each of the following propositions is used distributively or collectively.
   (1) The students surrounded the building.
   (2) The students filled up class enrolment forms.
   (3) Two wrongs don't make a right.
   (4) All the fish were caught in Loch Lomond.
   (5) All the fish weighed twenty pounds.
   (6) The American people have a prejudice against negroes.
   (7) All the angles of a triangle are less than two right angles.
   (8) All these claims upon my time overpower me.

3. Distinguish between (*a*) general, (*b*) collective, and (*c*) singular terms. Point out to which of these classes the subjects of the following propositions belong:
   (1) The President of the United States is a Republican.
   (2) The President of the United States is elected by the people.
   (3) Man was made to mourn.
   (4) Food is necessary to life.
   (5) All the passengers carried lifebelts.
   (6) All the passengers could not be accommodated in one boat.
   (7) One and three are odd numbers.
   (8) One and three are four.

(9) The men entered the room.
(10) The men carried all the furniture out of the room.
(11) All the angles of a triangle are equal to two right angles.
(12) A horse is a four-footed animal.
(13) The students in the University number over 1000.
(14) A horse has just entered the garden.

4. Discuss the nature and consider the logical value of the distinction between concrete and abstract terms.

5. " Strictly speaking there are no negative terms, for affirmation and denial belong to propositions." Consider this statement.

6. State (giving reasons) whether each of the following is (*a*) abstract or concrete, (*b*) singular or general, (*c*) positive or negative : green, deafness, ammoniated quinine, the number 4, imperfect vision, principality, the Prince of Wales, colourless glass, to-morrow morning.

CHAPTER IV

THE EXTENSION AND INTENSION OF TERMS

EVERY term, according to its definition as the subject or predicate of a proposition, may be regarded as the name of some thing or things—a thing about which something is said (subject) or that which is said about it (predicate). But every thing or class of things has some qualities or attributes, and every quality or attribute belongs to some thing or class of things. Accordingly every term may be regarded in two ways: (1) as indicating or referring to a certain object or objects, and (2) as indicating or referring to a certain quality or qualities which the objects possess. The things or objects to which the term refers are called its extension or denotation, while the qualities or attributes are called its intension or connotation. Every term denotes objects and connotes qualities, and the extension or denotation of a term may thus be defined as its reference to objects while its intension or connotation is its reference to qualities or attributes. For instance, in the proposition 'All lions are carnivorous,' the term 'lions' has extension or denotation in so far as it refers to each individual lion or to the different varieties of lions, *e.g.* the African and the Asiatic, and it has intension or connotation in so far as it refers to the qualities or attributes of lions, *e.g.* vertebrate, quadruped, mammal, feline, animal, etc. Similarly the extension or denotation of the term 'carnivorous' is its reference to the various things (*e.g.* the kinds of animals) which have this quality, while its intension or connotation is its reference to the quality itself. In this instance, the subject of the

proposition (' lions ') seems to be primarily denotative, and the predicate (' carnivorous ') seems to be primarily connotative. But every term has both aspects, although in particular instances one aspect may seem more obvious than the other.

Regarding this distinction, however, there has been much difference of opinion among logicians. For instance, the question obviously arises : Must the connotation or intension of a term include *all* the attributes belonging to the thing or things which the term denotes ? If so, a considerable part of the connotation will, in the case of many terms, be unknown to us. We do not know all the qualities of electricity or of radium or of bacteria or of fungi. And, on the other hand, if we are to include in the connotation of a term only the qualities which are known to us, we must ask—' known to whom ' ? People's knowledge varies, and a thing will suggest one set of attributes to one man, and another set of attributes to another. The term ' gas ' suggests one set of attributes to the chemist or physicist and an entirely different set to the ordinary householder or the plumber. The term ' metal ' may to the chemist mean ' an element which can replace hydrogen in an acid and thus form a salt ' ; but to the majority of men it does not suggest anything like that. To meet such difficulties as this Dr. Keynes[1] has pointed out that ' there are at least three different points of view from which the qualities ' of a class of things ' may be regarded.'

(1) The qualities to which the term refers may be ' those which are essential to the class in the sense that the name implies them in its definition ' ; *i.e.* the qualities without which anything would not be a member of the class to which the term refers. The term ' dog ' for instance, may apply to a great many animals which vary very much in their qualities, *e.g.* in their colour, their size, the roughness or smoothness of their coats, etc. But on this view the connotation or intension of the term ' dog ' does not include these varieties of qualities : it includes only the qualities without

[1] *Op. cit.*, pp. 23-24.

which an animal cannot be described as a dog. This interpretation of intension or connotation is called by Dr. Keynes the *conventional* view.

(2) The qualities to which the term refers may be those 'which in the mind of any given individual are associated with the name in such a way that they are normally called up in idea when the name is used.' On this view the intension or connotation may not include all the essential qualities and it may include some that are not essential. The intension or connotation will not be in any way fixed, but will vary in different persons and even in the same person at different times. It may even include all the qualities which at any given time are known to belong to the things denoted by the term. It may, *e.g.* consist of the qualities which the most expert chemist finds in 'metal' or those which the ironmonger knows or those which 'the man in the street' associates with the name. From this point of view the intension or connotation is *subjective* or relative.

(3) The term may be regarded as referring to 'the sum-total of qualities actually possessed in common by members of the class,' by all the things to which the term refers. 'These will include all the qualities included under the two preceding heads, and usually many others in addition. The standpoint here taken is *objective*.'

Dr. Keynes suggests that 'intension' should be used as a general name for any of these ways of interpreting the reference of terms to qualities, and he would call the conventional intension the 'connotation' and the objective intension the 'comprehension,' while he would describe the second view as the 'subjective intension.' Nearly all other logicians, however, have used the words 'intension,' 'connotation' and 'comprehension' as practically synonymous, and it is therefore more convenient to describe the three views respectively as the conventional, the subjective, and the objective intension or connotation.

It should, however, be observed that, while these distinctions are useful and interesting, they refer rather to *names* than to *terms*. They are valuable suggestions as to

the main ways in which it is possible to interpret the connotation of a name; but they do not adequately solve the problem as to what we mean by the connotation of a term used in any particular proposition or reasoning. In the great majority of cases the term is used in the sense of its conventional connotation, or something like its conventional connotation. But terms are not always so used in propositions, and no way is suggested by which we may decide whether any given term is so used or not. The difficulty, however, disappears if we observe that logic is really concerned not with the meanings of *names*, but with the connotations of *terms*. All that logic requires is that a term shall, when used as a term, imply certain qualities and be applicable to certain objects. The particular qualities which any term may imply will, in each case, depend on the context in which the term occurs, *e.g.* on the knowledge and intentions of the people who are taking part in the discussion, on the general nature of the book or article in which the term is used and on other similar circumstances. The term 'gas,' as we have seen, will have one connotation in a discussion between a householder and his plumber and another in a discussion between physicists. The term 'metal' will have one connotation in a treatise on chemistry and another in an article on the metal market. Now logic requires merely that in any connected discourse, discussion or reasoning a term shall have some common meaning or connotation, understood and accepted by all who are concerned in the discussion, and that this common meaning shall be maintained throughout the discussion. In most ordinary cases the actual connotation used is the dictionary definition of the word or something like it. But it is not necessarily so. In philosophical and scientific books we often find unusual or provisional definitions of words. The author announces at the outset the connotation he intends to give to a word, which he is going to use repeatedly as a term. He may give the word almost any meaning he likes, being limited only by the consideration that it is convenient to choose a meaning which his readers will probably under-

stand and accept. It should also be noticed that many words (*e.g.* play, bat, pipe, etc.) have several quite distinct dictionary meanings, and in these cases it is self-evident that the connotation depends on the use actually made of the word in some definite proposition or reasoning. All that logic requires is that we shall not, without notice, pass from one connotation to another in the course of the same reasoning.

Mill and some other logicians have held that certain terms are non-connotative, *i.e.* that they have no connotation. A non-connotative term, according to Mill,[1] is one which signifies a subject [2] only or an attribute only. It is purely indicative. A connotative term denotes a subject and implies an attribute or attributes. It is suggestive as well as indicative. It has a meaning other than that of merely marking off one thing or one attribute. Mill thus maintains that there are two kinds of non-connotative terms or names, (1) the proper name, which signifies a subject, without implying attributes, and (2) the abstract term, which signifies an attribute, without implying a subject. We have already considered the question whether a proper name is a mere mark, having a purpose but no meaning, when we discussed the distinction between a proper name and a singular term (pp. 25-26). The proper name, as a mere name or word, has, like many other words, no definite meaning or connotation. The name 'William' is applicable to a great many individuals and it does not imply that these individuals have any attributes in common. It does not even imply that they are men, for one has known women and dogs called by that name. But when it is used as a term in a proposition, it has always some more or less definite connotation. In short, it is a singular term. The name 'Samuel Butler' may, by itself, mean either 'the author of *Hudibras*' or 'the author of *Erewhon*' or one of many other people ; but in any particular context it has a definite meaning, and is accordingly a connotative term.

[1] *Op. cit.*, bk. i. ch. ii., sect. 5.

[2] Mill defines a subject as anything which possesses attributes.

As to abstract terms, we have also seen (p. 28 ff.) that the distinction between them and concrete terms is a distinction, not between terms, but between names, and that all terms are really concrete. But if a term is concrete, it is generally admitted that it has both denotation and connotation. It must be allowed, however, that, when an abstract name is used as a term, there is a certain difficulty in distinguishing between its denotation and its connotation. 'Whiteness is dazzling.' 'Honesty is a virtue.' 'Whiteness' and 'honesty' signify qualities and apparently nothing else. But a term is connotative in so far as it refers to qualities. What, then, do 'whiteness' and 'honesty' denote? Mill would say that they denote the qualities and that they have no connotation, because they cannot be used as predicates. We can say 'These things are white,' but not 'These things are whiteness' (*v.* p. 29). The difficulty disappears when we recognize that denotation and connotation are not separate elements in a term, but differences in its aspect or its reference. An abstract name, when used as a term, denotes an attribute and it also connotes the same attribute. The difference of aspect is this, that when the attribute is taken as the denotation of the term, it is regarded as something about which something else may be said (*i.e.* as a subject of attributes), and, on the other hand, when the attribute is taken as the connotation of the term, it is regarded simply as the meaning of the term, a quality which may appear in various things. Regarded as a subject of attributes the term has denotation; regarded as having meaning, indicating qualities, the term has connotation. And it may be said in general that all terms have both connotation and denotation.

This may be shown to be true even in cases which, at first sight, seem inconsistent with it. Thus adjectives like 'white' cannot be used by themselves as subjects of propositions, and abstract terms like 'whiteness' cannot be used as predicates. It might thus appear that, as we cannot say anything about 'white,' it can have no denotation, and as 'whiteness' cannot be predicated of anything,

it can have no connotation. But 'white' and 'white things' or 'whiteness' do not differ in meaning. They are the same term regarded (*a*) specially in its aspect of connotation ('white') and (*b*) specially in one or another form of its aspect of denotation ('white things' and 'whiteness'). Thus we regard the proposition, 'Some men are white,' as equivalent to the proposition, 'Some white things are men,' and there is no essential difference of meaning between 'This thing is white' and 'Whiteness is a quality of this thing.' The term is essentially the same in each of the three forms in which we express it.

There has been considerable discussion among logicians as to the relation between the denotation and the connotation of a term. This has been concerned mainly with two points: (1) Is there a quantitative relation between denotation and connotation? (2) Is denotation prior to connotation, or connotation to denotation, or is neither prior to the other?

(1) It has been said by some logicians that the connotation and the denotation of a term are in inverse ratio to one another. In confirmation of this view it is pointed out that the greater the number of qualities connoted by a term, the fewer will be the objects to which the term is applicable, *i.e.* the fewer will be the things which come up to the required standard. On the other hand, the greater the number of objects to which the term is applicable, the fewer must be the qualities which these objects have in common. To take extreme cases, a term like 'being' is applicable to everything, *i.e.* to an indefinite number of objects, while it connotes the least possible number of qualities. On the other hand, a term which stands for an individual, *e.g.* 'Shakespeare,' connotes an indefinite number of qualities, while it denotes only one object. Between these extremes we may place terms like 'animal,' 'man,' 'European,' 'Englishman,' and when we pass from one to another in order, it is obvious that the denotation decreases and the connotation increases at each step from 'being' to 'Shakespeare.'

Now while it may be possible to arrange certain groups of names or terms in a series of this kind, so as to show some such relation between their denotation and their connotation, the relation ought not to be described as an inverse ratio. An inverse ratio holds between two calculable quantities; but, though the denotation of a term may occasionally (by no means always) be a calculable quantity, the connotation of a term can never be exactly calculated. Connotation is incommensurable with denotation. We may, in certain cases, be able to recognize one object (*e.g.* one man, one volume, one page), and to count the number of objects to which a term refers; but we cannot always say what one quality is. There are qualities, expressed by a single word, which really include many others. One language may express by a single word what in another language would require several. Such words as 'good,' 'beautiful,' 'rational,' 'intellectual,' may at first sight appear to stand for single qualities; but in reality they each represent a variety of qualities, which we cannot enumerate, because there is no common principle of enumeration. It is therefore impossible to determine any exact ratio between the denotation and the connotation of a term.

But it may be said (with Jevons [1]) that when we increase the connotation of a term, we decrease its denotation, and when we decrease its connotation, we increase its denotation. This is a very vague statement; but there is a certain amount of truth in it. If it means, 'when we pass from one term to another by merely adding some quality or qualities to the connotation, the denotation of the new term is less than that of the old, and when we pass from one term to another by merely removing some quality or qualities from the connotation, the denotation of the new term is greater than that of the old,' it may be maintained that the statement is perhaps true, though there may be exceptions. Of course, as Jevons points out, the decrease of denotation is not in exact proportion to the increase of

[1] *Elementary Lessons in Logic*, p. 40.

connotation. The addition of what seems to be one quality may decrease the denotation much more than the addition of another. If we pass from ‘ man ’ to ‘ red man ’ we decrease the denotation much more than if we pass from ‘ man ’ to ‘ white man.’ This is what Jevons really means ; but his words, and the similar words of other logicians, are open to a different interpretation, and this has brought much confusion into the discussion. If we take Jevons's words strictly, they imply that you can increase the connotation of a term and decrease its denotation, *i.e.* not that you can pass from *one* term to *another* by merely adding or subtracting qualities from its connotation, but that the connotation and denotation of the *same* term are changeable. Now if we are not careful in observing the distinction between names and terms, this may appear to be true. For instance, if by the connotation is meant the subjective connotation of a name, this may manifestly increase with the individual's increasing knowledge. In this case, however, the denotation does not necessarily decrease. It may remain the same (we may come to know more about the same things), or it may even increase (as indeed it usually does) through our coming to know more species or members of the class. But this is not increase or decrease in the connotation or denotation of a logical *term*, but change in the individual's knowledge or in the meaning which he attaches to a *name*. In the same way the conventional connotation of a name may change, owing to increase of knowledge or to a change in the things to which the name applies. The conventional connotation or definition of the name ‘ planet ’ was changed when the planets were found to be bodies moving in definite orbits with a definite relation to the sun. This was an increase in the connotation of the name, and at the same time there was an increase in its denotation, as the earth was now counted among the planets. But this also is a change in the meaning or reference of a name, and not change in the connotation or denotation of a term. If you take a word and use it, first with one connotation and then with a connotation differing from the former by one or more

qualities, you have really made a new logical term, although the word may be the same in both cases, just as the same word (*e.g.* 'ground') may stand as the verbal symbol for two or more different terms, according to the varying senses in which it is used. In an exposition of the philosophy of Aristotle the word 'planet' stands for one term; in an account of the astronomy of Kepler, it stands for a different term. Thus to talk of the connotation of a term 'being increased' is logically meaningless. If the phrase is interpreted strictly, it means that the term is ambiguous, *i.e.* that it is not a term in the logical sense. For one of the chief requisites of valid reasoning is that its terms should be unambiguous.

(2) The question has been raised whether terms are primarily connotative or primarily denotative. Mill[1] says that a name signifies a subject(*i.e.*denotes) directly, because it is predicable of the subject, and signifies the attributes (*i.e.* connotes) indirectly, because it is not predicable of them. Nevertheless he elsewhere[2] contends that a name or term is applied to things only in virtue of their possessing certain attributes, *i.e.* that the application of the name presupposes the recognition of attributes. On the first view the name would be primarily denotative, on the second it would be primarily connotative. He would probably reconcile this apparent conflict of opinion by suggesting that, in the history of language, individual or proper names, which he regards as having no connotation, were in the first instance applied to individual objects, merely denoting them or pointing them out; but that nothing like a common or general term, applying to various objects, could be used without a previous recognition of certain attributes as common to these objects. The common attributes would thus determine the denotation of the term by defining the class of things to which it is applicable. Whatever possesses these attributes is a member of the class; whatever does not possess them does not belong to the class. Connotation would thus be logically prior to denotation. But Mill is prevented from

[1] *Op. cit.*, bk. i. ch. ii., sect. 5. [2] *Ibid*, sect. 3.

saying this quite plainly by his view that there are some terms which have no connotation, and by the fact that historically it would appear as if denotation were the first function of the term or name.

Now, if it is admitted that names were first used to mark off individual objects rather than to indicate common qualities, does it follow that this use of names was merely denotative? It is obvious that, when we recognize one object as being the *same* as another in certain respects, we do so by recognizing certain qualities as common to both. But how is it that we recognize one object as *different* from another, *i.e.* how is it that we recognize one object as one object, something by itself, separate and distinct, not merely continuous with other objects and indistinguishable from them? Surely it is by the recognition of some quality or qualities in which the one object differs from others or from its surroundings in general. The one object must be so far identical with others that there is a basis of comparison between them, that we can recognize qualities which the one has and the others have not. They may differ in shape or in weight or in colour or in hardness or in taste or in numerous other ways. Their difference is always a difference in *some respect*, and thus it is not a pure difference, but a difference which implies a certain degree of sameness or identity, *e.g.* they may differ in the kind of colour they have, but, in order that they may so differ, they must agree in having colour of some sort, they may differ in the amount of their weight, but they must agree in having some weight, and so on (cf. pp. 13-14). Thus we cannot recognize one object as one object without recognizing certain characteristics in which it agrees with and certain in which it differs from others. It is in virtue of these characteristics or qualities that we give a name to the object, and that we are enabled afterwards to think of the object when we hear the name or to suggest the object to others when we use the name. Some sort of connotation, however vague and imperfect, is therefore already involved in the earliest use of names. In the beginnings of knowledge, however, our

recognition of qualities is very vague and indistinct, and our recognition of objects is correspondingly uncertain and vacillating. We observe general likeness and difference between objects, without being able to state precisely in what the likeness or the difference consists. We have not yet learned to make exact or minute distinctions. Indeed, the whole progress of knowledge consists in making more and more exact distinctions and at the same time coming to a more and more clear and definite recognition of the identities which these distinctions imply. No doubt in the early stages of knowledge it is the denotative rather than the connotative aspect of the term that seems the more prominent; but we cannot have the one aspect without the other.

It is most important to recognize that connotation and denotation are inseparable aspects of every term, because failure to grasp this has given rise to much confusion in the discussion of the question. Connotation and denotation have been implicitly assumed to be separate things, between which quantitative and other relations may be established. The denotation of a term has been regarded as a mere collection of things, and the connotation as a mere collection or aggregate of qualities. The thing, or subject of attributes, has been put on one side, and the attributes on the other, the thing being implicitly regarded (in Locke's words) as a "something I know not what," to which attributes may be attached. But the thing and its attributes are inseparable, and the attributes are not merely independent qualities added together into an aggregate. If you think of the attributes as a collection of independent qualities, like a number of labels, you must suppose that there is something independent of them to which they can be attached and which you call a thing, or subject of attributes. But this 'thing,' if it is really independent of the qualities, cannot be anything at all. Unless some quality or other can be recognized in it, it is nothing. Mill tried to escape from the difficulty by suggesting that the thing is merely another group of qualities. But this is of no avail,

for these new qualities require the supposition of another 'thing' to which they can be attached, and so on *ad infinitum*. Attributes have no actual existence except in things; and the attributes of a thing are not independent of one another but related together in a more or less complete system. That a thing has weight, for instance, depends upon its being a material thing, and the amount of its weight may depend on certain qualities of the particular material of which it is composed, or on its being waterlogged, etc. The colour of a thing also depends on the material of which it is made or on certain qualities of the pigment with which it is coloured. If the thing is organic (like a plant or an animal) its qualities are interdependent or interwoven with one another in countless ways. From a few of its characteristics we can infer the kind to which a particular plant or animal belongs, that is to say, we can infer a considerable number of its other characteristics. In general, then, it may be said that wherever we find actual qualities, we find them inter-related in some sort of system. The thing of which they are the qualities is just this system regarded as one, while the qualities are the system regarded as many. A thing having qualities is, in short, a unity in difference.

It follows that the connotation of a term is not its reference to a mere aggregate of independent qualities, but to a system of qualities related to one another so as to form a unity. Similarly the denotation is the reference of the term to one or more such systems, each considered as a unity. Connotation and denotation are thus inseparable aspects of the term. If the term connotes qualities, it must connote them as elements within the unity of a system, and so far as the term refers to such a system as one thing it is denotative.

EXERCISE IV

1. Distinguish between the extension and the intension of the following: man, white, friendship, Glasgow.

2. Define (*a*) a general name, and (*b*) a proper name; and examine

the statements, (*c*) that every general name is connotative, and (*d*) that every proper name is non-connotative.

3. 'Connotation and denotation are inseparable aspects of every term.' Explain this statement and indicate the difficulties to which neglect of the truth contained in it has given rise.

4. (*a*) In what sense, if in any, can we legitimately speak of increasing or decreasing the connotation of a term? (*b*) Discuss the doctrine that the connotation of a term varies inversely as its denotation.

5. What different senses of connotation have been distinguished? What do you consider the logical value of the distinction between them?

6. Critically consider the doctrine that terms are primarily denotative.

CHAPTER V

CATEGORICAL PROPOSITIONS AND THEIR KINDS

We have seen (p. 18) that a proposition is always a premise or conclusion of a reasoning. But for logical purposes this is not an adequate definition of a proposition. The premises and conclusions of our actual reasonings are not always expressed in the strict form of logical propositions. Accordingly the logical proposition must be defined more precisely, and we cannot do better than adopt the definition of Aristotle :[1] a proposition is a statement in which something is said regarding something else, either affirmatively or negatively. That about which the statement is made is the *subject* of the proposition, what is said about it is the *predicate*, and the saying of it is the *copula*, which is usually expressed by the words 'is' or 'is not,' 'are' or 'are not' etc. The real copula, however, is not the mere *words* 'is' or 'is not' ; but the connection or the discrimination (or both at once) which the proposition declares to exist between the subject and the predicate. The analysis of a proposition into three sets of words, standing respectively for subject, copula and predicate, is an artificial arrangement, the purpose of which is to avoid ambiguity in reasoning. For this purpose it is a useful, indeed almost a necessary, arrangement ; but we must not be misled by it into supposing that the copula is something separate from or independent of the terms, a mere external link or barrier between them. The sentence 'What can't be cured must be endured' does not contain the words 'is' or 'is not' ;

[1] *Anal. Pr.*, i. 24^a, 16.

but it is clearly an affirmative assertion, in which something is said about something else, and there is thus a copula involved in it. When it is stated in strict logical form as a proposition, it reads 'All things which can't be cured are things which must be endured.' Here the word 'are' appears as an *expression* of the copula ; but the real copula (the relation between subject and predicate) is the same in both forms of the statement.

The logical proposition must be carefully distinguished from the grammatical sentence. Whately described a proposition as 'an indicative sentence.' That is to say, a proposition is a sentence expressing or conveying information, whether the information be true or false. Thus several kinds of sentences are obviously not propositions. A question, for instance, is not a proposition, because it does not give information, but seeks for it ; it does not *say* something about something else, but *asks* whether something can be said about something else, or what can be said about a thing. Again a proposition is to be distinguished from a command, which does not give information but calls for action, and it is also to be distinguished from a mere exclamation, such as an involuntary cry, not intended to communicate any knowledge. While this is true, it must be noticed that many sentences, expressed in the form of questions, commands or exclamations, actually contain propositions and, in the context in which they occur, are intended to assert truths or convey information. For instance, the question 'Can the leopard change his spots?' is obviously intended to be an assertion that the leopard cannot change his spots; the command 'Do as you would be done by' is an assertion that our duty is to act towards others as we would that they should act towards us; and the exclamation 'Fire!' conveys the information that something is on fire. But in each of these cases the sentence as it stands is not a logical proposition. The proposition which it contains must be extracted from it and stated in the logical form—subject, copula and predicate.

And similarly, the ordinary grammatical sentence, which

is not a question, a command or an exclamation, is not always in the form of a logical proposition. The form of the grammatical sentence is determined in part by considerations of style, rhetoric, etc. The order of the words in a sentence may, for instance, be varied for the sake of emphasis or cadence. The sentence 'Great is Diana of the Ephesians' is rhetorically emphatic; while the logical proposition which it contains 'Diana of the Ephesians is great' is a tame, unrhetorical statement. Proverbs and idiomatic expressions often have a force and imaginative vividness, which is no part of their logical meaning as propositions. 'Give him an inch and he'll take an ell' is a vivid and emphatic way of saying that a certain person is of a grasping disposition. Again, the same logical proposition may be expressed quite differently in various languages, according to the differences of idiom. Where we say 'I bought it for an old song,' a Frenchman would say 'I bought it for a morsel of bread.' We speak of 'calling a spade a spade,' while the French express the same statement in the form, 'calling a cat a cat.' Again, the grammatical subject and predicate of a sentence are often quite different from the logical subject and predicate of the proposition which it expresses. The grammatical subject is generally the principal noun or pronoun of the sentence, while the predicate is almost always the principal verb. The logical subject is always that about which an assertion is made, while the logical predicate is that which is asserted about it. Thus, in the sentence 'We know what matter is by the evidence of one or more of our senses,' the grammatical subject is 'we' and the grammatical predicate is 'know' (or 'know what matter is'). But 'we' is not the logical subject, for the statement is not a statement about 'us' but a statement about 'our knowledge of matter.' This last phrase is thus the logical subject. Similarly the verb, which is the whole or part of the grammatical predicate, often contains the copula, as well as the whole or part of the logical predicate. If we say 'Our knowledge of matter comes to us on the evidence of our senses,' the word

'comes' is the grammatical predicate; but it includes the logical copula as well as a part of the logical predicate. The proposition, expressed in logical form, should be 'Our knowledge of matter is knowledge which comes to us on the evidence of our senses.' The logical predicate in this instance is therefore quite distinct from the grammatical predicate.

The general reason for this distinction between the logical proposition and the sentence is that language is a means by which we express not merely information and reasoning but also our feelings, our desires and resolutions, our inquiries and commands. It is also a means by which we try to influence the minds and wills of others, to persuade, convince, exhort or threaten. All these things may to some extent involve the conveying of information or the expression of thought. But language is not a pure medium of thought: it does have these other functions. In logic, however, we are concerned with language solely as the expression of information, reasoning, thought; and accordingly, in order to avoid ambiguity of meaning, we must for logical purposes make our statements as colourless as possible by removing from them those elements which are other than the pure expression of thought. And there is yet another reason why a sentence must be distinguished from a proposition. One sentence may contain more than one proposition. *E.g.* the sentence 'The cow, the horse, and the goat chew the cud' contains three propositions.

A logical proposition is sometimes called a judgment. But, strictly speaking, 'judgment' in logic has a wider meaning than 'proposition.' Just as all propositions are sentences, but not all sentences are propositions, so all propositions are judgments, but judgments are not necessarily propositions. A proposition is an expression of thought in words, a sentence is an expression of thought as well as, possibly, of other things (feeling, imagination, desire, etc.) in words; but a judgment is thought or knowledge which may or may not be expressed in words. Whenever I recognize (even in thought,

without speaking about it) that an object has a certain quality, I make a judgment. If, for instance, I observe by looking that it is raining or that it is getting dark, that the book I am reading has a red binding or that my pen requires more ink, I am making a judgment, even though I do not say any of these things in imagined words to myself or in spoken words to anyone else. When I express any of these ideas in imagined or in actual speech, the judgment becomes a proposition. As every proposition is the verbal expression of a judgment, it is often convenient to call a 'proposition' a 'judgment.' This is frequently done and there can be no objection to it, provided we remember that there may be judgments which are not expressed in words and which are therefore not propositions.

There are two main kinds of propositions, the categorical and the conditional. A categorical proposition is a proposition expressed absolutely, without conditions. A conditional proposition is a proposition stated subject to an expressed condition. Thus 'Light is a mode of motion' is a categorical proposition, while 'If the sky is clouded, the sun is invisible' is a conditional proposition, because it asserts that the sun is invisible, not absolutely but under the condition that the sky is clouded. We shall afterwards see (p. 90 ff.) that the distinction between categorical and conditional propositions is a relative distinction and that categorical propositions, although they are not subject to any *expressed* condition, are always stated under *implied* conditions. The distinction, however, is perfectly definite and useful for many of the purposes of logic. And it is convenient to limit our consideration, in the first place, to categorical propositions.

As we have seen, every proposition either asserts or denies, for a proposition is just a plain statement that something *is so* or that it *is not so*. Every proposition, then, is either affirmative or negative, and this is technically called the *quality* of the proposition. Again, when we affirm or deny something of any subject, we may affirm or deny it

of the whole subject or of a part of it. Thus we say 'All men are mortal,' where 'mortal' is affirmatively predicated of the whole subject 'men,' and 'No negroes are white,' where 'white' is negatively predicated of the whole subject 'negroes.' On the other hand, we may say 'Some men are white' and 'Some men are not white,' where 'white' is predicated affirmatively and negatively of an indefinite portion of the subject 'men.' When the predicate of a proposition is predicated of the whole subject (whether affirmatively or negatively), the proposition is said to be a *Universal* proposition ; when the predicate is predicated of an indefinite part of the subject, the proposition is said to be *Particular*. This distinction among propositions is called the distinction of *quantity*. Accordingly every categorical proposition is affirmative or negative in quality, and universal or particular in quantity.

In the case of certain propositions the subject is indivisible, *i.e.* it is used as a singular term, and it is therefore impossible to predicate anything of a part of it, *e.g.* 'The Speaker is the President of the House of Commons,' 'Socrates lived in Athens,' 'The United States was at war with Spain,' 'The Concert of Europe is ineffective,' 'Virtue is a condition of happiness,' 'Honesty is a virtue.' These propositions are called Singular propositions, and as their subjects must always be used in the whole of their extent, they are, for the sake of convenience, invariably classed as Universal. But, as we shall afterwards see, they have certain peculiarities, which make it necessary to mention them separately.

Some logicians, including Aristotle, have spoken of another class of propositions, which they call 'indefinite,' 'indesignate' or 'pre-indesignate (Sir W. Hamilton's terms), on the ground that their quantity is doubtful, *e.g.* 'Birds are feathered,' 'Crime is punished,' 'Japanese are not tall.' If a proposition of this sort is stated, apart from any context, it is usually impossible to say definitely whether it is intended to be universal or particular. Nearly all such propositions are approximately universal ; but taken apart from their context, they cannot be strictly classed as

universal. Their meaning might be expressed in some such formula as: 'S is (or is not) generally P', or, 'S as a rule is (or is not) P', where S stands for any subject and P for any predicate. Thus birds are usually feathered, and if the proposition occurs in a certain context (*e.g.* a discussion on ornithology) it may be possible, without error or ambiguity, to treat it as a universal proposition (All birds are feathered), whereas, if the context is a conversation about the birds in a poulterer's shop, it will probably be necessary to make it particular (Some birds are feathered). As we have already seen (p. 18) propositions do not stand by themselves; they are always parts of reasonings or trains of thought; they have a context. Accordingly, just as we cannot determine precisely the meaning (the connotation) of a term until we know its context, *i.e.* the proposition or reasoning in which it is used, so also in many cases we cannot determine the quantity of a proposition (*i.e.* the denotation of its subject) without knowing the context (the reasoning or train of thought) in which it appears. The indefinite quantity of a proposition is analogous to the indefinite connotation of a term. And the difficulty must be met in the same way in both cases. Just as terms (so-called) with an indefinite connotation are not terms, but *names*, so propositions (so-called) with an indefinite quantity are not propositions, but *sentences*. They are not propositions in the strict logical sense, until some definite quantity has been assigned to them. Thus, for the purposes of ordinary deductive reasoning, indefinite propositions do not form a separate class in respect of quantity. They must be either universal (including singular) or particular.

The only signs of quantity which are recognized by the Deductive Logic which is based on that of Aristotle, are the words 'all' and 'some'—'all' being the sign of universal quantity, and 'some' the sign of particular. The subject of the proposition must be used either in its entirety or in some indefinite part of its extent. There are no signs of degree of quantity in elementary logic. Thus, *e.g.* 'most' and 'many' must be represented merely as 'some.' The

sentence 'Most houses in Scotland are built of stone,' when expressed as a logical proposition, becomes 'Some houses in Scotland are built of stone.' Of course, if we happen to be discussing the geology of Scotland and other countries in relation to the habits and dwellings of their peoples, it may be possible, in that particular context, to treat the proposition 'Houses in Scotland are built of stone' as a universal proposition, on the ground that the exceptions to the statement do not affect the particular argument in which it occurs. But if the proposition stands alone we must regard it as particular. Again the word 'some,' when used as a sign of quantity in logic, is not absolutely exclusive of 'all.' The word 'some' in general is an ambiguous word. It may mean either *some only*, *i.e.* some but not all, or *some at least*, *i.e.* some and possibly all. As a sign of quantity in a logical proposition, it is always used in the latter sense, as equivalent to 'some at least.' It is a perfectly indefinite 'some' and may merely mean that we are not in a position to say 'all' with complete certainty.

We should also notice that in ordinary language there are various forms of expression which are practically synonymous with 'all'; *e.g.* 'every' ('Every cow is a ruminant' = 'All cows' etc.), 'any' in certain cases ('Anyone who goes there is a fool' = 'All who go there' etc.), 'he who' in certain cases ('He is gentle that doth gentle deeds' = 'All who do gentle deeds' etc.), 'whosoever (= 'all who'), 'the' in certain cases ('Blessed are the pure in heart' = 'All who are pure in heart' etc.), and sometimes even 'a' ('A man who would do that is not worth considering' = 'No man who would do that is' etc.). Similarly, there are various words in ordinary speech which must be expressed in logical propositions by the sign of particular quantity, 'some,' *e.g.* 'few,' 'a few,' 'hardly any,' 'certain,' etc. 'Few S are P' usually means 'Some S are *not* P,' *e.g.* 'Few men are learned' = 'Some men are not learned.' 'A few S are P' usually means 'Some S are P,' *e.g.* 'A few of the passengers were saved' = 'Some of the passengers were saved.'

Some logicians hold that ' Few S are P ' really stands for two propositions, viz., Some S are P, and Some S are not P. If the proposition is regarded abstractly, apart from any context, it is barely possible to interpret it in this way. But propositions are always parts of a reasoning or train of thought or discussion, and consequently the statement ' Few S are P ' must have one interpretation in every case in which it actually occurs. Now when we say, *e.g.* ' Few men are learned,' we clearly desire to emphasise the opinion that some men are *not* learned rather than to say that some men are learned. If we desire to say that some men are learned, it is unnatural and misleading to put this in the form ' Few men are learned,' while it is quite natural to express our meaning in the words 'A few men are learned.' Again the word ' certain ' may usually be interpreted as ' some,' *e.g.* ' Certain lewd fellows of the baser sort ' = ' Some lewd fellows ' etc. On the other hand, ' *a* certain ' generally indicates a singular (*i.e.* universal quantity), *e.g.* ' A certain man had two sons.'

In each case, however, the statement must be interpreted, not by reference to the mere words or the grammatical construction, but by consideration of the thought that is implied. We must ask ourselves : (1) what is the thing about which the proposition says something ? and (2) is the statement made with reference to the whole or a part of the subject ? When the meaning of the statement is not quite obvious, we should take into consideration the probable context in which it might be uttered and interpret it reasonably in relation to the context. Thus the statement ' He jests at scars who never felt a wound ' seems at first sight, from its grammatical structure, to mean ' All persons who never felt a wound are persons who jest at scars.' But a moment's consideration shows that this is a most unlikely interpretation. The proposition is too obviously false, for most persons who have never felt a wound would not dream of jesting at scars. In any intelligible context the sentence must mean that 'All who jest at scars are people who have never felt a wound ' (or that

Only those who have never felt a wound are people who jest at scars,' which is the same statement in another form).

All propositions, then, have one or the other quality (affirmative or negative), and they have also one or the other quantity (universal or particular). Thus every proposition, when expressed in logical form, must belong to one of four classes or kinds. It must be either universal and affirmative, or universal and negative, or particular and affirmative, or particular and negative. For convenience of reference these four kinds of propositions have for many centuries been symbolised by the first four vowels of the alphabet, A, E, I, O. Thus a universal affirmative proposition is called an A proposition and is of the general form, All S is P (S standing for any subject and P for any predicate); a universal negative proposition is called an E proposition and is of the general form, No S is P; a particular affirmative proposition is called an I proposition and is of the general form, Some S is P; and a particular negative proposition is called an O proposition and is of the general form, Some S is not P. Accordingly, when a sentence or statement is to be expressed as a proposition in logical form, it should be made to appear as an instance of one of these four general forms. In ordinary speech and writing propositions are, as a rule, not expressed in such a way that it is immediately evident to which of these classes any individual proposition belongs. Consequently there may be ambiguity in the statements, and this may lead to fallacies or erroneous inferences. The object of stating every proposition in strict logical form is to get rid of this ambiguity and thus to avoid fallacy.

The E proposition (No S is P) requires a little further notice. At first sight it looks as if the copula in this case were the word 'is.' But a proposition with the copula 'is' is an affirmative proposition, and E is negative. Again, there seems to be no sign of quantity ('all' or 'some') as in the case of the other kinds of proposition. It might seem, in short, as if the E proposition ought to be expressed in the form 'All S is not P,' 'all S' indicating that it is universal,

while 'is not' indicates that it is negative. But in many languages, including English, 'All S is not P' really means 'Some S is not P.' It is a rhetorical way of saying that although most S may be P, there are at least some S, however few, which are not P. 'All is not gold that glitters' means that some things that glitter are not gold. Consequently the form 'All S is not P' is at least ambiguous, and 'No S is P' is used instead. On the other hand, we must remember that the word 'no' in this form is a combination of 'all' and 'not,' and consequently the subject of the E proposition is 'all S' (not 'no S') and the copula is 'is not' (not 'is'). Again a singular negative proposition (which counts as an E proposition) is expressed not in the form 'No S is P,' but in the form 'S is not P' or 'The S is not P'; because in this case there can be no ambiguity, as the subject is used as indivisible and therefore 'Some S is not P' would be impossible, *e.g.* 'The House of Commons is not sitting.'

Some propositions have a subject limited by the words 'only,' 'none but' or some equivalent expression, *e.g.* Only S is P, None but S is P, 'Only graduates are eligible,' 'No admittance except on business.' These are usually called *exclusive* propositions. This is a convenient name for them; but it is not necessary to regard them as a special class, additional to the other four classes. These propositions may usually be expressed as E propositions having as subject a negative term and being thus of the form 'No not-S is P' (or 'No thing other than S is P'), *e.g.* 'Only graduates are eligible' = 'No not-graduates are eligible,' 'No admittance except on business' = 'No persons who are not on business are admitted.' The important thing to notice is that 'No not-S is P' cannot be regarded as necessarily equivalent to 'All S is P.' 'Only graduates are eligible' may, in certain contexts, mean that 'All graduates are eligible'; but we are not entitled to assume this from the mere form of the statement, *e.g.* 'Only matriculated students attend the logic class' clearly does not mean that 'All matriculated students attend the class.' On the other hand the statement 'Only S is P' can always be interpreted as 'All *P* is *S*,' *e.g.*

'All who are eligible are graduates,' or 'All who attend the logic class are matriculated students.'

Some logicians also mention a class of *exceptive* propositions. They are similar to exclusive propositions, but are affirmative, not negative, *e.g.* All S except N are P, 'All the books except the dictionaries are bound in calf.' These are evidently a special kind of affirmative propositions, having as their subjects 'S except N' (or 'S which are not N'), 'The books except the dictionaries' etc.

We have seen that the distinction between universal and particular quantity in a proposition depends upon whether or not the subject is used in its whole extent. Now when a term is used in its whole extent it is said to be *distributed*, and when it is not used in its whole extent it is said to be *undistributed*. Accordingly in every universal proposition the subject is distributed (though it may not be a *distributive* or general term), and in every particular proposition the subject is undistributed. But the predicates of propositions have usually no sign of quantity attached to them. Predicates are primarily adjectival, that is to say, they are, as a rule, used in a sense that is mainly connotative. They are generally intended to qualify the subject; the proposition is the attributing or refusing to attribute a certain quality or characteristic to the subject. And thus, while the subject, which is a thing or things, has naturally some sign of quantity, the predicate, which expresses a quality, has naturally no such sign. But the predicate also may be treated as a thing or things, and the proposition may be regarded as saying that the subject is or is not included in the class of things to which the predicate refers, *e.g.* 'All horses are quadrupeds' may mean either that all horses have the characteristic of being four-footed or that all horses are included in the class of four-footed things; 'No men are to be despised' may mean either that the quality 'despicable' is not to be attributed to any man or that all men are excluded from the class of things to be despised, etc.

Thus the affirmative proposition (whether universal or particular) may be regarded as stating that the subject is

included in the class denoted by the predicate ; while the negative proposition (universal or particular) states that the subject is excluded from the class denoted by the predicate. Now when we include one class in another, we are using the wider class only in a portion of its extent, *i.e.* in that portion which the wider class has in common with the narrower. If, *e.g.* we say 'All horses are included in the class of quadrupeds,' we are, for the purposes of this proposition, referring in the predicate only to that part of the class of quadrupeds which coincides with the class of horses. The remainder of the class of quadrupeds is not used or referred to in our assertion. Consequently in such a case the term denoting the wider class (*i.e.* the predicate) is undistributed, and we may therefore say that the predicate of every affirmative proposition is undistributed.[1] On the other hand, in the negative proposition the subject is excluded from the class denoted by the predicate. Now, if you exclude a thing or class (S or a part of S) from another class (P) you exclude S from the whole of P, *i.e.* you refer to P in its whole extent. Consequently P is a distributed term, and we may say that the predicate of a negative proposition is always distributed. We have thus obtained an invariable rule for the distribution of terms in any proposition. The subject of a universal proposition is always distributed ; the subject of a particular proposition is always undistributed. The predicate of a negative proposition is always distributed ; the predicate of an affirmative proposition is always undistributed. Thus in an A proposition, S is distributed, P undistributed ; in an I proposition, S and P are both undistributed ; in an E proposition, S and P are both distributed ; and in an O proposition, S is undistributed, P distributed.

Propositions have been distinguished from one another on other grounds than those of quantity and quality. The

[1] There are some cases of universal affirmative propositions in which the subject and predicate *happen* to have the same extension, *e.g.* 'Lord X is the Lord Chancellor,' 'All equilateral triangles are equiangular.' But we cannot discover this from their form alone. Cf. p. 128.

most important of these distinctions is that between *analytic* and *synthetic* propositions (sometimes described as a distinction between explicative and ampliative, or between verbal and real propositions). An analytic or explicative or verbal proposition is one in which the predicate says nothing except what is already implied in the subject, *e.g.* 'All triangles are figures having three angles.' A synthetic, ampliative or real proposition is one in which the predicate adds something new to what is said in the subject, or says something which is not already implied in the subject, *e.g.* 'A triangle was drawn upon the blackboard yesterday.' Kant expresses the distinction thus : "There are two ways in which the predicate of an affirmative judgment may be related to the subject. Either the predicate B is already tacitly contained in the subject A, or B lies entirely outside of A, although it is in some way connected with it. In the one case I call the judgment *analytic*, in the other case *synthetic*. Analytic judgments are those in which the predicate is related to the subject in the way of identity, while in synthetic judgments the predicate is not thought as identical with the subject. The former class might also be called *explicative*, because the predicate adds nothing to the subject, but merely breaks it up into its logical elements, and brings to clear consciousness what was already obscurely thought in it. The latter class we may call *ampliative*, as adding in the predicate something that was in no sense thought in the subject, and that no amount of analysis could possibly extract from it." [1] J. S. Mill,[2] again, regards the distinction as one between *verbal* propositions, which merely state the meaning or definition of the word which is the subject, and *real* propositions, which convey information, not about words, but about real things.

This distinction, however, is one in the application of which considerable difficulties arise. It is apt to involve all the difficulties which we have considered in connection

[1] Kant, *Critique of Pure Reason*, Introduction, translated in Watson, *Selections from Kant*, p. 13.

[2] *Op. cit.*, bk. i. ch. v., sect. 4.

with the connotation of terms. For it seems to imply that the subject of a proposition has always a fixed connotation, apart from its use in this or that proposition. An analytic proposition would be one in which the predicate is the whole or a part of the connotation of the subject and includes nothing outside of this connotation; while a synthetic proposition would be one in which the predicate includes something which is no part of the connotation of the subject. But this would make the analytic proposition a mere tautology; the predicate would simply repeat the whole or a part of the subject. And we have seen (p. 43 ff.) that a term has not a fixed connotation, apart from the proposition or reasoning in which it is used. It may be said, however, that a proposition, which states in the predicate the meaning or connotation in which the subject is to be used in a discourse or discussion, is analytic and yet it is not a mere tautology, because it conveys new information to anyone who does not know the meaning of the word defined. For instance we may say 'A triangle is a geometrical figure, having three angles,' our purpose being to show that we are not speaking of a triangle as a musical instrument. But, if we mean this, our proposition, properly stated, should be 'The term 'triangle' is to be used in this discussion as meaning a geometrical figure, etc.' That, however, is clearly a synthetic proposition, for the predicate is not implied in the subject, taken by itself.

Cases of this sort suggest an explanation of the distinction which is sometimes offered. It is stated that, though we cannot say that any proposition, considered in itself, is either clearly analytic or clearly synthetic, propositions may be analytic to one person or at one stage of knowledge and synthetic to another person or at another stage of knowledge. The chemical definition of a metal, for instance, is an analytic proposition to an expert in chemistry and a synthetic proposition to a beginner in the study; it is analytic to the chemist of to-day, but it was synthetic when it was first discovered. But if the distinction is interpreted in this way, it ceases to have any logical significance. It is

no longer a distinction between logical propositions ; it has become a distinction between the knowledge of one person and that of another.

We must, however, inquire whether, in the strict logical sense, there are any purely analytic or purely synthetic propositions. For this purpose we may take as a basis the statement of Kant : "Analytic judgments are those in which the predicate is related to the subject in the way of identity, while in synthetic judgments the predicate is not thought as identical with the subject."[1] A purely analytic proposition will thus be one in which the predicate is related to the subject solely by way of identity, *i.e.* one which adds nothing to what is contained or implied in the subject. Now such a proposition would give no new information whatever. It would be a mere tautology. In every genuine proposition there must be a reference to reality in the broadest sense, to experience, to a system or context of some kind within which the proposition claims to be true. Otherwise, what would be the use of stating the proposition ? To mention the subject without any predicate would be enough. Every genuine proposition does state a connection between two things which are not exactly, in all respects, identical (*v.* pp. 13-14). It is therefore synthetic, inasmuch as it involves going from one thing to another. It is not pure unity ; it implies a difference or differences. It does not state absolute identity between its subject and predicate, but identity in certain respects and difference in others. No actual thing is absolutely, in all respects, identical with any other. There must always be some difference which enables us to distinguish the one as one from the other as another. They may be identical in everything except their position in space or in time ; but that exception is enough to constitute a difference. If there were no such difference, the things would not be two but one. Thus the proposition 'A is A' (which is the most analytic proposition we can make) is not a statement of perfect identity without any difference. It means, not 'A is identical with A in all

[1] *Op. cit.*, p. 13.

respects,' but 'A is identical with A in all respects except its position in time or space.' In the very stating of the proposition 'A the predicate' is put in a different position from 'A the subject'—different in time, if the proposition be spoken, different in space if the proposition be written. What the proposition means is that A now is identical with A at some other time, or A in this place is identical with A in some other place. Thus even such a proposition as 'A is A' cannot be purely analytic. Its analytic aspect is certainly the predominant one; but the proposition has also a synthetic aspect, which it is misleading to overlook.

On the other hand, a purely synthetic proposition would be one in which there is no identity between subject and predicate, in which the predicate is not thought as in any way identical with the subject. As the purely analytic proposition would express pure identity so the purely synthetic proposition would express pure difference. It would be a proposition in which there is no necessary connection between the subject and the predicate, *i.e.* no real connection, but merely an arbitrary conjunction. But before we can, with any meaning, attribute a predicate to a subject, there must in the subject be something, some element or characteristic, which makes it at least possible to attribute this particular predicate to it, rather than another. Otherwise we might form propositions by attaching any predicate to any subject. But this is impossible, for to attach certain predicates to a subject may result in a statement which is neither true nor false, but meaningless; and such a statement is, of course, not a logical proposition, but an unintelligible jumble of words, *e.g.* 'The differential calculus is made of horse-radish,' 'The top of Ben Lomond is largely gladiatorial.' Now that in the subject which makes a particular predicate (or a particular set of predicates) possible constitutes a necessary connection between the subject and the predicate. In every proposition, as we have already seen (p. 13 ff.), there is implied some ground of connection between the subject and the predicate, and this ground is something which they have in common,

an element of identity. This applies, not merely to affirmative, but also to negative propositions. When we refuse to attribute a certain predicate to a subject, it is always implied that that predicate might conceivably be declared to belong to that subject. It is always a *possible* predicate of the subject ; for otherwise there would be no meaning in denying it. To say that the differential calculus is not made of horse-radish is just as absurd and meaningless as to say that it is. And yet the negative proposition is predominantly synthetic, for its main purpose is to express a difference. But, as we have seen (pp. 13-14), it may be taken as a general principle that no difference can exist or be expressed except on a basis of identity, however small or general.

We may, therefore, say in general that every proposition, inasmuch as it expresses an identity in difference, is both analytic and synthetic, the analytic element consisting in the identity between subject and predicate which it states or implies, while the synthetic element consists in the corresponding difference between subject and predicate. But in individual propositions one element or the other may appear to predominate, and accordingly, for the sake of convenience or for some special purpose, we may call a proposition synthetic, when the difference between its subject and its predicate is very remarkable and the identity between them somewhat obscure, while a proposition in which the identity between subject and predicate is the most obvious thing, and the difference is not recognized without some consideration, may be called analytic. But while some propositions may at once be seen to belong to one or other of these extreme types, there is a great mass of propositions intermediate between the two, in regard to which it would be very difficult to say whether the analytic or the synthetic element is the more prominent or essential. It therefore seems to be a mistake to use this distinction as a basis for classifying propositions.

EXERCISE V

1. Explain and illustrate the differences between the grammatical and the logical subject and predicate.

2. Distinguish carefully between a sentence, a judgment, and a proposition.

3. Put the following statements in strict propositional form, indicating the subject, predicate, and copula. State the quantity and quality of each, giving the logical symbol (A, E, I, or O). Point out whether the terms are singular, collective, or general:

(1) It is 100 miles to Dublin.
(2) Not every tale is to be believed.
(3) Few men are acquainted with themselves.
(4) When Caesar says do this, it is performed.
(5) Many rules of grammar overload the memory.
(6) All are not faithful that seem so.
(7) All's well that ends well.
(8) Two blacks don't make a white.
(9) Only the honest are respected.
(10) Heat and work are mutually convertible.
(11) He always succeeded.
(12) Every mark of weakness is not a disgrace.
(13) All the declarations of independence in the world will not render anyone really independent.
(14) That is not true.
(15) A triangle is drawn on the diagonal of a square.

4. Distinguish between a distributive and a distributed term. Put the following sentences into logical form and point out which of the terms are (*a*) distributive, (*b*) distributed, (*c*) undistributed:

(1) Anyone could do that.
(2) Where luck is awanting, diligence availeth nothing.
(3) Faint heart never won fair lady.
(4) Fain would I climb but that I fear to fall.
(5) What I have written, I have written.
(6) One swallow does not make a summer.
(7) Go a'borrowing, go a'sorrowing.
(8) Many hands make light work.
(9) The hands of the people of Glasgow are many.
(10) All men are not doctors.
(11) It does not always rain.
(12) One man in his time plays many parts.
(13) Only the ignorant affect to despise knowledge.
(14) Our sincerest laughter with some pain is fraught.
(15) It is never too late to mend.

5. Explain the nature and discuss the logical importance of the distinction between analytic and synthetic propositions.

6. Put the following statements into the form of logical propositions:

(1) Thieves!
(2) It rains.
(3) It is a fine day.

(4) On the hillside deep lies the snow.
(5) We cannot all do all things.
(6) A triangle cannot have all its angles obtuse.
(7) Only the educated are fit to vote.
(8) A few workmen were the only survivors.
(9) No man can be rich and happy, unless he is also prudent and temperate, and not always then.
(10) Am I my brother's keeper?
(11) A few drops of rain are not of much consequence.
(12) No man who witnessed that scene will ever forget it. The men fixed bayonets and the battalion extended and moved forwards in open order. A machine-gun from an embankment on the left-front opened fire. The majority fell—many never to rise again; a few continued to struggle bravely forwards, but very few reached their objective. A few stragglers from the right of A company were the only survivors at the end of the day. Almost all the others had been killed or fatally wounded.

CHAPTER VI

THE IMPORT OF CATEGORICAL PROPOSITIONS

WHAT is the meaning, the true nature of predication? What exactly do we do when we judge or make a proposition? To these questions there has been a considerable variety of answers. The problem which they raise, that of the import of propositions, may be put in two distinct, though not entirely separate, ways: (1) What kind of relation between its terms does the proposition express? (2) To what does the proposition as a whole primarily refer—to real things, to names or to ideas?

(1) All terms, as we have seen, have an extensive and an intensive sense, and they may be used primarily in either sense. In considering the first form of the problem of the import of propositions, we have to ask regarding each of the terms of a proposition whether it is to be taken in extension or in intension. Abstractly and apart from its use in a proposition, each term may be taken in either way. There are consequently four possibilities: (*a*) subject taken in extension, predicate in intension; (*b*) both subject and predicate taken in extension; (*c*) both subject and predicate taken in intension; and (*d*) subject taken in intension and predicate in extension. On the first three of these possibilities theories of the import of propositions have been founded; but the fourth, while it has been regarded as a possible interpretation of some propositions, has not been taken as the basis of a general theory. We may consider these theories in turn.

(*a*) The first theory is the 'subject and attribute' or 'predicative' view, according to which the subject stands for a thing or group of things, and the proposition either affirms or denies that a certain quality or set of qualities belongs to the subject. This evidently means that the subject is taken in extension and the predicate in intension. Although the problem was not definitely put in this form in the time of Aristotle, this may be taken as practically his view. He says, for instance, that a simple (or categorical) proposition is a significant sound which expresses that something does or does not belong to (or inhere in) something else.[1] Yet we find him speaking sometimes as if the proposition stated a relation between two things, sometimes a relation between a thing and an attribute, sometimes a relation between two names. It is evident that this theory applies to many propositions, *e.g.* 'This collar is white,' 'All men are mortal,' 'Some cats have no tails.' But it may be objected that there are propositions in which it is not the primary intention of the user to predicate a quality of the subject, *e.g.* 'Caesar crossed the Rubicon,' 'The batsman was bowled out.' We can hardly be said to mean that 'crossing the Rubicon' is a quality or attribute of Caesar, or that 'being bowled out' is a quality or attribute of the batsman. This is certainly true if by 'quality' we mean something comparatively permanent or essential. But many qualities are temporary (*e.g.* 'The boy's hands were dirty'), and all that is meant by the predicative theory is that the subject is considered primarily in extension and the predicate in intension. It is undoubtedly possible to interpret in this way all categorical propositions, though in many propositions (*e.g.* 'Tully is Cicero') the sense is greatly strained. The Aristotelian classification of propositions (A, E, I and O), according to which only the subject is quantified, is based on this theory. It is a defective and incomplete classification, and its imperfection is due to the imperfection of the theory.

[1] *De Inter.* 5, 17*a*, 23.

(*b*) The second theory, according to which both subject and predicate are taken in extension, is usually called the 'class' theory. On this view the proposition expresses a relation of inclusion or exclusion between two classes, the class denoted by the subject being included in or excluded from the class denoted by the predicate. Thus 'All men are mortal' means that the whole class of men is included in the class of mortal beings; and 'No dogs are bipeds' means that the whole class of dogs is excluded from the class of bipeds. This theory was, explicitly or implicitly, the ruling theory of the Scholastic Logic, and in modern times it has been widely accepted and considerably developed. It is substantially the theory of those who hold that logic is concerned with the form, to the exclusion of the matter, of thought. For many logical purposes it is a useful theory. If classification is taken as the ideal of science, *i.e.* if it is supposed that there are certain fixed classes (genera and species) to which all things belong, and if knowledge is regarded as consisting in placing particular things in their proper classes, the class view is obviously the right one. It is also the theory on which we proceed when we lay stress on the distribution of terms, whether in mediate or in immediate inference. It may, with some distortion of language, be applied to all categorical propositions; but it is certainly not the natural interpretation of most of our ordinary judgments. If I say 'George fell from a ladder,' I can hardly be said to mean that George is included in a class of things which fell from a ladder. Classification, in short, is not the primary function of knowledge, for classes are always formed on the basis of attributes, and if we say that a thing is included in a class, we do so because the thing is found to possess the attributes which determine the class.

The class view has in modern times been developed into the equational theory of the proposition. This development has taken place through the quantification of the predicate, proposed by Sir William Hamilton.[1] The

[1] *Discussions on Philosophy* (Ed. 3.), p. 646 ff. Cf. Mill, *Examination of Sir W. Hamilton's Philosophy*, ch. 22.

principle which Hamilton lays down in regard to the logical statement of propositions is that we ought in the proposition "to state explicitly what is thought implicitly." "From the consistent application of this postulate, on which Logic ever insists, but which Logicians have never fairly obeyed, it follows that, logically, we ought to take into account the *quantity*, always understood in thought, but usually, and for manifest reasons, elided in its expression, not only of the *subject*, but also of the *predicate* of a judgment."[1] This particular application of the principle is valid only on the class view of the proposition, which Hamilton acccpted. In a proposition the subject and predicate are related together, not absolutely but in a certain respect. For instance, in the proposition 'This door is blue,' the subject and predicate are related together only in respect of the quality, colour; in the proposition 'Nelson was brave,' subject and predicate are related in respect of moral quality, etc. Now in these propositions, as stated, we do not implicitly think the predicate in terms of quantity. The relation in each case is one of quality, not of quantity. But if we regard all propositions as relating classes of things together, we think of subject and predicate, not as related in respect of quality, but only in respect of quantity. If we interpret the proposition 'All cows are ruminants' as meaning that the whole class of cows is included in the class of ruminants, we mean that the quantity (*i.e.* number) of cows is a part of the quantity of ruminants, that the one class is smaller or larger than the other.

Now on the class view of the proposition, Hamilton's demand that the predicate be quantified is quite just. If a judgment expresses merely a relation of quantity, the relation ought to be made as definite as possible. If you have never seen an elephant and someone tells you that it is bigger than a mouse, you do not get much information about its size. It might be merely the size of a rat or it might be as big as a mountain. If you are to get any real knowledge of its size, you must be told approximately how many times larger the elephant is than the mouse. Simi-

[1] Hamilton, *loc. cit.*

larly if you are told that all cows are ruminants, the relation of quantity between the two classes is still indefinite. All cows may be all ruminants or they may merely be some ruminants. The classes may coincide in extent or the one may be wider than the other. Hamilton held that when we say 'All cows are ruminants,' we think implicitly that the whole class 'cows' is not as large as the class 'ruminants,' whereas when we say 'The Earth, Mars, Mercury, Venus, Jupiter, etc. are the planets,' we mean that the subject class and the predicate class coincide. Accordingly, in order to state explicitly this, which he says we think implicitly, Hamilton proposed the quantification of the predicate, *i.e.* that a definite sign of quantity, 'all' or 'some,' should be attached to the predicate as well as to the subject of every proposition. If this be done, we get, instead of the traditional four forms of the categorical proposition (A, E, I, and O), the following eight forms: [1]

A	All S is some P	I	Some S is some P
U	All S is all P	Y	Some S is all P
E	No S is any P	O	Some S is not any P
N	No S is some P	W	Some S is not some P

The quantification of the predicate leads directly to the equational theory of the proposition. Hamilton[2] himself says that "a proposition is always an equation of its subject and predicate," and propositions may therefore be stated in the form All S = some P, All S = all P, etc.

Before considering the further development and value of the equational theory, we must inquire whether the four new forms of propositions are a satisfactory and useful addition to the old. We have seen that the traditional logic interprets the word 'some,' the sign of particular quantity, as meaning 'some at least, possibly all.' It is a quite indefinite 'some.' Now one reason for suggesting the quantification of the predicate is that 'some' ought to be made more definite. We 'think implicitly' either 'all'

[1] The letters indicating the additional forms are those suggested by Archbishop Thomson.

[2] *Op. cit.* p. 647.

or 'some,' and we ought to state this explicitly. But this clearly means that 'some' should be interpreted as 'some only,' 'some to the exclusion of all.' Otherwise the relation of the terms to one another, considered as quantities, would be as indefinite as ever. If, *e.g.* 'All S is some P' might possibly mean 'All S is all P,' we cannot maintain that these two propositions are quite distinct. But if 'some' means 'some only,' each proposition which has an undistributed term is really two distinct propositions expressed as one. For instance, 'All man is some animal' implies that 'All man is not some (other) animal' (*i.e.* logically expressed, 'No man is some animal'). Hamilton's A proposition thus contains also his N proposition. Similarly 'Some mortals are all men' (Y) implies 'Some (other) mortals are not any men" (O). As to the W proposition, 'Some (but not all) men are not some (but not all) rational animals,' it is at first a little difficult to see what it means, but on consideration it will appear that it is equivalent to saying that a certain portion of the class 'men' is not equal to a certain portion of the class 'rational animals,' but is equal to the remainder of the rational animals, while the remaining portion of the class 'men' is equal to the first-mentioned portion of the class 'rational animals,' since some men only, not the remainder, are excluded from that portion. But this is merely a round-about way of saying that all men are rational animals and all rational animals are men, and these two statements combined constitute a U proposition. Consequently A and N, Y and O, U and W are each two ways of saying the same thing, and the eight propositional forms may be reduced to five: U (or W), A (or N), O (or Y), I and E. The only really new form is the form U. The same result may be obtained, in a slightly different way, if we interpret 'some' as meaning 'some at least.'

Probably no one now maintains that the form W has any value; but it is said that Y, N and U are convenient for certain purposes. Y and N are said to be useful for expressing exclusive propositions, such as 'The virtuous alone

are happy' (Y) and 'Not men alone are good' (N). But instead of expressing the first of these as 'Some virtuous beings are all happy beings' (Y), it is surely more natural to take it as 'All happy beings are virtuous' (A) or 'No beings who are not virtuous are happy' (E). And surely, 'Not men alone are good' is less evidently equivalent to 'No men are some good beings' (N) than to 'Some good beings are not men' (O) or 'Some beings other than men are good' (I). Y and N seem to be inconvenient rather than convenient forms, and the need of them arises only if we insist on regarding the *grammatical* subject as the *logical* subject of the proposition. The form U is regarded as useful for the expression of definitions, identical propositions, propositions in which both terms are proper names, etc. But, as De Morgan and Mill have pointed out, the proposition 'All S is all P' is really a combination of the two propositions 'All S is P' and 'All P is S.' And these two propositions may be so distinct that not merely can we know one without knowing the other, but each of them may require a separate proof. For instance, the proposition 'All isosceles triangles are all triangles in which the angles at the base are equal to one another' is a combination of the two propositions, 'All isosceles triangles are triangles in which the angles at the base are equal, etc.' and 'All triangles in which the angles at the base are equal, etc. are isosceles triangles.' But these propositions (*Euclid*, bk. i. props. 5 and 6) require separate proofs. And Mill pertinently asks: "If 'All equilateral triangles are all equiangular' is only one judgment, what is the proposition that 'All equilateral triangles are equiangular'? Is it half a judgment?"[1] The U proposition is thus ambiguous, and though it may be convenient to take note of it for certain purposes (*e.g.* in immediate inference, p. 128), it would be misleading to treat it as a distinct form of proposition, analogous to A, E, I and O). We may, therefore, conclude that the four new forms of proposition are not a satisfactory addition to the four recognised by the older logic.

[1] *Examination of Sir W. Hamilton's Philosophy*, p. 440.

We have seen that the quantification of the predicate is the first step in the development of the equational theory of the proposition, and that it follows inevitably from the class view. But even if the predicate is quantified, the relation of quantity between subject and predicate becomes only a shade less indefinite than it is when the predicate is not quantified. Between 'All hawks are birds' and 'All hawks are some birds,' the only quantitative difference is that in the second proposition the possibility of the predicate 'all birds' is excluded. Except for this distinction 'some birds' is quite indefinite. The denotation of 'some birds' is indeed more indefinite than that of 'all birds.' We can, *e.g.*, assign classes into which 'all birds' may be divided; but we cannot do this in the case of 'some birds,' for 'some birds' may mean any one of several groups of birds. Thus 'All hawks = some birds' is as unsatisfactory an equation as 'X = any number or group of numbers except 1 or infinity.' Accordingly, if the proposition is to be an equation between two classes, it must be still further modified.

There are two ways in which we may try to increase the definiteness of the predicate's denotation. (1) We may put, instead of 'some' a definite number. For instance, we may say 'All S = 2/3 P' or '4/5 S = 3 P.' But when we have done this, we have (except in the simplest cases) destroyed the possibility of direct reasoning. We have turned the proposition into an arithmetical or algebraic equation, and we cannot reason from this in the same way as we can reason from 'All S is P,' etc. We must add to the expression of our reasoning at least one arithmetical or algebraic principle, some such principle being implied in the reasoning itself. Otherwise logic becomes merely a branch of mathematics, for we must proceed according to the laws of algebra and not according to the laws of logic. But logic cannot be, as Boole contended,[1] a branch of mathematics, for mathematics, like all other sciences, uses reasoning without giving general laws or standards of

[1] v. Jevons, *Elementary Lessons in Logic*, p. 191 ff.

reasoning, while logic gives forms and tests for all reasonings including those of mathematics. Further, even if we quantify numerically the predicate of the proposition, we do not make its denotation much more definite than when we use the sign 'some.' '2/3 P' or 'two-thirds of all birds' may mean any one of various groups of P's or of birds. We cannot, therefore, make the denotation of the predicate definite in this way. (2) The second way is to qualify the predicate, instead of quantifying it. This is the method adopted by Jevons.[1] To qualify the predicate is to say precisely what part of it the subject is. For instance, 'All hawks are birds' is stated, on the Hamiltonian theory in the form 'All hawks are some birds.' But *what* 'birds' are 'all hawks'? Evidently 'hawk birds.' We therefore qualify the predicate by adding to it the word 'hawk,' and the proposition becomes 'All hawks are hawk-birds.' And the general form of the proposition becomes $A = AB$, or $S = SP$. The denotation of the predicate has thus been made quite definite, and the proposition is expressed as a real equation. But in this equation P is either an analysis of what is already given in S or it is an addition of meaning to what is given in S. If it is an addition of meaning, the equation is destroyed, because the addition is not made to both sides. If 'hawk birds' meant more than 'hawks' there would be no true equation or absolute identity between the two. On the other hand, if the predicate adds nothing to the subject, there is no proper predication, and the proposition suggests a difference which does not exist. To remove ambiguity it would be necessary to qualify the subject as well as the predicate, and to say $SP = SP$, 'Hawk-birds = hawk-birds,' which would, of course, entirely destroy the proposition. As a *mathematical* equation $S = SP$ has a meaning; but that meaning is $P = 1$, which is certainly not the meaning of the logical proposition.

The equational theory accordingly turns out to be the *reductio ad absurdum* of the class view of the proposition; for, if we interpret the proposition as meaning that both

[1] *The Principles of Science*, ch. iii.

subject and predicate are taken in denotation or extension, we are practically compelled to adopt the equational theory; and the equational theory reduces the proposition to a meaningless tautology. We have already seen (p. 46 ff) that extension and intension are inseparable, and this is confirmed by the failure of the class theory of the proposition.

(*c*) The third theory, according to which both subject and predicate are taken in intension, is expounded by J. S. Mill [1] and is usually called the Attributive view. According to Mill, in every proposition in which subject and predicate are both connotative, the subject does not primarily refer to any definite class of things, each of which is known. It primarily means certain attributes, in virtue of possessing which an indefinite number of beings is referred to. And "the assertion" made by the proposition is "that the attributes which the predicate connotes are . . . possessed by each and every individual possessing certain other attributes; that whatever has the attributes connoted by the subject has also those connoted by the predicate; that the latter set of attributes *constantly accompany* the former set. Whatever has the attributes of man has the attribute of mortality; mortality constantly accompanies the attributes of man." [2] Mill, as we have seen (p. 46) regarded things as merely groups of attributes, and his view may therefore be expressed in the form: Every proposition states that certain attributes do or do not constantly accompany certain other attributes. But attributes, according to Mill, are all "grounded on some fact or phenomenon either of outward sense or of inward consciousness," and "to possess an attribute is another name for being the cause of, or forming part of, the fact or phenomenon upon which the attribute is grounded." [2] Things and their attributes are thus reducible to phenomena and groups of phenomena, and ultimately the proposition may be regarded as saying that

[1] *System of Logic*, bk. i. ch. v. § 4.
[2] *Loc. cit.*

one phenomenon or set of phenomena does or does not constantly accompany another phenomenon or set of phenomena.

The class view of the proposition implied that there is an identity or unity between subject and predicate so complete that the one may be said to be equal to the other, and that in the end they become indistinguishable (SP = SP). The attributive view is the exact contrary of this. Subject and predicate have no unity or identity with one another: the one just happens to accompany the other. This view, in short, lays full stress on the difference between subject and predicate. Its ideal is the purely synthetic proposition, while on the class view, the ideal proposition is purely analytic. But just as we found that the class view cannot eliminate difference without destroying predication, so we shall find that the attributive view cannot eliminate unity or identity without a similar result. For Mill's statement of the attributive view really implies a unity or identity between subject and predicate, which it does not explicitly recognize, just as the class view implies a difference which it does not explicitly admit. This will appear if we consider the meaning of the phrase, 'always or constantly accompany.' If we say that a group of attributes or phenomena (or even one attribute or phenomenon) 'always or constantly' does anything, we clearly imply that this group has a unity or identity which appears in a variety of different cases or instances. And if we say that one group of attributes 'always accompanies' another group, we must imply that they accompany one another within some more or less definite unity or system, *e.g.* in my mind or in all minds, at some place or in all places, in Greek mythology or in prose fiction or in poetry, etc. Otherwise predication becomes meaningless. The attributes or phenomena constituting the top of Mont Blanc constantly accompany those which constitute the top of Ben Nevis; but we cannot therefore say that the top of Mont Blanc is the top of Ben Nevis. And even accompaniment within a definite unity or system is not enough by itself to warrant predication.

For instance, attributes or phenomena frequently accompany one another in my mind, while I recognize that they do not belong to the same object and are not predicable of one another. Again, in the case of a particular proposition (*e.g.* Some S is P), interpreted by the attributive view as meaning 'The attributes or phenomena S sometimes accompany the attributes or phenomena P,' it should be possible to predicate any phenomenon occurring anywhere at any time of any other phenomenon occurring at the same time, *e.g.* that a typhoon in the Pacific Ocean is a motor-bus running in London. Such statements as these are, as we have seen (pp. 65-6), not propositions, but meaningless combinations of words ; and they are as meaningless when the predication is negative as when it is affirmative. Accordingly the proposition, if it is to be a real predication, must do more than state that certain attributes or groups of attributes, independent phenomena, do or do not accompany one another. The proposition, in short, must express a real unity in difference.

It is evident that only the first of the three views we have considered is consistent with the requirement that a proposition should express a unity in difference. And even the first view (the predicative view) is unsatisfactory in this respect, if it means that the subject is to be taken purely in extension and the predicate purely in intension. For the unity in difference must permeate the whole proposition. The unity cannot be found in one part of it and the difference in another. In other words, as we have seen, extension cannot be separated from intension, thing (or subject) from attributes. The three theories have all the additional defect that they are applicable only to categorical propositions. If we attempt to apply them to conditional propositions (*e.g.* If A is B, C is D), we must regard conditional propositions as merely combinations of categoricals, and the theories fail to give us any explanation of the relation between the categoricals (A is B, and C is D). Yet this relation is the very essence of the conditional proposition. We must, therefore, regard all these theories as inadequate,

although each of them may, within limits, be useful for various practical purposes.

(2) We may now consider another set of theories, suggested in answer to the questions : To what does the proposition as a whole primarily refer—to names, to ideas or to things ?

(*a*) Hobbes held the 'Nominalist' view, that the proposition expresses a relation between two names. He holds that the import of the proposition is the belief of the speaker that the predicate is a name of the same thing of which the subject is a name. And he even goes so far as to say that "the first truths were arbitrarily made by those that first of all imposed names upon things, or received them from the imposition of others. For it is true (for example) that *man is a living creature*, but it is for this reason, that it pleased men to impose both these names on the same thing."[1]

This view apparently implies that truth is merely verbal consistency, consistency in the use of names. This is certainly an element in truth, and it has a high importance in reasoning. But it has this importance only when we regard names, not as mere labels, but as having definite meanings. To be consistent in the use of names is to use names unambiguously, *i.e.* not to use the same name *as a term* in different senses in the same reasoning. This, however, is not mere verbal consistency, but consistency of meaning, consistency in our application of a name to things, ideas, qualities, meanings of any sort. No proposition is a relation between mere names. Every proposition refers to meanings. And the reference to things or meanings appears even in Hobbes's statement of the import of the proposition.

It is, of course, possible to express any proposition in accordance with the nominalist formula. But such an expression is manifestly inadequate. The proposition clearly means more than a statement about the relation

[1] Hobbes, *Computation* or *Logic*, ch. iii. sect. 8 (quoted by Mill, *op. cit.* Bk. I., ch. v., sect. 3).

between the *name* S and the *name* P. Even such propositions as ' Tully is Cicero ' are statements about the *meaning* of names, and not about names alone. It does not merely say that Tully is a name of the same thing of which Cicero is a name. If that were all, Tully and Cicero might mean anything. It is not a proposition except to someone who knows what Cicero *means*, and to such a person it is a statement of the meaning of the name Tully.

(*b*) The ' conceptualist ' theory, as it has been called by Mill, is the view that the proposition states a relation between concepts or ideas, *i.e.* that the proposition states a connection or a distinction between two ideas. The ground of this theory is the view that the knowledge and thinking of each person consist primarily of his own ideas. In some propositions we undoubtedly mean to refer to real things ; but the question whether we can ever be justified in making statements about real things, and if so in what sense we can make these statements, is a question of metaphysics or theory of knowledge, about which there is great difference of opinion. Some metaphysicians have maintained that there are no real things, except sensations and ideas, and others have held that we can only refer our ideas to real things by an act of faith. The judgments of such thinkers do not profess to be, in the first instance, statements about anything except ideas. On the other hand, every judgment is at least an idea, a thought, and many propositions are explicitly statements about thoughts, ideas, notions or concepts. If, for example, I say, ' The thought of it makes me shudder,' I am speaking of my own personal experience, of ideas in my own mind. And every proposition is concerned with notions or ideas *at least*. It may or may not be concerned with real things, and, as a general theory of the proposition must apply to all propositions, we can only say that all propositions state a relation between ideas.

Now, while one or more ideas are involved in every judgment, a judgment is not a combination of two independent

ideas. There are no such things as ideas unjudged; for such ideas, if they were possible, would be floating ideas, having no relation to any other ideas. It would be impossible for us to know such ideas or to assign any characteristics to them. For to know is to judge, and to assign any characteristics to them is to think that they *are* so-and-so and not something else. But if no characteristics can be assigned to such ideas, they are indistinguishable and are therefore as good as nothing. They are contradictory of the very nature of our mental life. In any judgment therefore we have one idea, appearing as a unity in difference; not one complete idea tacked on to another complete idea, but two fragments or elements held together as one.

Mill and Bosanquet have interpreted the conceptualist view in a way which does not do it justice. Mill,[1] for instance, supposes that the conceptualist statement of the proposition 'Fire causes heat' is 'My idea of fire causes my idea of heat.' And Bosanquet[2] speaks of 'My idea of the earth going round my idea of the sun.' This, of course, would make the theory absurdly self-contradictory, for there is an obvious reference to things other than ideas in the words 'causes' and 'going round.' Yet, if we are to avoid any reference to things, as distinct from ideas, we must interpret the proposition, on the conceptualist theory, as being 'My idea of S includes my idea of P' or 'The idea of S includes the idea of P,' 'The idea of fire includes the idea of causing heat,' etc. But if we have the idea of fire and it already includes the idea of causing heat, why should we say so? The judgment is tautologous. The whole of it seems to be already present in the idea of fire. And when we say, 'Fire causes heat,' we clearly mean more than an assertion about our idea of fire; we mean to speak of fire itself. If we intended to refer specially to our own idea, we should say so: 'My idea of fire is so-and-so.'

The judgment or proposition, then, always involves an idea or ideas. So far the conceptualist theory is right.

[1] *Op. cit.*, bk. i., ch. v., sect. i.

[2] *Logic* (Ed. 2), vol. i., p. 76.

But it is inadequate in so far as it fails to recognize that every idea must be an idea *of* something, that there are no such things as mere ideas, and that the judgment therefore cannot refer to ideas alone.

(*c*) We may now turn to the third view, that the proposition refers to things. Mill is a strong supporter of this view. Almost all our propositions, he holds,[1] are made with reference to reality—not with reference to ideas, classes or names, but with reference to phenomena, real things. By a 'thing,' as we have seen, he means a phenomenon or a group of phenomena. And all phenomena, he holds, are phenomena of 'outward sense or inward consciousness.' But as the phenomena of outward sense are, according to him, reducible to sensations and 'permanent possibilities of sensation,' all phenomena may be ultimately regarded as 'feelings or states of consciousness.' And, as we have seen, Mill regards the proposition as asserting or denying that certain of these phenomena or 'feelings' always or sometimes accompany one another.

We may agree with Mill that the proposition always refers to things. But we have already found that a thing cannot be satisfactorily regarded as an aggregate of phenomena, attributes or 'feelings' (pp. 46-7). A thing is more than a mere 'bundle of attributes,' as Mill[2] calls it. And, that being so, it is unnecessary for our present purpose to discuss the question whether or not phenomena are reducible to feelings or states of consciousness.

A more satisfactory view is that of F. H. Bradley, who says that "judgment proper is the act which refers an ideal content, recognised as such, to a reality beyond the act."[3] Every judgment involves an idea or concept, a universal meaning, and it refers this meaning to an objective reality. This view may be regarded as an enlargement of the predicative theory, so as to include what is best in the

[1] *Op. cit.*, bk. i., ch. v., sect. i.

[2] *Examination of Sir W. Hamilton's Philosophy*, p. 424.

[3] *Principles of Logic* (Ed. 2), vol. i., p. 10.

conceptualist theory and in Mill's view. The predicative view was that in judgment we have the attribution of a quality (predicate) to an individual object or class of objects (subject). But the subject of a proposition is not always a definite object or class of objects, *e.g.* 'Fire!', 'It rains,' etc. In these instances the subject is a vague undefined totality; but it is always treated as having reality of some sort. Further, in all propositions, even the most developed, those with the most definite subjects and predicates, the statement is always made with reference to a reality beyond the proposition itself.

Every proposition has a context, in which it is an element. It is made on certain grounds or in a certain universe of discourse, *e.g.* in the universe of physical reality, the universe of Greek mythology, that of mathematics, that of poetry, that of a particular novel, etc. The proposition is so bound up with its universe of discourse, that the very meaning of the proposition, its 'ideal content,' is determined by its universe of discourse. The proposition 'Man is mortal' has different meanings when it is made in the universe or context of biology and in that of religion. It may be true in the one universe and false or doubtful in the other. But all universes of discourse, so far as they are systematic (*i.e.* so far as they have a meaning, for all meanings are to some extent systematic), are parts of reality. As an extreme instance we may take the children's rhyme:

'If all the world were chocolate,
And all the sea were ink,
And all the trees were bread and cheese,
What would there be to drink?'

The assertion that there would be nothing to drink depends on the real systematic nature of chocolate, ink, and bread and cheese, in which the quality 'undrinkable' is an element. Biology, religion, mathematics, mythology, poetry, fiction, etc. are all real. They are elements in the one all-comprehensive system of reality. Thus in every judgment the whole meaning is referred to reality, or is regarded as an adjective or qualification of reality.

Proximately the reality to which it is referred is a particular system or universe of discourse; ultimately it is reality as a whole. Thus, with the conceptualists, Bradley maintains that the proposition is concerned with ideas: every proposition has an ideal content. But against the conceptualists he holds that the essence of the proposition is not the connecting together of two ideas, but the reference of one idea (an 'ideal content' or meaning, which is a unity in difference) to reality. Similarly, Bradley agrees with Mill in holding that the proposition is not concerned with mere ideas, but with things, with reality. But he differs from Mill in maintaining that the real is not to be regarded as a mere collection of particular phenomena, quite independent of one another, but as one system (including lesser systems), and accordingly he differs also from Mill's view that the essence of the proposition is to connect together independent phenomena, just as he differs from the conceptualist view that its function is to connect together separate ideas.

Those who hold this theory do not, of course, mean to imply that propositions have no *explicit* subjects. The explicit subject of the categorical proposition indicates some point or element in the context (in reality as already known) which the maker of the proposition desires specially to insist on. Thus the explicit subject of a proposition may vary according to the particular context, *e.g.* 'That cloud is like a hill,' 'That object like a hill is a cloud.' In order to avoid misunderstanding on this point, Bosanquet has modified Bradley's statement, and he defines judgment as "the reference of a significant idea to a subject in reality by means of an identity of content between them." [1] This definition has the advantage that it makes the reference of the judgment to reality more precise, (*a*) by suggesting that the reference is not merely to reality as a whole, but to a 'subject,' a more or less definite element, in reality, and (*b*) by indicating the nature of the reference, viz., an identity of content between the significant idea and the

[1] *Essentials of Logic*, p. 79.

'subject.' This last feature is important, because it prevents us from supposing that, on this theory, judgment is something external to reality or that in judging we may refer ideas to reality without any ground of connection between the two. When I judge that this horse is white, I am asserting that the meaning of the idea 'white horse' has an identity with the object to which I refer, and in virtue of this identity another person, who understands English, can grasp the meaning and reference of my idea, and can verify or challenge my judgment by observation of the object. If there were no such identity of content, anything might be predicated of anything else, and ideas would have no meaning.

This seems to be the most satisfactory solution of the problem of the import of the proposition. It recognizes that names or words as used in propositions must stand for realities grasped by thought. It implies that judgments are not isolated acts of thought, but that each is an element in a rational system, in which it finds its ground. Further, as it gives a deeper interpretation to the predicative theory by going beneath the distinction of the proximate subject from the proximate predicate (S and P), it applies not merely to categorical, but also to conditional propositions (which are not of the S-P form), and it offers a basis for a much more complete and satisfactory classification of propositions than that of the traditional logic.

EXERCISE VI

1. Indicate briefly the main theories of the import of propositions and discuss any *one* of them in detail.

2. Explain the nature and discuss the adequacy of the predicative view of the import of propositions.

3. Do you consider the theory that a proposition expresses a relation between classes, (*a*) useful for any purpose, (*b*) a satisfactory account of the nature of propositions?

4. Explain and examine the doctrine of the quantification of the predicate.

5. Explain, illustrate, and consider critically the theory that the proposition can be regarded as an equation.

6. What is meant by a concept? Explain the conceptualist view of the import of propositions. In what sense, if any, are there two concepts in every proposition?

7. By an analysis of two or three typical propositions bring out Mill's view of the import of propositions. Consider its adequacy.

8. Discuss the doctrine that all propositions refer to reality.

9. Which theory of the import of propositions do you consider most satisfactory, and why?

CHAPTER VII

CONDITIONAL PROPOSITIONS AND THEIR IMPORT

In Chapter V we discussed categorical propositions, *i.e.* propositions expressed absolutely, without conditions. We have now to consider propositions which include a condition or conditions. Of these two main kinds are generally recognized, the hypothetical proposition and the disjunctive proposition. The hypothetical proposition is usually expressed in one of two forms : (*a*) If A is B, C is D ; (*b*) If A is B, A is C. 'A is B' expresses the condition and is called the antecedent ; 'C is D' or 'A is C' expresses what is asserted under the condition and is called the consequent ; *e.g.* 'If Aristotle is right, slavery is justifiable' ; 'If life is full of distraction, it is exhausting.' The first of these propositions asserts that slavery is justifiable, not absolutely, but under the condition that Aristotle's opinions on the subject are right ; the second asserts that life is exhausting, under the condition that it is full of distractions. These two forms are, of course, merely typical forms, to which it is convenient to refer for purposes of discussion. In these forms the antecedent and the consequent are both affirmative ; but we may have propositions of the same general type, in which the antecedent or the consequent or both are negative, *e.g.* If A is not B, C is D ; If A is B, C is not D ; If A is not B, C is not D ; If A is not B, A is C, etc. A hypothetical proposition is affirmative, when its consequent is affirmative, whether or not its antecedent is also affirmative ; and it is negative when its consequent is negative, whether or not its antecedent is negative.

There are two corresponding typical forms of the disjunctive proposition: (*a*) Either A is B or C is D; (*b*) Either A is B or A is C. 'A is B' and 'C is D' (or 'A is B' and 'A is C') are, in this case, not antecedent and consequent but alternatives. The proposition asserts at least that if one of the alternatives is not true, the other is true, *e.g.* 'This is either a moth or a butterfly' means at least that if it is not a moth it is a butterfly, and if it is not a butterfly it is a moth; 'Either slavery is justifiable or Aristotle is wrong' means at least that if slavery is not justifiable Aristotle is wrong, and if Aristotle is right slavery is justifiable. But some logicians maintain, while others deny, that the disjunctive proposition also asserts that if either of the alternatives is true, the other is not true, *e.g.* 'If this is a moth, it is not a butterfly,' etc.; 'If Aristotle is wrong, slavery is unjustifiable,' etc. This difference of opinion among logicians will be discussed later (pp. 99-102). This interpretation of the disjunctive proposition, however, makes it clear that it is really a conditional proposition, although this is not quite apparent in its form. It should also be noted that a disjunctive proposition may include more than two alternatives, and that any of the alternatives may be negative. We cannot, however, speak of the whole proposition as negative.

We are now in a position to consider the import of conditional propositions, about which there is considerable difference of opinion. We must first inquire how they are related to categorical propositions. Is the categorical proposition entirely unconditional? A little inquiry will show that it is not. Take any categorical proposition, *e.g.* 'I am going to London on Tuesday.' This statement is evidently not unconditional. It implies such conditions as that I shall be alive on Tuesday, that I shall not have a serious illness at that time, that there are means of transport to London and that they will not be completely disorganized by war, strikes, etc. These conditions and many others are not expressed in the proposition, because

they are so obvious and normal that it is unnecessary to state them. When the conditions are not obvious and normal, we do state them, and the proposition becomes conditional, *e.g.* 'If a certain meeting is called for Tuesday, I shall go to London on that day.' Categorical propositions are always made under some implied conditions. No statement can have meaning without a context expressed or implied ; and even when there is an expressed context (as in a conditional proposition), there is always a further context implied. The context conditions the statement and includes its ground or premise. Thus the proposition, 'All men are mortal,' has no meaning unless one knows to some extent what a man is, *i.e.* unless one realizes a man more or less definitely as a being belonging to some particular world or system of things. The statement, for instance, implies that men are not mere creatures of the imagination, like the Struldbrugs in *Gulliver's Travels*, that they are not mere ideas in our minds without any corresponding reality and so on. If we say, 'Centaurs are combinations of man and horse,' we are now referring to an entirely different system. We make the statement on the understanding that centaurs are not to be regarded as beings which exist in the ordinary physical world, but as beings which exist in the world of Greek mythology. In short, every proposition, as we have already seen, is made with reference to some universe of discourse, some system of connected objects of thought, within which alone the proposition is asserted as true, beyond which it is either false or doubtful. If the truth of the proposition is challenged, the question can be decided only by examining the implied grounds on which it is stated, in order to discover whether it is consistent with the system or universe of discourse which conditions it. Thus the existence of a centaur is manifestly inconsistent with the conditions of physical life, as a system or universe of discourse, because (taking only two out of many points) the body of a horse develops more quickly than that of a man and a horse is short-lived in comparison with a man, so that to suppose

the two actually combined would be inconsistent with our knowledge of biology. But in the universe of mythology the conditions of physical life do not count; and the centaur is consistent with that universe, for we read of centaurs in mythological stories.

In most cases the implied conditions of a categorical proposition may be assumed as evident, and they are therefore not stated. The proposition usually occurs in some connected conversation or writing about a more or less definite 'subject' which is assumed throughout. Even when the proposition is the first statement in a book or a conversation, it usually contains some element which points its reference in a certain direction. But occasionally the universe of discourse is misunderstood and it is necessary to state it explicitly. 'I think Maeterlinck is wonderful' may get the response 'I have never been there' or 'Is Maeterlinck a man or a place?' The first of these responses assumes the wrong system of discourse, the second asks that the system may be explicitly indicated. Had the original statement been 'I think Maeterlinck is a wonderful writer,' the implied reference would have been sufficiently indicated. Again, if I say 'A straight line is the shortest distance between any two points,' the implied condition is that the line connecting the points lies in the same plane as the points themselves. If I mean to refer to points in the surface of a sphere, this condition should be explicitly stated. And, of course, the proposition is true in the one case and false in the other.

Accordingly, the difference between a categorical and a hypothetical proposition, while it is a real difference, is a difference of degree. The hypothetical proposition differs from the categorical in being more fully developed, more definite and explicit. 'All men are mortal,' for instance, might, so far as its form is concerned, be regarded as meaning merely that all men who have been actually observed are mortal. It is indistinguishable in form from such propositions as 'All European nations have armies' or 'All the States in the American Union elect senators

who believe in the Monroe doctrine.' But there is a very important difference between the first of these propositions and the other two. 'All men are mortal' means that if there is a man anywhere and at any time, past, present or future, he is mortal. But the other propositions do not mean that every European nation has always had and will always have an army or that every American State has always elected and will always elect senators who believe in the Monroe doctrine. They are statements of limited application, based on the observation of a definite number of individuals (nations or states) at a particular time. There is something in the very nature of man as a living being which makes him mortal; there is nothing in the very nature of an American State to make it elect senators with certain opinions. 'All men are mortal' is thus a truly universal proposition, while the other two propositions are merely general or enumerative. This distinction was first pointed out by Aristotle, and some modern logicians indicate it by describing truly universal propositions as 'generic.'

Now if we state the generic or truly universal proposition in the hypothetical form, we make clear its universal nature, *e.g.* 'If man, then mortal,' or 'If any being has the characteristics of man, it has the characteristic of being mortal.' The characteristic of mortality is asserted to be dependent on some characteristic of humanity. On the other hand, we cannot say that if there is anything having the characteristics of a European nation, it must have the characteristic of possessing an army. The antecedent of the hypothetical proposition indicates the ground or some part of the ground on which the consequent may be asserted, *e.g.* 'This being is mortal, if or because it has the characteristics of man.' But the general proposition, *e.g.* 'All European nations have armies,' is also dependent on some ground or reason, which is not indicated in the proposition itself. This ground may be merely that we have enumerated the nations and found that each happens to have an army, just as we might count a dozen men and find that each of them is wearing a silk hat. Or it may be our recognition that the present inter-

national relations of the European nations, including their relations to other nations, make it necessary for each of them to maintain an army. In either case the proposition is implicitly conditional. In the case of the first suggested ground, the condition is 'If our enumeration and observation is correct'; in the case of the second it is 'If or when the international relations of the European nations have such and such characteristics.' If the proposition is stated on the first ground, the condition may be assumed without being expressed, until the proposition is challenged; but if it is stated on the second ground, the condition ought properly to be expressed and the proposition should be stated in a hypothetical form. For in this case the characteristic of having an army is regarded as essentially depending on certain other characteristics.

It is thus evident that the hypothetical proposition is a development of the categorical. In short we have, in the transition from the categorical to the hypothetical proposition, what may be described as an evolution of the copula. The real copula of the categorical proposition is the connection between its subject and predicate. This connection is the ground on which the proposition is asserted. In the categorical proposition, the copula or connection is implicit; in the hypothetical it becomes to some extent explicit. This evolution of the copula is a gradual development or explication of the reasoning, of which the categorical proposition is the conclusion. For instance, consider Aristotle's example, 'The moon, being overshadowed by the earth, is eclipsed.' This proposition may be regarded as a development of the proposition, 'The moon is a luminary which suffers eclipse,' by stating the copula explicitly. 'Being overshadowed by the earth' is the real copula, and it is evidently the ground on which the proposition is made, a condition of its truth. But the proposition, 'The moon, being overshadowed, etc.' is really a hypothetical proposition = 'If (or when) the moon is overshadowed, etc. it is eclipsed.' And, as we saw (p. 12), such a proposition can be put into the form of a reasoning.

This instance shows that it is possible to express hypothetical propositions in the categorical form. 'If A is B, it is C,' becomes 'A, being B, is C.' But most hypothetical propositions are less easily reduced to the categorical form and the reduction can usually be effected only by considerable distortion of language. We have, *e.g.* to say 'The fact of A being B involves (or is a condition of) A being C' or 'The truth of A being B involves that of A being C.' The hypothetical proposition, though it differs only in degree from the categorical, is really a higher type, and as a general rule the reduction of the hypothetical to the categorical form makes the proposition more obscure and unnatural.

The reason of this may throw further light on the relation between the two kinds of proposition. The subject of the categorical proposition usually stands for an individual object or group of objects. Now ordinary, unscientific, knowledge is to a large extent concerned with individual objects and groups of objects. In the early stages of knowledge we are mainly engaged in trying to find out the qualities of things, and we are in the main content to know their more obvious qualities and those which it is useful to know for practical purposes. We do not inquire deeply into the reasons of things ; and we are mainly occupied in acquiring certain kinds of skill and in accumulating materials for scientific investigation and reflection. To this stage of knowledge the categorical proposition is most appropriate. It is sufficient for the expression of ordinary facts, for the attribution of qualities to objects. But scientific knowledge is not content to deal with objects as individual things, having qualities or characteristics. The aim of science is to unify, to connect, to go beyond the individual object to the general type of which it is an instance, and thus to pass from the limited class, consisting of a group of objects, each of which is separately observed and known, to the universal class of indefinite extent, the class which is fixed and limited by the type alone and which may include countless individual objects and instances that

have never been actually observed. This general type is not an individual object or a group of individual objects. It is not a thing which we can ever find in nature or in our own minds, not a thing which can be presented to our senses or pictured in our imagination. It is really a system of relations, having a certain unity or essential connection.

The difference between knowledge of individual objects and knowledge of general types may be put in this way. When we attribute various qualities to particular objects as mere particular objects, we do not necessarily mean to suggest that there is any relation or connection between these different qualities themselves, except the fact that they happen to belong to the same object. If I say of a man that he is white, bearded, dressed in a frock coat, tall, reading, mortal, courageous, stupid, hot-tempered, etc., I do not mean to suggest that there is any special connection of any one of these qualities with another, *e.g.* that his wearing of a frock coat is dependent on his being mortal, or that his hot temper is a consequence of his having a beard. I confine myself to particular, independent statements. The subject may be the same throughout ; but each predicate stands by itself, and accordingly there are as many categorical propositions as there are predicates. This state of mind, in which knowledge is of independent facts, is the state of mind or knowledge which expresses itself adequately in categorical propositions. On the other hand, when we regard individual objects, not as mere individuals, but as instances of a certain type, we mean that certain characteristics or qualities are connected with one another, in such a way that one is dependent on another or that they are interdependent. And this systematic relation of qualities to one another requires for its expression a hypothetical proposition. If, *e.g.*, we mean to suggest that there is a connection between a man's hot temper and his being rash, we do not merely say that he is rash *and* hot-tempered, but we say, 'If he is hot-tempered, he is rash.' The hypothetical proposition is thus the expression of know-

ledge at a higher stage than that which appears in the categorical.

In the disjunctive proposition we have a form that is still higher than the hypothetical. Indeed the knowledge to which the disjunctive proposition is appropriate is knowledge in its highest form. In the hypothetical proposition the consequent follows from (or is dependent on) the antecedent; but the antecedent does not necessarily follow from the consequent. When we say, 'If A is B, C is D,' we mean that A being B is *a* condition of C being D; but we do not mean to exclude the possibility that there are other conditions of C being D. 'If a match is applied to gunpowder, the gunpowder will explode,' means that the application of a match to gunpowder is *one* condition of its exploding, but not necessarily that it is the only one. A spark from a flint or a discharge from an electric battery might explode it. In short, the antecedent of the hypothetical proposition expresses a condition, but not a condition *sine qua non*, *i.e.* it expresses a condition, which being given, something else follows, but not a condition, which being absent, the other thing is absent also. Now our knowledge of a thing is not complete until we can assign, not merely one or another condition under which it happens or exists, but *the* condition of its happening or being what it is, not merely *a* cause, but *the* cause. In other words, the hypothetical proposition, as we have seen, implies some knowledge of a system. But a system always implies elements which are mutually conditioned. The famous Indian explanation of the world as resting on an elephant, and the elephant on a tortoise, and so on, fails because it is unsystematic. It is an example of one-sided dependence or conditioning. If we say that A is the effect of B, and B is the effect of C, and C is the effect of D, and so on, we have started on an infinite regress and we can never find the cause of anything. Similarly if we suppose that the evidence for a proposition is another proposition, and the

evidence for that is a third proposition, and so on, we make all proof impossible.

We may put the same thing in another way. 'If A is B, C is D,' means that always when A is B, C is D, and sometimes when C is D, A is B. But evidently our knowledge of the matter is not complete until we can say exactly on what occasions, in what circumstances, or under what conditions, the fact of C being D implies that of A being B. Take, for instance, the case of a scientific inquiry into a death by poisoning. If a man takes arsenic in certain quantities death will follow. But death may be caused in other ways, and death by poisoning may be the result of other poisons. The knowledge expressed in the hypothetical proposition is therefore imperfect. We have full knowledge only when we can say 'If this particular kind of death has happened, the dead man has taken a certain quantity of arsenic.' We must not merely know the conditions under which death from arsenical poisoning takes place; we must, if our knowledge is to be complete, know also the conditions under which a particular kind of death must have been caused by arsenic. In short, we must be able to say 'If A is B, C is D, and if C is D, A is B.' Antecedent and consequent must be reciprocal, mutually dependent; we must be able to say, when either is, the other is, and when either is not, the other is not.

The disjunctive proposition expresses knowledge which has attained to this degree of definiteness. The hypothetical proposition means that if A is B, C is D, and also that if C is not D, A is not B; for evidently if 'A is B' is a condition of C being D, and if we deny that C is D without denying that A is B, then A is B will be a condition both of C being D and of C not being D. In other words, 'A is B' will both be and not be a condition of C being D. On the other hand, the hypothetical proposition does not mean that if C is D, A is B or that if A is not B, C is not D; for the antecedent expresses *a* condition, but not necessarily the only condition, of the consequent. But if we take a disjunctive proposition, such as 'Either A is B or C is not D,'

we can argue from the affirmation of either alternative to the denial of the other, or from the denial of either to the affirmation of the other. That is to say the given disjunctive proposition means *(a)* If A is B, C is D; *(b)* If C is D, A is B; *(c)* If A is not B, C is not D; *(d)* If C is not D, A is not B. It should be noted, however, that there are propositions which are apparently, but not really, disjunctive. In English the word ' or ' is used in two distinct senses, *(a)* as corresponding to the Latin *aut*, meaning a relation of exclusion between two things or statements, and *(b)* as corresponding to the Latin *vel*, meaning that the statements are practically equivalent to one another, and that you can take whichever you please. In this sense, ' or ' is similar in use to the phrase, ' that is.' An example of *(a)* is ' The train will either arrive in time or be late.' ' This is a common ranunculus or buttercup ' is an example of *(b)*. In disjunctive propositions ' or ' must, of course, be interpreted in the first sense, as indicating exclusion between the alternatives; and the disjunctive proposition is usually expressed in the form ' either '—' or.' When ' or ' indicates equivalence, we do not as a rule say ' either.'

There has been considerable difference of opinion on the question whether the alternatives in a disjunctive proposition must be completely exclusive of one another or whether they may be only partially exclusive. If the alternatives are completely exclusive of one another, they are the only possible alternatives (*e.g.* Either A is B or A is not B), and the disjunction is therefore exhaustive. If the alternatives are only partially exclusive of one another, then there is another possible alternative, and the disjunction is not exhaustive; *e.g.* in the proposition ' He is either a knave or a fool,' the disjunction is not exhaustive, for there is a third possibility, namely, that he is both a knave and a fool. If we include this third possibility, the disjunction becomes exhaustive: ' He is either a knave or a fool or both.' We have assumed so far that the disjunction must be complete

and exhaustive; but many logicians (including Mill, Jevons, Keynes and others) maintain that this is not necessary, and they point out that we often use disjunctive propositions or statements in which the alternatives are neither mere equivalents, on the one hand, nor completely exclusive opposites, on the other, *e.g.* the proposition, 'He is either a knave or a fool.' This is a very important question, because, if this second view is true, the disjunctive proposition does not express a reciprocal conditioning between its alternatives. For on this view we must interpret the proposition 'Either A is B or C is D' as meaning that if A is not B, C is D, and if C is not D, A is B, but not that if A is B, C is not D, and if C is D, A is not B.

Now, if we regard it from a purely formal point of view, without any reference to its matter, to the particular meaning it expresses, it is undoubtedly true that to insist on a complete disjunction is to make the disjunctive proposition useless. For formally the only complete disjunction between two alternatives must be the disjunction between A is B and A is not B. And we hardly ever use a disjunctive proposition of that kind, *e.g.* 'A horse is either a quadruped or it is not a quadruped.' But this merely shows that the purely formal view of the proposition is a wrong view. For, even supposing that the alternatives in a disjunctive proposition are only partially exclusive, you cannot ascertain this in any individual proposition by a mere reference to its form. It is the actual meaning or matter of the proposition that shows it. 'Either A is B or C is D' might (so far as its *form* is concerned) mean 'Either I am too old a bird to be caught with chaff or decortications of the golden grain are spread to allure the aged fowl in vain.' And these two alternatives are obviously not exclusive in any way: denial of the one would not involve affirmation of the other. This points to the fact that every actual disjunctive proposition states alternatives within a system of some sort, and the nature of the system determines whether or not the alternatives are completely exclusive. *In other words, the disjunction is*

exhaustive when the alternatives are the only alternatives which the nature of the system allows. The propositions (*a*) 'This is either a moth or a butterfly' and (*b*) 'He is either a fool or a knave,' are exactly the same in form; but we regard the first as stating completely exclusive alternatives, while the second allows a third possibility. The reason is that proposition (*a*) obviously refers to a system of biological classification which distinguishes sharply between moths and butterflies, so that no one thing can be both, while the proposition (*b*) refers to the system of human character and intellect, in which knavery and folly are not altogether incompatible.

Propositions like (*a*) are quite unambiguous, provided that we know exactly the nature of the system to which they refer. In the case of (*a*) the reference is obvious. But propositions like (*b*) are ambiguous, until we know their exact reference. The context of (*b*) is not made obvious by the statement itself. If the proposition means merely to assign a person to one of two classes of men (as in 'He is either a Frenchman or a German'), the alternatives are not completely exclusive. But if that is the meaning of the proposition, it is obviously untrue, and no one who knows what he is talking about would make it, except for purposes of deception. In actual use the proposition (*b*) is most probably interpreted as an explanation of the conduct of a certain person or as the basis of an inference as to his probable conduct. In the first case, it may mean that either alternative accounts for what he has done, and the alternatives are treated as equally satisfactory explanations. You may choose whichever you like. The question whether he is merely a knave or merely a fool or both is irrelevant to the particular problem. And this is also irrelevant in the second case. If he is a knave, he will steal my money, and if he is a fool he will lose it. But he is either one or the other, and I shall therefore lose my money if I entrust it to him. If he is both knave and fool it makes no difference; but, of course, it would make a difference if he were neither. Propositions like (*b*) are thus, when

taken by themselves, ambiguous, just as many categorical statements are. But if they are to be treated as logical propositions, they must be made unambiguous by being put into strict logical form. And it is surely more logical to make the ambiguous unambiguous than to take the ambiguous as your standard and classify unambiguous propositions as if they were ambiguous. If you treat proposition (*b*) as the standard, of which (*a*) is an instance, you make it impossible to infer from (*a*) that if this is a moth, it is not a butterfly. Yet this is obviously a legitimate inference if (*a*) is true.

It has been pointed out by Bradley [1] that exclusion is really common to all the senses in which we use the word 'or.' Even when we take it as meaning *vel*, or practical equivalence between alternatives, it has to some extent an exclusive sense. You may for instance call a thing by either of two names, but not by both at once. 'This is a common ranunculus or buttercup,' but it is not a 'ranunculus-buttercup.' And exclusion is clearly implied in all other cases. Accordingly if a disjunctive proposition is to be *unambiguous, which it must be if it is part of a reasoning* which claims to be valid, the alternatives must be stated as really exclusive, *e.g.* if the proposition, 'He is either a knave or a fool' is intended to mean that he may possibly be both, it should be stated in the form 'He is either a knave or a fool or both.'

Just as the hypothetical proposition may be expressed in categorical form, so the disjunctive may be resolved into hypotheticals. If the disjunction is regarded as complete or exhaustive, the proposition is reducible to four hypotheticals. Thus 'Either A is B or C is D' may be reduced to: (*a*) If A is not B, C is D; (*b*) If C is not D, A is B; (*c*) If A is B, C is not D; (*d*) If C is D, A is not B. If the view of Mill, Jevons and Keynes is taken, the proposition is reducible only to (*a*) and (*b*). But (*a*) and (*b*) are not independent propositions. Each is really a form of the

[1] *Principles of Logic* (Ed. 2) vol. i. p. 131 ff.

other. For in any hypothetical proposition denial of the consequent implies denial of the antecedent. Thus 'If A is not B, C is D' implies that 'If C is not D, A is B.' (*c*) and (*d*) are also forms of one another for the same reason. Thus, if the disjunction is regarded as complete, the disjunctive proposition is really reducible to two hypotheticals, viz. 'If A is not B, C is D' and 'If C is D, A is not B,' or 'If A is B, C is not D' and 'If C is not D, A is B.' On the view of Mill, it is equivalent only to one hypothetical proposition, 'If A is not B, C is D,' or 'If C is not D, A is B.' *E.g.* 'He is either a knave or a fool' becomes 'If he is not a knave, he is a fool,' which implies that if he is not a fool, he is a knave. This is an additional reason for rejecting the view of Mill; because this view evidently implies that the disjunctive proposition is merely equivalent to a hypothetical proposition. It leaves out the peculiar characteristic of the disjunctive and practically makes it a superfluous form.

Though the disjunctive proposition may be analysed into two or more hypotheticals, it is nevertheless one statement and expresses one relation. All relation implies system. It is the connection and distinction of elements in a system. The categorical proposition expresses relation within a system which is comparatively indefinite or imperfectly known. In the hypothetical proposition the relation expressed and the system within which it is expressed are more definitely known. But the relation is still known only on one side. It is like the knowledge of Charles Lamb's Chinaman, 'If my house is burned down, I shall get roast pig.' In the disjunctive proposition, the system is so fully known that the relation is known on both sides, *i.e.* it is known as reciprocal, which all relations must really be. We know not merely that we may get roast pig by burning the house or putting the pig in a furnace or in some other way; but that, if certain definite conditions are present, we get roast pig, and if these same conditions are absent we do not get it.

On the basis of the view we have taken of the import of

the various kinds of propositions, it is possible to make what may be described as an evolutionary classification of propositions, proceeding from the most rudimentary type of categorical proposition to the fully-developed disjunctive. Bosanquet in his *Logic*, has set forth and elaborately expounded such a classification.[1]

EXERCISE VII

1. (*a*) Explain and illustrate the difference between an enumerative and a generic universal. (*b*) Discuss the relation of the proposition 'All S is P' to the proposition 'If S, then P.'

2. How far can the hypothetical proposition be reduced to a categorical form? Reduce the following propositions to their categorical equivalents and discuss the adequacy of the categorical form to express their meanings:
(*a*) If the weather is foggy, the train is late.
(*b*) If a metal is heated, it expands.
(*c*) If a tariff is introduced, the price of imported articles will rise.
(*d*) If you ask him, he is sure to refuse you.
(*e*) If you had come last night, you could have seen for yourself.

3. (*a*) In what sense is the disjunctive proposition an advance on the hypothetical? (*b*) Express the following, as far as possible, in their equivalent hypothetical forms:
(*a*) Either he has forgotten or he is deliberately lying.
(*b*) A line is either straight or curved.
(*c*) The man who did that was either drunk or stupid.
(*d*) He is either a Protestant or a Catholic.

4. What different views have been taken as to the meaning of 'or' in disjunctive propositions? Explain the logical point at issue in the controversy and indicate your own view on the matter.

5. 'All propositions are both categorical and conditional.' In what sense, if any, do you consider this statement true?

6. Discuss the relation between the categorical, the hypothetical, and the disjunctive proposition. What kind of knowledge is each form of proposition specially suited to express?

7. (i) Consider carefully whether there is any difference of implication between the following: (*a*) If A is not B, then it is not C; (*b*) Either A is not C or it is B.
(ii) If B is substituted for C in the above, (*a*) becomes a tautology, and (*b*) becomes the Law of Excluded Middle. Consider the significance of this for your answer to (i).

8. 'Hypothetical propositions are really inferences.' Explain and discuss.

[1] For his general scheme, *v.* vol. i. (Ed. 2), p. 86.

CHAPTER VIII

THE LAWS OF THOUGHT

HAVING in previous chapters considered some of the chief characteristics of the elements of reasoning (terms and propositions), we may now turn our attention to the problem of the basis of reasoning, *i.e.* the principles on which reasoning proceeds. These principles are usually expressed in the Laws of Thought. It is important to keep in mind that these laws are principles of reasoning, because considerable confusion has entered into the discussion of them, as a result of assuming that they refer primarily to terms or to propositions. The Laws of Thought are usually stated in the form of propositions; but they are principles of all knowledge, and, as we have seen (p. 18), propositions are not independent pieces of knowledge but elements in reasoning.

Three Laws of Thought are generally recognized, the Law of Identity, the Law of non-Contradiction (frequently but unhappily called the Law of Contradiction) and the Law of Excluded Middle. These laws were indicated in part by Plato and more fully set forth by Aristotle, and in modern times Leibniz added a fourth law, that of Sufficient Reason. The Law of Identity is frequently stated in the form: A is A; the Law of non-Contradiction: A cannot both be A and not-A; the Law of Excluded Middle: Everything is either A or not-A. Stated in this way the laws appear at first sight to be mere tautologies, which say nothing. But their real meaning has been obscured by these forms of expression, which differ considerably from those used by Aristotle. The Law of non-Contradiction,

for instance, is expressed by him in some such form as this: The propositions A is B and A is not B (or S is P and S is not P) cannot both be true together. Similarly the Law of Excluded Middle is put by him in the form: Of the two propositions A is B and A is not B, one must be true and the other false.

The significance of the laws will most easily appear if we consider their origin, taking as our guide a remark of Aristotle: "It is impossible for the same thing to be at the same time predicated affirmatively and negatively of the same thing in the same sense. This is the most certain of all principles. . . . For it is impossible for anyone to maintain that the same thing is and is not, as some think that Heraclitus says."[1] Now the characteristic tenet of Heraclitus was that the world is in a continual flux. He held that we cannot draw sharp lines between things. We cannot say that this or that object is unchangeably what it is. We must rather suppose that the world is like a river, which indeed is one, but of which no part ever remains the same for more than an instant of time. The world is continuous, all things are in ceaseless change, and the appearance of permanence is an illusion. On the other hand, Parmenides, a contemporary of Heraclitus, maintained the exactly opposite view, that everything is permanent, that all apparent change, motion and multiplicity in things is an illusion, that the only real thing is the abiding, unchangeable One, "a finite, spherical, motionless corporeal *plenum*."[2] In short, Parmenides held that there is an absolute distinction between 'what is' and 'what is not.' What is, is, and what is not, is not at all. "It cannot be thought nor uttered that 'what is not' is."[3] A view closely akin to that of Heraclitus was afterwards taken by some of the Sophists, who held that "man is the measure of all things," that there is no absolute and eternal truth, and that it is therefore impossible to assign to anything a definite permanent nature,

[1] *Met.* Γ, 1005*b*, 19.
[2] *v.* Burnet, *Early Greek Philosophy*, (Ed. 3), p. 182.
[3] *v.* Ibid. p. 175.

to say that certain characteristics essentially and universally belong to it or to give it a complete definition. The definitions of things, according to the Sophists, are entirely relative. A thing may be defined in one way at one time and for one purpose, and in another way at another time and for another purpose. Heraclitus held that a thing has no permanent qualities, because it is in continual change from moment to moment; while the Sophists maintained that a thing has no essential qualities, for its qualities are what men take them to be, and our ideas of the qualities of things are continually varying with the changes in our knowledge and the variety of our purposes.

Now the fundamental presupposition of Plato and Aristotle was that things have a definite nature, which can be described or stated, that each thing (or rather each species of thing) has certain essential characteristics which universally belong to it, and that accordingly a real definition of the species is possible, a definition in which these qualities are stated. Things do not vary indefinitely. In spite of such continual changes in the world that nothing remains exactly the same in any two successive moments; in spite of the variety of ways in which things appear to us, each thing (or species of things) has a definite nature, in virtue of which it is the same for different people or for the same person at different times. The mistake of the previous thinkers consisted in assuming that a thing must either be always absolutely the same, without a shadow of difference from moment to moment or from person to person, or that it must have no identity at all, but be really a succession of entirely different things. The view of Plato and Aristotle was that a thing may remain the same through certain changes or differences, that a thing may be permanent, although it changes. The world is not a continual flux, if by that is meant that there is in it no sameness or permanence. If the world were such a flux, there would be no distinction between truth and error. Predication would be impossible. We could say nothing about anything, for before the words were out of our mouths, the thing we are

speaking about would have changed, and our statement would therefore be no longer true.

The Law of Identity is the formal statement of this principle, on which all continuous speech, thinking and reasoning depends. 'A is A' means that a thing remains the same (or must be assumed to remain the same) throughout our discussion or thinking about it. The Law of non-Contradiction is a negative way of expressing the Law of Identity. If a thing has a definite nature, if it remains the same in various circumstances, it cannot at once have and not have the same quality. A term, for instance, must not be used in two or more different senses within the same context of reasoning. A tree in summer (with leaves) is very different from a tree in winter (without leaves), and yet it may be the same tree. The Law of Identity means that if we are reasoning about a tree, we must take it definitely in one sense or another. If we speak of it barely as a tree (without reference to its appearance in summer or in winter) we must refer only to those characteristics of the tree which are independent of the seasons. If our reasoning implies that the tree is in leaf or that it is not in leaf, we must make it clear that this is so. And the Law of non-Contradiction requires that we must not begin, say, by referring definitely to the tree as in leaf and afterwards go on to speak of it in the same reasoning as if it were not in leaf.

It is important to notice the grounds on which these laws are accepted as valid. Many logicians say that they are valid because they are self-evident or axiomatic. But this is a very insufficient ground for the validity of laws which are so universal and fundamental. What is self-evident to one man is not self-evident to another, and many people regard as self-evident propositions statements which are not even true. In this particular case, the contrary arguments of Heraclitus and Parmenides are sufficient to show that the laws are not self-evident to everyone. Others maintain that the Laws of Thought are generalizations or inferences from experience, which is the ground of their validity. But all generalization or inference is grounded on these laws

and presupposes their validity, and it is absurd to say that the ground of all inference is itself a result of inference. The error of those who hold that the laws are generalizations from experience arises from a failure to distinguish clearly between two different questions: (1) Are these laws actually the principles on which all accurate thinking proceeds? and (2) How do we come to *formulate* these laws? We can and do observe, judge and reason without being able to state these laws and without having ever heard of them; and the formulation of these laws is no doubt the result of considerable thinking. But this does not mean that the laws themselves are apprehended only when we are able to formulate them. On the contrary, we cannot think accurately or know truly except on the assumption (which may be unconscious) that these laws are valid; and this is the real ground of their validity. If these laws do not hold, all coherent speech, all knowledge and thought and all rational communication between different persons are impossible. This is the strongest possible ground for the Laws of Thought, for it means that these laws are so woven into experience that if they are supposed to be false all experience falls to pieces.

The significance of the Laws of Thought may be made clearer by reference to some possible misunderstandings. As regards the Law of non-Contradiction, for instance, it may be noted that in saying that a thing cannot both be A and not A, we do not mean that a thing cannot have more than one quality; but merely that it cannot both have and not have the same quality at the same time and in the same sense. Kingsley's 'Be good, sweet maid, and let who will be clever,' is apt to suggest to some minds the ironical conclusion, 'for it is certain that you cannot be both.' But evidently this does not necessarily follow. A man may be generous in some ways and mean in others; but he cannot both be and not be generous at the same time and in the same context. The Law of Excluded Middle has also been sometimes misunderstood in a similar way. This law is simply another way of stating the same principle as appears

in the Laws of Identity and non-Contradiction. The Law of non-Contradiction says that the propositions A is B and A is not B cannot both be true. The Law of Excluded Middle says that they cannot both be false, but one of them must be true and the other false. If they could both be false, we should evidently be able to say that the same characteristic B can be both negatively and affirmatively predicated of the same thing A, which would be a denial of the Laws of Identity and non-Contradiction and therefore a denial of the possibility of predication.

The main misunderstanding of the Law of Excluded Middle arises from assuming it to mean, not merely that one of the two propositions must be true and the other false, but that one of the two propositions must be *known* to be true and the other *known* to be false. This misunderstanding appears in various forms. It is often difficult to say whether at any given moment a certain quality B belongs to a certain thing A. Many qualities of things come and go by a gradual, almost imperceptible process, so that there are moments when one can hardly know whether the quality is there or not. Such moments, for instance, are the moment of death, when the quality of being alive passes almost imperceptibly away, the moment of waking or of sleeping, the moment of dawn, etc. At a given moment we may not know whether a man is awake or asleep ; but this does not really affect the Law of Excluded Middle, which merely asserts that of the two propositions 'He is awake' and 'He is not awake,' one must be true and the other false, whether or not we happen to know which of the two propositions is true. This misunderstanding of the Law of Excluded Middle appears in another form in certain fallacious arguments, *e.g.* in the fallacy of the inevitable. Someone might conceivably argue that, in the case of a dispute with a foreign country, diplomatic negotiation is useless, for either war is going to come or it is not going to come, and if war is going to come, talking is useless, while if it is not going to come, the talking makes no difference. The fallacy of this argument consists in unconsciously

identifying two distinct propositions: (1) 'Either war is going to come or it is not'; and (2) 'Either war is known to be coming or it is known not to be coming.' The first of these is a case of Excluded Middle; but the second is not, for there is a third or middle possibility, namely, that it is not known whether war is coming or not. The Law of Excluded Middle is also sometimes erroneously interpreted as meaning that anything A must have one of two opposite qualities, B or C, *e.g.* that everything must be either wise or foolish, either moral or immoral, either black or of some other colour. But the Law of Excluded Middle declares merely that everything must either have a certain quality or not have it. As we have seen, however, in dealing with negative terms (p. 31), there are certain terms which, in particular contexts, are entirely exhaustive, though in other contexts there is a possible third alternative. Thus, *e.g.* every piece of stone must be either black or of some other colour; but if the subject of our propositions is 'thought' or 'courage,' etc., there is the third possibility that it has no colour at all.

From what has been said so far it must be clear that the three Laws of Identity, non-Contradiction and Excluded Middle are really different forms of one law, which we may describe as the Law of Identity or, more correctly, of Identity in Difference. The special forms of this law, which are described as the Laws of non-Contradiction and Excluded Middle, we shall find to be useful in connection with the opposition of propositions, and immediate and mediate inference. But the law as a whole may be regarded as asserting that the principle of identity in difference is the basis of all reasoning, all knowledge, all experience, since if this principle be denied, reasoning, knowledge, experience are all impossible. Now we have already seen (p. 14) that wherever we find identity in difference we have what is called a universal. The Laws of Thought are thus declarations that all knowledge and reasoning are concerned with universals. Yet if we are rightly to understand these laws we must not be content with the words

'universal' and 'identity in difference,' for these words have been variously interpreted. We must, therefore, make further inquiry into their meaning.

Many logicians have regarded the term as the unit of thought, and even at the present day the unit of thought is said by most logicians to be the proposition. On either of these views it is natural to regard a universal as being a subject (a thing, species, concept or term) having or implying attributes. If the subject is a general term (*e.g.* man), it is considered to be a universal because it implies certain common or essential qualities (rational, animal, etc.) which appear in a variety of particular instances (individual men). If the subject is a singular term (*e.g.* Julius Caesar), it is also considered to be a universal because it implies certain essential qualities (Roman, general, statesman, etc.) which remain the same in a variety of times and circumstances. In such cases as these the universal is an identity in difference, in the sense that the identity consists of the essential or common qualities, while the differences are the various particular instances, times, circumstances, etc. Now at certain stages of knowledge and for certain practical purposes (such as the placing, say, of man in a classification of living beings or the identification, say, of Julius Caesar in a general historical narrative) this account of the universal may be convenient and useful. But if it is offered as an explanation of the real meaning of a universal, it is inadequate and it leads to serious difficulties. For it implies that identity in difference means a fixed, permanent identity, which is 'in difference' only in the sense that differences accompany it. There is no attempt to show how the differences are related to the identity. They are practically treated as if they were outside of the identity, just as my watch apparently remains the same whether it is in my pocket, on a table, under my pillow or in the hands of the watchmaker. This view of identity is practically the same as the view of Parmenides (p. 106). It leaves change and difference entirely unexplained, and if carried out logically it must in the end deny

both. For if all things are fixed, unchanging universals, it is manifest that there can be no change or difference. Similarly, on this view, all predication and all reasoning become arbitrary and meaningless. We have no right to say 'This man is writing,' because we have no means of relating 'writing or not writing at any particular moment' to the essential qualities of 'this man.' As Antisthenes pointed out long ago, we can only say 'This man is this man' and 'Writing is writing.' Of course those who hold the view that identity consists in essential qualities really mean by 'identity in difference' something more than this. But their wrong theory has given rise to great confusion and many futile discussions.

We may get a better understanding of the true meaning of a universal or identity in difference, if we ask ourselves how it is that we are able to make the true judgment 'This man is writing.' This proposition expresses an identity in difference, while the sentence 'This man is this man' expresses pure identity without difference and therefore says nothing. If the proposition 'This man is writing' is taken by itself and regarded as having no relation to anything else, it is an arbitrary collocation of words. But if it is a real judgment, it is the conclusion of an inference. It is based on grounds or reasons (such as actual observation), which may or may not be valid, but which the maker of the judgment regards as sufficient. The judgment is part of a rational system, in which its subject, its predicate, its grounds or premises, and the relations between these are all elements (p. 18). Every such system is an identity in difference. The identity, however, is not one part of the system and the difference another. The identity is the unity of the system, that which, as it were, holds it together, and the difference is the variety of its elements. The identity is inseparable from the difference, the unity from the variety, though they may be distinguished in thought. The identity permeates the system, and the differences are differences of the identity. If you could separate the identity from the differences, you would

destroy the system. For mere identity without difference, mere relation without any things related, cannot be a system, nor can mere differences, mere terms without any relation to one another.

Now propositions and terms, which are elements in this system (which may be called a reasoning), are themselves systems, subordinate universals, and accordingly we may find in them also identity in difference. In the case of the proposition, the identity is the connection or relation between the terms, while the difference is the distinction between the terms. We have already seen that these are inseparable, though they may be distinguished. As regards terms, we have already seen that they also are systems or identities in difference (pp. 46-7). The connotation of a term is a system of qualities, not a mere aggregate or sum, and the identity of the term is the unity of this system, the inter-connection of the qualities, while the difference is their variety. The difficulties which have given rise to criticism of the Laws of Thought proceed entirely from a failure to observe that terms and propositions are systems within some more comprehensive systems. Terms have been regarded as units, which are externally combined so as to make propositions, while these again are put together to form reasonings. The identity of a term is then supposed to consist in its having a fixed connotation of certain essential qualities, and the identity of a proposition or of a reasoning is regarded as ultimately reducible to the identity of its terms. But the critics of the Laws of Thought quite reasonably ask: Why are some qualities of a thing essential, while others are not? and, supposing that the thing (or the term) is an identity in difference, what degree of difference would destroy the identity? To these questions those who hold this view can give no satisfactory answer. Their theory presupposes that identity and difference are separate things, and the questions ask: Where, then, do you draw the line between them? To this there can be no answer, if terms are supposed to be independent of propositions and reasonings, and if the connotation of a term is a

mere aggregate of qualities. But if terms and propositions are elements in wider systems, their identities and differences are determined by the context, *i.e.* by the nature of the system in which they are elements. The ancient logicians, for instance, defined man as a rational animal. That is to say, the qualities of 'rational' and the qualities implied in being an animal were the essential qualities constituting the identity of man. But why should not the quality 'biped' and many others be included? Nowadays the biologist defines man in one way and the theologian in another. In each case the definition depends on the nature of the rational system in which man is regarded as an element. For the biologist the identity of man is determined by man's position in the system of living, organic beings; for the theologian it is determined by his position in the system of spiritual beings. And, of course, man might be defined in many other ways, by reference to other systems in which he is an element.

These systems are, of course, not necessarily static. They may be in process of evolution. Individual living beings are systems which grow and decay, and the whole world of living beings is a developing system. This does not, however, necessarily mean that these systems have no identity or permanence, though it does mean that their identity cannot consist in essential common qualities. If we define man only by the qualities which belong to every individual man who has ever existed, we leave out of our definition some of the most important characteristics of humanity. But it does not follow that man is so variable and changing as to have no identity at all. Man may become civilized or savage, cannibal or vegetarian, honest or dishonest, clever or stupid; but a stone or a sea, a star or a microbe cannot become any of these opposites, nor can man become a stone or a sea, a star or a microbe, without losing his identity. Further, man may pass from savage to civilized, from cannibal to vegetarian, etc. and still retain his identity; but he cannot become anything which is inconsistent with his nature as a system within

wider systems, such as the biological, the social, the intellectual, the moral. His identity is not an identity of qualities, but an identity which reveals itself in and through a vast but limited field of differences. And the limits of his identity are determined by his position in some wider system.

This is the real significance of the fourth law of thought, the Law of Sufficient Reason. Leibniz, who was the first logician to formulate it, saw clearly that the Law of Identity, interpreted as asserting the self-identity of common or essential qualities, was inadequate.[1] He did not, however, suggest a reinterpretation of the law but proposed to supplement it by the addition of the Law of Sufficient Reason, which he expressed thus : " Everything must have a sufficient reason why it is so and not otherwise." The Law of Identity has sometimes been expressed (*e.g.* by Bishop Butler) in the form " A thing is what it is and it is not another thing." Leibniz pointed out that there must always be some sufficient reason why it is what it is and not another thing. He did not see the full significance of his law, and he still regarded it as different from the Law of Identity. But it is clear from what has been already said, that the Law of Sufficient Reason is simply the Law of Identity rightly interpreted. It evidently means that every proposition presupposes premises or grounds and is therefore an element in a reasoning, and that every thing or event is determined by its place in a system, as the effect of a cause, as the result of a development, as the end of means or as means to an end, etc. We shall realize more fully the significance of this law when we examine the principles of inductive inference (ch. xx.).

The Laws of Thought are thus ultimately reducible to one great principle, that of the systematic nature of thought and of things. This principle is, in a sense, incapable of proof ; because it is the principle which makes all proof possible. But it is sufficiently guaranteed by the consideration that, if it be denied, it is impossible to know or to prove anything.

[1] *v.* Latta, *Leibniz*, p. 58 ff.

EXERCISE VIII

1. Explain the historical circumstances which led to the first formulation of the Laws of Thought. What light do these circumstances throw on the nature of the laws?

2. Consider the nature of the Law of non-Contradiction and discuss the view that it denies the possibility of change.

3. Explain the nature and function of the Laws of Thought. Can they be proved? If so, how?

4. What is the Law of Sufficient Reason? Consider the view that it is the most adequate expression of the principle on which all thought depends.

5. What is meant by calling a universal an identity in difference? Discuss the part which universals play in reasoning and consider the relation of universality to the possibility of change.

CHAPTER IX

THE OPPOSITION OF PROPOSITIONS

As we have seen (p. 110) the Law of Excluded Middle declares that of the two propositions ' S is P ' and ' S is not P ' one must be true and the other false, *i.e.* from the truth of either we can infer the falsity of the other and from the falsity of either we can infer the truth of the other. But these propositions are ambiguous, except in the cases in which ' S ' is a singular term. The propositions ' This man is clever ' and ' This man is not clever ' or ' Parliament is prorogued ' and ' Parliament is not prorogued ' are instances of Excluded Middle. In each of these pairs the truth of one of the propositions implies the falsity of the other and *vice versa*. But if the two propositions are general and universal in quantity (*e.g.* All S is P, No S is P, or ' All men are clever,' ' No men are clever ') they are not instances of Excluded Middle. There are two other possibilities, viz. ' Some S is P ' and ' Some S is not P,' or ' Some men are clever ' and ' Some men are not clever.' Consequently, though the two propositions ' All S is P ' and ' No S is P ' cannot both be true (*i.e.* if the one is true the other must be false), we are unable to say that they cannot both be false (*i.e.* that if one is false the other must be true). It cannot be true both that all men are clever and that no men are clever ; but evidently both propositions *may* be false. In propositions of this kind, therefore, we can infer from the truth of either to the falsity of the other, but not from the falsity of either to the truth of the other. We can, however, say that if ' All S is P ' (' All men are clever ') is false, ' Some S is not P ' (' Some men are not clever ') is true ;

and that if 'No S is P' ('No men are clever') is false, 'Some S is P' ('Some men are clever') is true.

This distinction was observed by Aristotle,[1] who describes it as the distinction between a contradictory and a contrary opposition between two statements or propositions. Two propositions are in contradictory opposition when the one simply denies what the other asserts or asserts what the other denies (*e.g.* S is P and S is not P, All S is P and Some S is not P, No S is P and Some S is P). A mere denial, which does not imply or assert anything new, is a contradictory opposite. On the other hand, two propositions are in contrary opposition when the one not merely denies what the other asserts, but goes beyond the bare denial and, by implication, asserts something else (*e.g.* All S is P and No S is P). The distinction may be regarded as arising from the question of what exactly is meant by negation or contradiction, what is implied in the answer 'No' to a question. 'Are all the windows closed?' 'No.' This may mean either that none are closed or that some are not closed. But from the bare denial we are not entitled to infer more than that some of the windows are not closed, which is the contradictory of 'All the windows are closed,' and is therefore the true interpretation of the 'No.' If the answerer had intended to state the contrary, and not merely the contradictory opposite, he should have said not merely 'No,' but something more, *e.g.* 'No, none of the windows are closed,' which is, of course, the contrary opposite.

Aristotle[2] recognized four forms of opposition, (1) that between an A and a corresponding[3] O proposition; (2) that between an E and a corresponding I proposition; (3) that between an A and a corresponding E proposition; and (4) that between an I and a corresponding O proposition. (1) and (2) are forms of contradictory opposition; (3) is the form of contrary opposition; and (4) is now described as the form of sub-contrary opposition. By the opposition

[1] *An. Pr.* ii. 15, 63^b 28. [2] *Op. cit*, 63^b 23.

[3] By 'corresponding propositions' is meant propositions with the same subject and the same predicate.

of propositions Aristotle meant opposition in its ordinary sense, namely, incompatibility or repugnance. Accordingly he hardly admits the fourth form as really a form of opposition, for both the I proposition and the corresponding O proposition may be true. Indeed the assertion of the one very often seems to imply the assertion of the other, *e.g.* 'Some of the people were afraid' seems to imply that some of them were not afraid, and 'Some men are not white' seems to imply that some men are white. But the truth of the one cannot be definitely inferred from that of the other, because 'some' means 'some at least, possibly all' and not 'some only.' As the I and the corresponding O proposition may be true together, Aristotle says[1] that they are opposed 'only verbally' and not really. On the other hand, they have an indirect claim to be regarded as in opposition, because we can say that if one is false the other must be true. If they were both false their contradictories, No S is P and All S is P ('No men are white' and 'All men are white') would both be true, and this is of course impossible.

Later logicians developed certain implications of Aristotle's view and thus gave a new and rather artificial sense to the term 'opposition.' It is evidently implied in Aristotle's doctrine that the opposed propositions have in each case the same subject and the same predicate. This fact, along with the differences of quantity and quality of the propositions, was taken by later logicians as the ground of the definition of opposition. Accordingly opposition is usually defined in this way: Two propositions are logically in opposition when they have the same subject and predicate, but differ in quantity or in quality or in both. Opposition in this sense does not necessarily imply incompatibility. Many propositions are really opposed, which are not technically in opposition, *e.g.* 'Lord Haldane is Lord Chancellor' and 'Lord Loreburn is Lord Chancellor.' These are not technically opposed, because they have the same quality and quantity and different subjects. Again 'No whales are fishes' and 'Some whales are not fishes' are

[1] *Loc. cit.*

quite compatible with one another (indeed the first includes the second), but they are said to be logically opposed, because they have the same subject and predicate, and differ in quantity.

On this view the kinds of opposition are defined and named as follows :

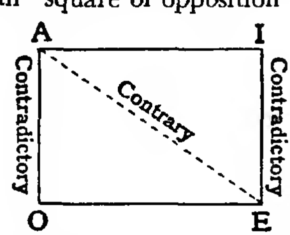
(1) Two propositions which differ both in quantity and in quality are in contradictory opposition (A and O, or E and I).

(2) Two propositions which differ in quality only are in contrary opposition (A and E) or in sub-contrary opposition (I and O).

(3) Two propositions which differ in quantity only are in subaltern opposition (A and I, or E and O).

Contradictory propositions cannot both be true, nor can they both be false. Contrary propositions cannot both be true, although they may both be false. Sub-contrary propositions cannot both be false, but may both be true. In the case of subaltern propositions, the truth of the particular follows from that of the universal, but the truth of the universal does not follow from that of the particular.

Aristotle illustrated the difference between contradictory and contrary opposition by suggesting that, while contradictories are opposed by a distance comparable to that of the side of a square, contraries are opposed by a distance comparable to that of its diagonal.[1] We should thus have, as the Aristotelian 'square of opposition'

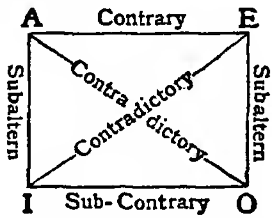


This is, however, defective, in so far as it suggests that the sub-contraries, I and O, are opposed in the same way as

[1] *De Interp.* 19*b* 35.

A and E, while, from Aristotle's point of view, they are not really opposed at all.

If we accept the view of later logicians, the 'square of opposition' takes this form.



The function of this square is not to illustrate the difference between contradictory and contrary opposition, but merely to provide a diagram in which the various relations of technical opposition between the four forms of categorical propositions (A, E, I, O) may be conveniently represented. Accordingly in this square the difference between side and diagonal has no significance, and the representation of the contradictories as diagonally apart is merely a matter of convenience.

In the case of the singular proposition the distinction between contrary and contradictory opposition does not arise, and there can be no sub-contrary or subaltern opposition. For the subject of the singular proposition cannot have particular quantity. Formally in the singular proposition we can only have opposition between A and E, *i.e.* contrary opposition. But this is really indistinguishable from contradictory opposition, and it is therefore usual to say that in the case of the singular proposition the contradictory and the contrary coincide.[1] If, however, we take account of *real*, as distinct from merely *formal*, opposition,

[1] It is preferable to call the opposite of a singular proposition its contradictory; for it merely denies the given proposition and does not assert anything else. The singular proposition is always a source of difficulty in Formal Logic, because Formal Logic presupposes the class view of the import of propositions, and that view is an unnatural interpretation of the singular proposition (*v.* p. 71).

we can distinguish between the contradictory and the contrary of a singular as well as of a strictly universal proposition. 'That is a weasel.' 'No', meaning simply 'That is not a weasel.' These are real contradictories. 'That is a weasel.' 'No. It is a stoat.' These are real contraries. The contradictory merely denies : the contrary makes an additional assertion. In the case of such real contraries, the additional assertion is not implied in the proposition which denies, but requires a new proposition. The question whether or not, in any particular case, the propositions are real contraries depends, of course, on the context or the nature of the system within which the statements are made. The instance just given is a case of real contraries, only if it is true that no weasels are stoats. And in general 'S is P' and 'S is Q' are real contraries, only if S's being P is incompatible with S's being Q. We frequently use these contraries in discussion, and they have various degrees of validity, *e.g.* (*a*) 'You told me he was an agnostic and I find he is an ironmonger.' (*b*) 'You said he was a philosopher and he is really a lawyer.' (*c*) 'He was described as a Unionist ; but he is a Conservative.' (*d*) 'He called that ship the *Agamemnon* ; but it is the *Lord Nelson*.' (*a*) is obviously invalid ; because it is clearly untrue that a man cannot be both an agnostic and an ironmonger. (*b*) is valid only if 'philosopher' means a professional philosopher, in the sense of one whose whole life is devoted to philosophical work. The validity of (*c*) depends on the question whether the distinction, say, between a Liberal Unionist and a Conservative is important in the actual context. (*d*) is probably valid because the same ship cannot have two different names. But if the names happened to be those of ships of the Armada, in which the same ships were often called by apparently different names, or if the ship's name had been changed (*e.g.* 'That is the *Vectis*.' 'No, it is the *Rome*'), the propositions would not necessarily be contraries. In all such cases, then, we have really an inference, having its ground in the context or system of discourse. The argument in every case might be

put in the form : Either S is P or S is Q ; S is Q ; therefor it is not P.

Contradictory and contrary opposition are evidently the most important kinds for logical purposes. The contradictory of a proposition denies that it is wholly true ; the contrary asserts that it is wholly false. A contradictory opposite is thus more easily defended and more difficult to refute in argument than a contrary opposite. Suppose that someone maintains that extravagance is the cause of poverty. It would be easier to prove the contradictory, 'Some extravagance is not a cause of poverty,' than the contrary, 'No extravagance is a cause of poverty.' If the contrary were used by way of reply, it would be easy for the original assertor to refute this contrary by its contradictory, 'Some extravagance is a cause of poverty.' Much confusion is thus caused in ordinary discussion, especially when the opponents become excited. The angry man usually refutes by contraries, *e.g.* 'Nothing of the sort !' 'There is not a word of truth in it,' etc.

EXERCISE IX

1. (*a*) Distinguish carefully between contrary and contradictory opposition. Are there any propositions to which this distinction does not apply ? (*b*) State the following in logical form and give the contradictory and (where possible) the contrary of each :

(1) There's many a true word spoken in jest.
(2) Ignorance is not always bliss.
(3) Few citizens of this country failed to support the prosecution of the war.
(4) The best of men are but men at the best.
(5) Only ignorant people hold such opinions.
(6) A miss is as good as a mile.
(7) Life is not all pleasant.
(8) Violets grow on the chalk downs in April.
(9) Not all the crags are accessible.
(10) No man is good enough to be trusted with unlimited power. Unless he is a saint—perhaps even if he is a saint—he is sure to abuse it.

2. Explain the four types of logical opposition ; and show how the account of opposition would be affected if 'some' meant in logic 'some only.'

CHAPTER X

IMMEDIATE INFERENCE[1]

WE have already seen (pp. 11-12) that in all reasoning or inference there are three elements: (*a*) one or more grounds or premises, (*b*) a consequent or conclusion depending on or following from the grounds or premises, and (*c*) the connection between them. In a fully developed and expressed inference the connection between (*a*) and (*b*) is made explicit and is sometimes described as the 'middle term' or that which mediates between the ground or premise and the conclusion. It is, in short, a universal relation between them, which has its roots in both. But we may and do frequently infer one proposition from another without expressing or making explicit the connection between them. Then we have immediate inference. It is thus usual to define immediate inference as inference in which from a given proposition we infer another proposition which is necessarily implied in it. The inference is called immediate because it takes place without the explicit use of a middle or mediating term. In mediate inference, on the other hand, there is usually more than one premise, and the middle term appears in both premises.

When two propositions are in logical opposition we can,

[1] It is sometimes held that the operations considered in this chapter are not inferences at all, but verbal changes, the expression of the same judgments in different verbal forms; and that therefore they belong to grammar, not to logic. Whether they are to be regarded as inferences or not is not, as is sometimes supposed, a mere matter of terminology. It involves a question of principle; it depends on what we take to be the essence of inference. For a full discussion of the question see Joseph, *An Introduction to Logic* (p. 240 ff.) and the references there given.

in certain cases, infer the truth or falsity of one of them from the falsity or truth of the other (p. 118). This has sometimes been described as a form of immediate inference, depending on a direct application of the Laws of Thought. But it is more usual to confine the name of immediate inferences to those reasonings in which we infer the truth of one proposition from the truth of another, *i.e.* to reasonings in which the conclusion is not a negation of the opposite of the given premise, but is necessarily implied in the given premise.

The chief kinds of immediate inference, which have been generally recognized by logicians presuppose the 'class' theory of the import of propositions, viz. that the subject and predicate of a proposition are to be taken in denotation and that the proposition states a relation of inclusion or of exclusion between them (p. 71 ff.). A relation of this kind has two aspects. It is a relation of S to P; but it is also a relation of P to S. If S is included in P, as a part in a whole, then some part of P is identical with S. If S is wholly excluded from P, then P is wholly excluded from S. In other words, a certain relation of S to P implies necessarily a corresponding relation of P to S. Thus from a proposition of the form SP we can in most cases immediately infer a proposition of the form PS. This kind of immediate inference is called *conversion*, which may be defined as the process by which from a given proposition we infer another, having the subject of the given proposition as its predicate and the predicate of the given proposition as its subject. The given proposition, the proposition to be converted, is called the *convertend*, and the converted proposition is called the *converse*. The quality of the converse is the same as that of the convertend.

Conversion, however, does not consist in merely substituting P for S and S for P. We must take into account the quantity of the terms, and the general principle of conversion is that, in interchanging the terms, we must not make a term distributed in the converse, if it is undistributed in the convertend, *i.e.* we must not make a statement

about the whole on the ground of a statement about a part. Applying this principle to the conversion of the four forms of categorical propositions, we get the following results :

A All S is P (All dogs are animals)—Converse I, Some P is S (Some animals are dogs).
(P is undistributed in the convertend and must therefore be undistributed in the converse).

E No S is P (No dogs are ruminants)—Converse E, No P is S (No ruminants are dogs).
(S and P are both distributed in the convertend and may therefore be distributed in the converse).

I Some S is P (Some bridges are made of stone)—Converse I, Some P is S (Some things made of stone are bridges).
(S and P are both undistributed in the convertend and must therefore be undistributed in the converse).

O Some S is not P (Some bridges are not made of stone)—No converse.
(The subject of the convertend is undistributed, and cannot therefore be the predicate of the converse; for the converse, being negative, must have a distributed predicate).

The A proposition states that all S is a part of P, and its converse states that a part of P is S. The E proposition states that all S is excluded from P, and its converse states that all P is excluded from S. The I proposition states that some S is a part of P, and its converse states that a part of P is S. In each case we have two aspects of the same relation. The O proposition states that some at least of S is excluded from P. In this case the relation is indefinite, for it covers two possible cases, viz. (*a*) some S is excluded from P and some S is part of P, and (*b*) all S is excluded from P. It is therefore impossible to infer a converse necessarily implied in the O proposition. The I proposition may mean either (*a*) that some S is a part of P and some S is not a part of P or (*b*) that all S is a part of P. But in this case a converse is possible, for on either alternative we can

infer necessarily that a part of P is S. On the other hand, it is obvious that from the O proposition we cannot infer either that P is included in S or that it is excluded from S.

The conversion of an E or an I proposition is called simple conversion because it consists in simply transposing subject and predicate without changing the quantity of the proposition. The conversion of an A proposition is called conversion *per accidens*. It is also called conversion by limitation because the quantity of the converse is particular, while that of the convertend is universal.

The most common fallacy in this form of immediate inference is the simple conversion of an A proposition (*i.e.* the conversion of All S is P into All P is S). Such a conversion is strictly legitimate only in the case of a proposition of which the subject and predicate are both singular terms, *e.g.* 'The shortest of these essays is the best.' Neither of the terms can be undistributed, and consequently simple conversion is possible. But there are many of our universal judgments which we know to be simply convertible, but which are not immediate inferences from one another. The most obvious of these is the geometrical converse, *e.g.* "An isosceles triangle is a triangle which has the angles at the base equal to one another," and "Every triangle which has the angles at the base equal to one another is an isosceles triangle" (*Euclid*, bk. i. props. 5 and 6). We cannot immediately infer one of these propositions from the other, because each requires a separate proof. Thus the geometrical converse must be carefully distinguished from the logical converse. And there are many other universal affirmative propositions which are actually true both in the form All S is P and in the form All P is S, *e.g.* 'All islands are pieces of land completely surrounded by water.' But in all these cases the universal converse does not follow immediately from the convertend.

The second main form of immediate inference depends on the general principle that every proposition can be expressed either affirmatively or negatively. It is simply a

way of obtaining a negative equivalent for an affirmative proposition or an affirmative equivalent for a negative one. This form of immediate inference is called *obversion*.[1] The inference in obversion, like that in conversion, depends on a double aspect in the relations of inclusion and exclusion between terms. The difference is that in conversion we infer from inclusion in one aspect to inclusion in another and from exclusion in one aspect to exclusion in another; but in obversion we infer from a relation of inclusion to a relation of exclusion which is necessarily implied in it, and from a relation of exclusion to the necessarily implied relation of inclusion. The relation which is the basis of obversion is that which is expressed in the Law of Excluded Middle, S is either P or not-P. It follows from this that if S is P, it is not not-P, and if S is not P, it is not-P. 'S is not not-P' is the obverse of 'S is P,' and 'S is not-P' is the obverse of 'S is not P.' In other words, if S is included in P, it is necessarily excluded from the contradictory of P, and if S is excluded from P, it is necessarily included in its contradictory. If men are animals, they are excluded from the class of things which are not animals, and if men are not quadrupeds, they are included in the class of things which are not quadrupeds.

It is evident that obversion involves the use of contradictory terms (pp. 31-2). We have already considered the theoretical objections to such terms; but modern logicians regard them as practically useful, and without them obversion would not be possible. Accordingly obversion may be defined as an immediate inference in which from a given proposition we infer another, having for its predicate the contradictory opposite of the predicate of the given proposition. This inference is legitimate if the quality of

[1] It has sometimes been called Permutation; but this name is open to the objections, (*a*) that it is used in mathematics for a process having no analogy to the logical process, and (*b*) that it yields no convenient name for the result of the operation. Jevons calls the process 'immediate inference by privative conception,' a phrase which is too clumsy for ordinary use.

the proposition be changed. Applying this rule to the four forms of proposition we get the following results :

A All S is P (All birds are feathered)—Obverse—E, No S is not-P (No birds are not-feathered).

E No S is P (No birds are mammals—Obverse—A, All S is not-P (All birds are not-mammals).

I Some S is P (Some birds are migratory)—Obverse—O, Some S is not not-P (Some birds are not not-migratory).

O Some S is not P (Some birds are not migratory)—Obverse—I, Some S is not-P (Some birds are not-migratory).

For 'not-' we may, of course, use equivalent phrases, such as 'other than'; and the practice is to be recommended to beginners, who usually find difficulty in managing a statement with more than one 'not' in it. Similarly 'Only S is P' may be interpreted as 'No not-S is P.' It should also be observed that as not-P is the contradictory of P, so P is the contradictory of not-P. Consequently 'All S is P' is the obverse of 'No S is not-P' and in general the given proposition is the obverse of its own obverse. Prefixes such as 'in-' 'un-' etc. sometimes indicate contradictory terms, *e.g.* 'unobserved' is the contradictory of 'observed,' 'infallible' is the contradictory of 'fallible.' But these prefixes sometimes indicate contrary, not contradictory terms. For instance 'immoral' is not in all cases the contradictory of 'moral'; it is usually its contrary. The contradictory of 'moral' is 'non-moral.' But in certain contexts 'immoral' and 'moral' may be contradictories (*v.* p. 32). In each case the real contradictory must be ascertained by reference to the context, *i.e.* to the meaning of the given proposition.

Of the immediate inference which depends on relations of inclusion and exclusion between the terms of a proposition conversion and obversion are the two fundamental forms. From a given proposition of the form SP conversion yields

a proposition of the form PS, and obversion yields a proposition of the form S not-P. But there are five other possible combinations of any subject (S) and any predicate (P) and their contradictories (not-S and not-P), viz. (1) P not-S, (2) not-PS, (3) not-P not-S, (4) not-S P, and (5) not-S not-P. By applying alternately the processes of conversion and obversion we can get, from given propositions of certain kinds, immediate inferences in each of these five forms, as follows :

(1) By obverting the converse of SP we get the *obverted converse* P not-S.

A All S is P ; converse, Some P is S ; obverted converse, Some P is not not-S.

E No S is P ; converse, No P is S ; obverted converse, All P is not-S.

I Some S is P ; converse, Some P is S ; obverted converse, Some P is not not-S.

O Some S is not P ; no converse ; no obverted converse.

(2) By converting the obverse of SP we get the *partial contrapositive* not-P S.

A All S is P ; obverse, No S is not-P ; partial contrapositive, No not-P is S.

E No S is P ; obverse, All S is not-P ; partial contrapositive, Some not-P is S.

I Some S is P ; obverse, Some S is not not-P ; no partial contrapositive.

O Some S is not P ; obverse, Some S is not-P ; partial contrapositive, Some not-P is S.

(3) By obverting the partial contrapositive of SP we get the *full*[1] *contrapositive* not-P not-S.

[1] The terminology 'partial' and 'full' in the case of the contrapositive and the inverse is that adopted by Dr. Keynes (*Formal Logic*, 4th ed., p. 134 ff.). The forms called 'partial' are the more natural and common inferences, but, as Dr. Keynes points out, the forms called 'full' have been preferred by many logicians on the ground that they restore the quality of the original proposition.

A All S is P ; partial contrapositive, No not-P is S ; full contrapositive, All not-P is not-S.

E No S is P ; partial contrapositive, Some not-P is S ; full contrapositive, Some not-P is not not-S.

I Some S is P ; no partial contrapositive ; no full contrapositive.

O Some S is not P ; partial contrapositive, Some not-P is S ; full contrapositive, Some not-P is not not-S.

(4) By alternate processes of obversion and conversion we get from SP the *partial inverse* not-SP.

A All S is P ; full contrapositive, All not-P is not-S ; partial inverse (by conversion and obversion), Some not-S is not P.

E No S is P ; obverted converse, All P is not-S ; partial inverse (by conversion), Some not-S is P.

I Some S is P ; no partial inverse, because there is no contrapositive and the obverted converse, being an O proposition, cannot be converted.

O Some S is not P ; no partial inverse, because there is no converse and the full contrapositive, being an O proposition, cannot be converted.

(5) By obverting the partial inverse of SP we get the *full inverse*, not-S not-P.

A All S is P ; partial inverse, Some not-S is not P ; full inverse, Some not-S is not-P.

E No S is P ; partial inverse, Some not-S is P ; full inverse, Some not-S is not not-P.

I and O have obviously no full inverse.

These are all the forms of immediate inference which are now generally recognized. But some logicians have drawn attention to other apparently immediate inferences, which are dependent on more complex kinds of relation than those of inclusion and exclusion. For instance Jevons[1] describes 'immediate inference by added determinants' and 'immediate inference by complex conception.' In the former

[1] *Op. cit.* pp. 86-7.

of these an adjective or other determinant is added to the subject and the same determinant is added to the predicate, *e.g.* 'All negroes are men, therefore all honest negroes are honest men.' In the latter the subject and predicate are each modified by the addition of the same words so as to form more complex conceptions, *e.g.* 'All bees are insects, therefore all wings of bees are wings of insects,' or 'All metals are elements, therefore a mixture of metals is a mixture of elements.' The basis of both of these forms of immediate inference is either (*a*) that the proposition is an equation between its terms (a view held by Jevons and others, p. 77 ff.), and therefore we may add the same thing to each term or each side of the equation; or (*b*) that whatever characterises an individual by itself or a species in a class will equally characterise it in any wider class in which it may be included. The former of these considerations is insufficient to justify an immediate inference by added determinants, because, though it may be permissible to add the same quantity to both sides of an equation, it is not necessarily permissible to add the same quality or other determinant to both. A quality attached to one thing has very often a different value from the same quality attached to another. A musician is a man; but a good musician is not necessarily a good man. A British stamp is a European stamp; but the most rare of British stamps is not necessarily the most rare of European stamps. These instances are also sufficient to show that the second suggested basis does not justify an immediate inference. A thing or species which has a quality as member of a class does not necessarily have the same quality as member of a wider class. A tortoise may be swift-footed in comparison with other tortoises; but it is not therefore swift-footed as an animal. Jevons himself practically admits the imperfection of these inferences when he makes the strange statement that "the truth of the new proposition necessarily follows from the truth of the original in almost all cases."[1] To say that a proposition follows from another 'in

[1] *Op. cit.* p. 86.

almost all cases' is to say that it does not follow 'necessarily,' for 'necessarily' implies 'in all cases.' And every logical immediate inference must follow necessarily.

A more important form of immediate inference is described by Keynes as 'immediate inference by converse relation.'[1] All immediate inferences, including those which depend on relations of inclusion and exclusion, may be described as inferences by converse relation; but the term is convenient as a description of inferences which depend on various reciprocal relations other than those of inclusion and exclusion. These relations may generally be described as the reciprocal relations of terms which are due to their place in one or another system, *e.g.* family relations, such as husband and wife, parent and child, etc.; relations of time and space, such as before and after, above and below, right and left, north and south, etc.; and relations of degree, such as greater and less, hotter and colder, narrower and wider, better and worse, etc. Thus from 'A is the father of B' we can infer that B is a child of A, from 'A is a day before B' we can infer that B is a day after A, from 'A is north-east of B' we can infer that B is south-west of A, from 'A is greater or hotter or better than B,' we can infer that B is less or colder or worse than A. These inferences are made by substituting for the relation expressed with reference to A its correlative expressed with reference to B. Their validity, of course, depends on our knowledge of the system within which the particular relation holds. If they are true interpretations of the system, they are necessary inferences.

EXERCISE X[2]

1. Express the following in logical form and give the converse and the obverse of each:
   (1) Boys will be boys.
   (2) All the doctors in the world could not save him.

[1] *Formal Logic* (Fourth Edition), p. 149.

[2] The sentences given in Exercise V (Nos. 3, 4 and 5) may be used for further practice in the various forms of immediate inference.

(3) No tale-bearer is to be believed.
(4) He was not the only one who cheated.
(5) Him justice leads and truth attends
Whose means are pure and spotless as his ends.
(6) What can ennoble sots or fools or cowards?
Alas! Not all the blood of all the Howards.
(7) Few were saved from the shipwreck.
(8) With stupidity even the gods cannot contend.
(9) None are so blind as those who will not see.
(10) The most may err as grossly as the few.

2. Give, where possible, the obverted converse and the contrapositive of the propositions contained in the following sentences:
(1) All men are not equally wise.
(2) Only the impartial reason.
(3) Many a flower is born to blush unseen.
(4) It is not given to everyone to rise to the highest positions in life.
(5) The more, the merrier.
(6) You cannot expect anybody but a schoolboy to remember dates.
(7) All but he fled.
(8) No news is good news.
(9) Fortune favours the brave.
(10) It never rains but it pours.

3. Give, where possible, the inverse and the obverted inverse of the following:
(1) The good die young.
(2) The good is sometimes the enemy of the best.
(3) Faith without works is dead.
(4) There are more ways of killing a cat than choking it with cream.
(5) Only a philosopher can fail to see the difference between a post and my idea of a post.
(6) All bloodless lay the untrodden snow.
(7) None think the great unhappy but the great.
(8) Liquids always tend to find their level and having found it to keep it.

4. All Jews are clever.
What is the logical relation of this proposition to each of the following?
(1) Some dull persons are Jews.
(2) If a man is clever you may be sure he is a Jew.
(3) Some very clever people are not Jews.
(4) If a man is not clever you may be sure he is not a Jew.
(5) If you meet a clever person you cannot be sure whether he is a Jew or not.
(For the purpose of this question dull = not-clever.)

5. Explain and illustrate immediate inference by converse relation. Does this process satisfy your idea of the requirements of real inference?

CHAPTER XI

THE PREDICABLES, DEFINITION, DIVISION AND CLASSIFICATION

I. THE PREDICABLES

IN an affirmative categorical proposition we attribute certain characteristics to the subject of the proposition. But when we consider various instances of categorical propositions we find that there are various ways in which the characteristics expressed in the predicate are regarded as belonging to the subject. Some qualities belong to a given subject as part of its nature. It would not be the thing it is, if it did not have these qualities. A triangle would not be a triangle if it did not have three sides. A cow would not be a cow if it were not a ruminant. Other qualities belong to a subject at all times, without being part of its nature. All white cats with blue eyes are deaf; but if we found an animal having all the other characteristics of a white cat with blue eyes and yet not deaf, we should not deny that it was a white cat with blue eyes. Again there are many other qualities which belong to a subject only for a time. When we say that a man is singing or that he is wearing a frock coat, we do not mean that these qualities belong to him in the same sense as qualities like 'mortal' or 'rational.'

Aristotle [1] made a classification of these relations between qualities (or predicates) and the subjects to which they are attributed, and these main classes of relations are called the Predicables. The Predicables, then, may be defined as

[1] *Top.* I, 4, 101^b 16.

a classification of general terms used as possible predicates of an affirmative proposition, the basis of the classification being their relation to the subject. Aristotle [1] classifies these relations by asking, in the case of each proposition, whether or not the proposition is simply convertible. A universal affirmative proposition is simply convertible if (to use the language of later logic) the denotation of the predicate is identical with that of the subject. We thus get two main classes of predicables, according as the denotations of the terms in a given affirmative proposition are or are not identical.[2] In each of these main classes two other classes may be discriminated. (1) If the denotations of the terms are identical we may have (*a*) cases in which the predicate expresses the whole essential nature of the subject, 'what the subject really is,' (in modern language, the whole connotation of the subject), and (*b*) cases in which the predicate expresses something which, though not the essence of the subject, is peculiar to it and an inseparable concomitant of it. In case (*a*) the predicate is the definition of the subject ; in case (*b*) it is a proprium of the subject. In the proposition, 'Man is a rational animal,' the predicate is a definition ; in the proposition, 'Man is capable of civilization,' the predicate is a proprium. (2) If the denotations of the terms are not identical, we may have (*c*) the case in which the predicate is a part of the definition (or connotation) of the subject, and (*d*) the case in which the predicate is not part of the definition or essential nature of the subject. In case (*c*) the predicate is a genus or differentia of the subject ; in case (*d*) it is an accident of the subject. A genus is a predicate which, though part of the essence (or connotation) of the subject, is applicable to other subjects. A differentia is a predicate which distinguishes one species of things from other species in the same genus. In the proposition, 'Man is an animal,' the predicate is a genus ;

[1] *Top.* 1, 5, 101^b 37.

[2] It is better to take this as the basis of classification, because the proposition Some S is P, though it is simply convertible, has terms which differ in denotation. Aristotle was thinking only of A propositions.

in the proposition, 'Man is rational,' the predicate is a differentia; and in the proposition, 'Barristers wear wigs,' the predicate is an accident. In Aristotle's classification there is no real distinction between genus and differentia. 'Rational' may be a genus of 'man,' if we are thinking of man as a species of the genus 'rational beings,' which may include other kinds. 'Animal' may be a differentia of 'man,' if we are thinking of man as a species of the genus 'living being,' which includes plants, etc. Accidents are usually predicates of singular or particular subjects, *e.g.* 'This man was born in London'; 'Some men are ambitious.'

According to Aristotle's classification there are thus four Predicables—Definition, Proprium, Genus or Differentia, and Accident. But Porphyry (about 600 years after Aristotle), in an introduction to Aristotle's book on the *Categories* gave a revised classification which, through the mediaeval logicians, became the traditional account of the Predicables. Porphyry's classification is based on the view that a proposition states a relation between classes. He enumerates five Predicables: (1) Genus; (2) Species; (3) Differentia; (4) Proprium; (5) Accident. In this classification, as compared with that of Aristotle, definition is omitted, species and differentia are added as separate predicables, and the meaning of proprium and genus is somewhat changed. The reason for this difference between Aristotle and Porphyry is that Aristotle was thinking primarily of the *definition* of a subject, while Porphyry was thinking primarily of the *division* of a class. Aristotle, having definition in view, considered the various *qualities* of the subject, which may be (1) the essential qualities (definition); (2) the peculiarities, qualities which always accompany the essential qualities (propria); (3) essential qualities which the subject has in common with others (genus or differentia); and (4) qualities which may or may not belong to the subject at any one time, non-essential qualities (accidents). Porphyry, on the other hand, having in view *division* rather than *definition*, takes (1) genus as the whole or class to be

divided, (2) species as the subordinate divisions, (3) differentia as the attribute or attributes by which each species is distinguished from others in the same genus, and (4) propria, and (5) accidents as other qualities belonging to all or some of the individuals in a genus or species, but not reckoned for purposes of division and definition. Porphyry thus omits definition as a distinct predicable, and makes definition = genus + differentia, *e.g.* Man = rational (differentia) animal (genus), 'Man' being, of course, a species of 'animal.' Aristotle did not include species as a predicable, because he regarded species as the thing to be defined, the subject.

Porphyry's account of the predicables was accepted by logicians for many centuries, and it is the most useful account for purposes of division and classification. In conformity with it, the predicables may be defined as follows: (1) A genus is any wider class which is made up of narrower classes. (2) A species is any narrower class included in a genus. (3) A differentia is the attribute or attributes by which one species is distinguished from all others contained under the same genus (or the excess of the connotation of a species over that of its proximate genus). (4) A proprium is an attribute which does not form part of the connotation (or definition) of a term, but which follows from it, either as effect from cause or as conclusion from premise. (5) An accident is an attribute which is neither part of the connotation of a term nor is necessarily connected with its connotation.

It should be noted that genus and species, as logical predicables, are entirely relative to one another. Any class which contains sub-classes is a genus. Any sub-class is a species of the wider class in which it is contained. Thus the same class may be a genus relatively to its sub-classes and a species relatively to a wider class in which it is contained. The class 'vertebrates' is a genus, relatively to the class 'mammals,' and a species, relatively to the class 'animals.' Logically there are no fixed genera or species. But in botany and zoology the terms 'genera' and 'species'

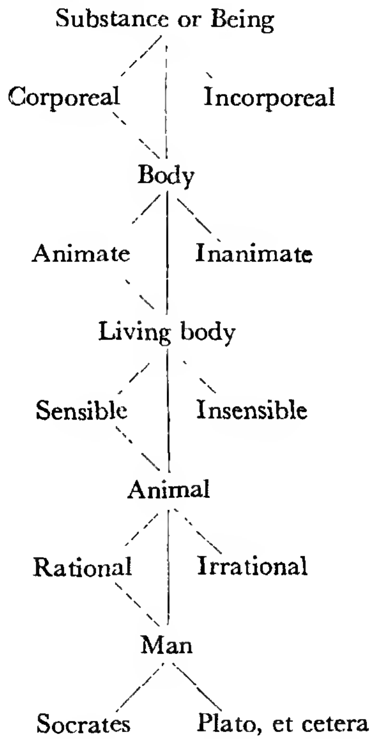
have a fixed and limited meaning. Until the time of Darwin it was supposed that there were fixed species or 'real kinds' of animals and plants. A biological species, in this sense, was defined as a class of animals or plants descended from common ancestors and having a very large number of essential attributes in common, which remained the same from generation to generation. 'Varieties' were sub-classes of species, differing from them in a few assignable and unessential attributes, *e.g.* species, 'man,' varieties, 'black man,' 'white man,' 'red man,' etc. Similarly 'genera' were groups of several species, 'families' or 'tribes' were groups of genera, 'orders' were groups of families and so on. Thus a zoological line of classification would be: kingdom (animal); sub-kingdom (vertebrates); class (mammals); order (carnivora); tribe (digitigrada); genus (felidae); species (lion); variety (African, Syrian, etc.). But, according to the theory of evolution, many genera and species are descended from a common stock, and accordingly the biological distinctions become merely names of groups which are convenient for classification.

The scholastic logicians used the term *summum genus* to describe a class which cannot be included as a species in any wider class. Similarly they applied the term *infima species* to a species which can only be divided into individuals, *i.e.* a species which has no sub-classes constituted by *essential* differentiae. A line of division beginning with a *summum genus* and ending with an *infima species* was called a 'predicamental line.' The stock instance of a predicamental line is 'Porphyry's tree,' which may be represented in the way shown on the opposite page.

In this illustration 'substance' is the *summum genus*, 'man' the *infima species*, and 'corporeal,' 'animate,' etc., are the various differentiae.

Porphyry also made a change in the meaning of *proprium*. According to Aristotle, a proprium is peculiar to its subject and is an inseparable concomitant of it. According to Porphyry, a proprium is not necessarily peculiar to its subject, but it follows from the definition (essence or conno-

tation) of its subject. *E.g.* 'Man is an animal which cooks its food.' The ability to cook food would for Aristotle be a proprium of man. But unless it can be shown that this power follows necessarily from man's rationality, it would



for Porphyry be an accident of man. Later logicians introduced a further difficulty by distinguishing between separable and inseparable accidents. Separable accidents are accidents which belong to an undistributed subject or to a subject only at certain times, *e.g.* 'wearing a white hat,' 'standing at the corner of the road,' etc., while inseparable

accidents are those which belong to a distributed subject or to a subject at all times, *e.g.* 'born in London,' 'six feet high,' etc. The predicate of the proposition 'All white cats with blue eyes are deaf' would be an inseparable accident, because we do not know why such cats should be deaf, *i.e.* we cannot show that deafness is a consequence of their other characteristics. It is evident that the distinction between separable and inseparable accidents is not easy to apply, and it is of minor importance.

II. DEFINITION

We have seen that, in making the distinctions indicated by the Predicables, Aristotle had in view the definition of things (or of species). He rightly regarded definition as being both the beginning and the end of science. In every scientific inquiry we begin with some knowledge of the objects we intend to study. We know some of their qualities, and this knowledge is a provisional definition of them. It enables us to mark them off roughly from other things. But our early definitions are inevitably superficial. *We distinguish and describe things by their most obvious* attributes, *e.g.* their shape, size, colour, etc., rather than by their essential attributes, which are often far from being obvious. At this stage, our powers of analysis and synthesis, of discrimination and connection are undeveloped. But gradually, through errors and difficulties of identification, we discover that some attributes of a thing are more important, more significant, more essential than others, and in this discovery we have the beginnings of science. The whole business of science, as conceived by Aristotle, is to penetrate, by means of analysis and synthesis, into the essential nature of the various species of things, to ascertain their essential qualities, the qualities without which the species would not be what it is. These essential qualities are the definition of the species, and the aim of science is to obtain more and more exact definitions. To this end the predicables are manifestly an important aid, for they indicate the general lines on which we may discriminate

between qualities, according to the degree in which they approximate to the whole essence of the species to which they belong.

A definition is thus a statement of the whole essential nature of the thing defined, or, in modern language, it is the explicit statement of the connotation of a term. This modern way of defining a definition is the more convenient of the two ; for it more obviously includes definitions which are not strictly scientific, *e.g.* legal and other definitions which are not primarily statements of the essential nature of the thing defined, but declarations of the exact sense in which the term defined is to be understood in a particular context. 'Man,' for instance, is defined in one way in biology, in another in theology. A 'person' within the meaning of a Franchise Act is defined differently from a 'person' in ethics.

According to the traditional logic, definition should be made *per genus et differentiam*, *e.g.* 'Man is a rational animal,' where 'animal' is the genus and 'rational' the differentia. In this instance, the differentia is one quality ; but it may, of course, include more than one quality. The differentia is the quality or qualities by which a species is distinguished from its genus. In such a definition the genus must be the *proximate* genus. Otherwise the differentia is incomplete. For instance, 'Man is a rational being' is not a definition ; for 'being' is not the proximate genus, and consequently the proposition omits the quality or qualities of 'animal,' which are a part of the definition of 'man.' In other words, the differentia is incomplete, for there may be other rational beings than man. Most definitions may be interpreted in this way ; but, as we shall afterwards see (p. 147), it is a very imperfect method of definition. For it is based on classification, which does not necessarily express the essential nature of the things classified or defined. It merely enables us to identify an object or a species of objects as distinct from all others. The definition 'Man is a rational animal' does not tell us the whole essential nature of man, and it is therefore not a scientific

definition, although it may be useful as marking off man from other things.

Certain rules or principles of definition have been generally recognized by logicians. These rules are simply negative ways of stating the traditional principle of definition, *i.e.* they are warnings against the most usual errors in definition. They may be stated as follows :

1. A definition must state neither more nor less than the whole connotation of the term defined ; or the definition and the term defined must be simply convertible, *i.e.* they must have the same denotation.

A definition which states more than the connotation of the term defined is too narrow, because it includes qualities which are not essential and therefore suggests that the denotation of the term is less than it would be if these qualities were not included in its definition. If you define man as a featherless rational animal, you suggest that man is a particular species of rational animal, and your definition is therefore too narrow. In particular cases the added qualities may be propria and may thus make no difference in the denotation of the term defined, *e.g.* 'An equilateral triangle is a triangle with three equal sides and three equal angles.' But even in such cases the definition is too narrow because it suggests that the denotation is restricted, *e.g.* that triangles with three equal sides and three equal angles are a species of triangles with three equal sides. In general it may be said that a definition is too narrow when it defines a term by qualities which belong, or appear to belong, only to a species of the term defined.

Similarly a definition which states less than the connotation of the term defined is too wide, for evidently it consists of either the genus or the differentia, each of which is wider than the term defined, *e.g.* 'A triangle is a three-sided figure,' 'Speech is vocal sound.'

2. A definition must not be obscure, metaphorical or ambiguous.

It is evident that a term cannot be explained by a defini-

tion which is as vague and obscure as the term itself. A well-known instance of a breach of this rule is Dr. Johnson's definition of a net as "a reticulated fabric, decussated at regular intervals, with interstices between the decussations." But what is obscure to one man may be clear to another, and accordingly in order to determine whether or not a definition is obscure, we must take into account the context in which it is made. Mathematical and scientific definitions may be perfectly sound, though they may be unintelligible to a man who knows nothing of mathematics or science.

3. A definition must not contain the term defined or a term which is synonymous with it or implies it.

To define a term by means of the term itself is evidently meaningless. 'A conic section is a section of a cone,' 'Eleemosynary assistance is charitable aid,' 'A ruler is a person who has subjects,' are instances of a breach of this rule. The most important error against which it gives warning is the *circulus in definiendo*, which consists in defining or explaining a term and proceeding, at some later stage of the discussion, to define or explain your previous explanation by means of the original term. For instance, sensations may be defined as impressions made on the mind by external objects, and subsequently the mind may be defined as a heap or collection of sensations. The rule, of course, does not mean that a word which is part of the term may not be included in the definition, *e.g.* 'A reciprocating engine is an engine which etc.' In such cases, the question of the validity of the definition depends on whether or not the repeated word (*e.g.* 'engine') has already been defined. In this case the definition is really a definition of the term 'reciprocating' as applied to engines.

4. A definition must not be negative, if it is possible to make it affirmative. We must not, for instance, define things by their contraries or contradictories, *e.g.* 'Knowledge is the opposite of ignorance,' 'Sleeping is the opposite of waking.' There are, of course, terms, such as 'non-elector,' 'alien,' 'indivisible,' etc., which it is hardly, if at

all, possible to define affirmatively. But a negative definition should not be given, if it is possible to make an affirmative one. A definition professes to give the essence of the thing defined, and this essence is always something positive. Even a negative definition implies something positive as its basis; but the positive or affirmative element is not made clear, and the definition is therefore defective. It is really the first step in an attempt to define. For instance, 'A non-combatant is one who does not take part in the fighting' is an imperfect definition, which ought to be made more perfect by indicating some or all of the characteristics which discriminate a non-combatant from a combatant, *e.g.* age, sex, occupation, etc.

Description has been distinguished from definition, in so far as in a description we state, not the essence or connotation of what is described, but characteristics of the nature of 'accidents,' which are sufficient to enable us to distinguish what is described from other things. A certain house, for instance, might be described as 'No. 22 High Street, Little Peddlington.' Millet's 'Angelus' was described by a framemaker as 'a picture of two people looking sad-like at some potatoes,' and, in the special circumstances of the case, the description was sufficient for identification. It is, however, difficult to draw a sharp line between description and definition. Descriptions are really imperfect attempts at definition.

A distinction has also been drawn between nominal and real definitions, nominal definitions being definitions of names, and real definitions being definitions of things. John Stuart Mill maintains that all definitions are nominal. If the existence of a thing corresponding to the name is implied, that is a postulate and not part of the definition.[1] Definitions, he maintains, are analytic, identical, verbal propositions. This distinction, however, disappears if we accept the view that logic is concerned, not with *names*, but with *terms*, which are names having a meaning, elements

[1] *System of Logic*, bk. i. ch. viii., § 5.

in a system or universe of discourse, and that terms always imply existence in some such system. A *mere* name cannot be defined. But we may indicate (*e.g.* in a dictionary) various senses in which it may be used as a term. In every such use the term refers to things or objects, *e.g.* physical objects, mathematical objects, mythological objects, psychical objects, etc. And the definition of the term is always the meaning of the term as standing for these objects or (what comes to the same thing) the nature of the objects for which the term stands.

The traditional account of definition, which we have so far considered, is in many ways defective. It presupposes the existence of fixed species or real kinds, for only thus is it possible to give any definite meaning to the 'essence' or 'essential qualities' of the species which is defined. The essential qualities are just the qualities of the fixed species. There is a definite core of such qualities, and all other qualities which the species or individual members of it may have are more or less accidental. But we no longer believe in the existence of real kinds, in the original sense of the term, and therefore the old theory of definition gives rise to endless difficulties and to much confusion, even when we modify it by declaring that the definition of a term is the statement of its whole connotation. We have already seen the difficulties involved in determining what *is* the connotation of a term (pp. 36-7). And the traditional view seems to involve the idea that the qualities of a thing are more or less independent, loose and separate, or at least that they are externally related to one another. If we define *x* by the qualities *a*, *b*, *c*, our definition is imperfect unless we can define *a* by the qualities *d*, *e*, *f*, etc., *b* by another set of qualities and *c* by a third; and in order to make the definition perfect we must go on until we arrive at certain qualities which are indefinable. Similarly, if we define by genus and differentia, we must go on to define the genus by a new genus and differentia, and so on until we come to an indefinable genus. The foundation of all

definition must thus be either qualities or classes which are isolated or externally related. We cannot by this method define such terms as 'red' or 'sweet' or 'being' or 'quality.'

Scientific definition does not consist in an enumeration of qualities. Its aim is really to show the position of the thing defined in one or another system, *i.e.* its relation to other elements in the system and to the system as a whole. The definition of a mathematical figure is not an enumeration of its properties, but the determination of its place in relation to other elements in the system of tri-dimensional space. The definition of a species of animal is not a statement of the qualities common to all members of the species, but an account of its position in the whole system of animal life, including environment. The definition of a circle or a lion may be made genetically, by showing how it has come into being, *e.g.* by the movement of a point under certain conditions or by a complex evolutionary process. The movement or the process, however, is not the important element in the definition. It is merely the way in which the relations of the figure to other spatial elements, or of the animal to other forms of life, may be exhibited. And in defining any element in the system we are defining the system as a whole. On this view nothing is indefinable. 'Red' is defined by its place in the system of pure colours. 'Being' and 'quality' are defined by their positions in the system of categories, which includes such other elements as 'substance,' 'relation,' 'quantity,' etc.

Definition, in short, is really the method and aim of the whole process of knowledge; and, as all knowing is a process both of analysis and of synthesis, of discrimination and connection, definition cannot be merely analytic or merely synthetic. The traditional view of definition is an imperfect attempt to express this ideal. It is imperfect because it tends to over-emphasise either the analytic or the synthetic factor. Definition by qualities tends to become mere analysis, reducing the things defined to a number of ultimate indefinables, and thus destroying the systematic

nature of the world of experience. Definition by genus and differentia (*i.e.* by means of classification) tends to become mere synthesis. It begins with the widest genus (say, ' being ') which it limits by a differentia (say, ' corporeal '), and it proceeds to limit or define this by adding new differentiae, until it reaches the *infima species*, the thing defined. This tends to become mere synthesis, for the differentiae are simply added, without any explanation why *this* differentia rather than *that* should be chosen. Of course, both qualities and classes (or genera) are actually relative and inter-related. They constitute systems of an imperfect kind, or rather systems whose unity in difference is imperfectly grasped or expressed. We have seen, *e.g.* (p. 46), that the connotation of a term is not a mere aggregate of qualities, but a system of inter-related qualities; and classes or genera are constituted by such systems of qualities. System is the basis of definition, and the end of definition is to make explicit the systems within systems which constitute the world. Failure to recognize this clearly is the root of the difficulties and defects in the traditional view.

III. DIVISION

As definition is a statement of the connotation of a term, so division is an analysis of its denotation. It is not a mere enumeration of the objects denoted by the term, but it is the analysis of a genus into its various species and subspecies. As the qualities constituting the connotation of a term are inter-related in a system, so the objects denoted by the term can be arranged in classes and subdivisions. Every division must have a principle or basis, which must ultimately have its root in the definition of the genus we are dividing. This basis or principle is usually called the *fundamentum divisionis*. Thus if we divide men into white, black, yellow and red men, the principle of division is evidently their colour, which is a part of the connotation of the term ' men.' But colour, regarded as a characteristic of all men, would not of itself be a sufficient principle

of division. All men have colour, but not any particular colour. The real principle of division in this case is the varieties of colour. And in general it may be said that we divide or analyse the denotation of a term on the basis of the varieties or differences in some quality or qualities which are part of its connotation. The basis of every division, in short, is a universal becoming explicit, a unity revealing itself in its differences or varieties. The species are not separate from the genus : they are a fuller expression of it.

It follows from this that the same term or genus may be divided in various ways, according as we use one or another principle of division. Books, *e.g.* may be divided on the basis of size, shape, colour, weight, subject-matter, language, binding, author's names, titles, publishers' names, places of publication, date, etc. Men may be divided according to colour, race, nation, varieties of skull, occupation, age, etc. The varieties of quality which are the principles of division cannot, of course, be ascertained from the connotation of the term, taken by itself, but only from the connotation in its relation to the denotation. Colour, *e.g.*, as a quality has many varieties ; but all of these varieties are not necessarily involved in any division on the basis of colour. There are many varieties of colour in books ; but few in men.

In a logical division the genus or term to be divided must be predicable of each of the species into which it is divided, *e.g.* white men, red men, etc. are men. In this way logical division is readily distinguished from other processes which may superficially resemble it. Examples of such processes are (*a*) physical partition or the analysis of a thing into its physical parts, *e.g.* a tree into stem, roots, branches, leaves, etc. ; (*b*) 'metaphysical' or conceptual analysis, *i.e.* analysis of a thing or class of things into its attributes, *e.g.* glass into something hard, brittle, transparent, etc.

The rules of division follow directly from the nature of the process. They are as follows :

(1) Each act of division must be founded on one principle

or *fundamentum divisionis*. Breach of this rule is called Cross-Division, *e.g.* the division of men into civilized, red, yellow, white and negro, which evidently involves three principles of division, viz. varieties of culture, of colour and of race.

(2) The division must be exhaustive, *i.e.* the species or sub-classes must be together co-extensive with the whole denotation of the genus. This rule is broken (*a*) if any species is omitted from the division, *e.g.* if animals are divided into carnivorous and herbivorous, and (*b*) if anything is included in the division which is not a species of the genus to be divided, *e.g.* the common, but erroneous division of the estates of the realm into King, Lords and Commons, instead of into Lords Spiritual, Lords Temporal and Commons.

(3) If the division be continued beyond one step, each step must as far as possible be proximate, *i.e.* each species must emerge directly from its proximate genus. *Divisio ne fiat per saltum.* An instance of a breach of this rule would be a division of religions into Polytheism, Catholicism, Protestantism, Judaism, etc. Judaism is a species of monotheism, which is a species on the same level as polytheism. Catholicism and Protestantism are species of Christianity, which is, like Judaism, a species of monotheism.

It is evident that division, so far as we have considered it, is not a purely formal process. It presupposes some knowledge of the matter, *i.e.* of the actual genus to be divided. For instance, rule (2) is inapplicable unless we know the genus so completely that we know all its species. But many formal logicians, adopting a suggestion of Plato, have developed a method of division which appeared to be purely formal. This is called *division by dichotomy.* It consists in dividing at each step into two contradictory terms, *e.g.* A is divided into B and not-B, not-B into C and not-C, and so on. It is evidently analogous to the series of divisions in Porphyry's tree (p. 141). A concrete

example would be the division of the term 'triangle' thus:

- Triangles
  - Equilateral
  - Non-equilateral
    - Isosceles
    - Non-isosccles
      - Scalene
      - Non-scalene

The division seems to be formally exhaustive, because it is made at each step on the principle of Excluded Middle. By this method also it seems possible *a priori* to get a complete classification of all things.

The method of dichotomy, however, is really far from being purely formal.

(1) At each step something *a posteriori* or material is assumed, namely, a real *fundamentum divisionis*. There is nothing in the process itself to indicate whether the attribute we choose as *fundamentum divisionis* is one which can lead to a real division or, on the other hand, is one which belongs without differentia to the whole genus to be divided.

(2) There is nothing to show why the genus should be divided into just these species.

(3) In every such division we must at least end with a term about the reality of which the process leaves us entirely ignorant, *e.g.* there are no non-scalene triangles which are also non-isosceles and non-equilateral; but we know this, not from the method of division, but only from our knowledge of triangles and their species. The negative or infinite term (not-A, not-B, etc.) does not denote any definite class.

(4) This method of division represents co-ordinate species as if they were subordinate to one another, *e.g.* equilateral, isosceles and scalene are co-ordinate species of triangle, the differentia being the equality or inequality of the sides.

IV. CLASSIFICATION

Classification is a development of division, from which it is distinguished mainly in being more elaborate and systematic. The chief object of classification is to reduce numerous varieties of things to unity and order in our minds. This may be illustrated by various expressions of the impulse to collect, which is often the starting-point in scientific inquiry. A boy who collects eggs may begin by arranging them (*a*) according to size or colour. Finding this unsatisfactory he may rearrange them (*b*) according to the species of birds which produce them. As his knowledge of birds increases he may arrange the various kinds (*c*) according to the relations of the species of birds to one another, so far as he knows them, *e.g.* the eggs of birds of prey, sea-birds, song-birds, etc. If (*d*) the eggs of foreign as well as of home birds are included, more complex arrangements will follow, and (*e*) the arrangements will be still more complex if the eggs of other oviparous creatures are included as well as those of birds. Thus the arrangement of the collection, having at first as its basis some simple and obvious common quality, will become gradually more elaborate and systematic until ultimately its basis may be, not the common qualities of the eggs themselves, but the whole evolutionary system of oviparous creatures. The collection itself is, of course, not a classification. It is an arrangement of physical objects on the basis of a classification, which is itself a system of unity and order produced or discovered by mind.

Every classification is made with a purpose. Two main kinds of classification are generally recognized, (1) artificial (sometimes called diagnostic) classification and (2) natural (sometimes called general or scientific) classification. These two kinds of classification are determined by two main kinds of purpose, (1) a more purely practical purpose, which results in artificial classification, and (2) a more purely theoretical purpose, which results in natural classification. A natural classification may be described

as a classification made for the purpose of expressing the actual order, unity or system of the things classified, their real fundamental relations to one another and the unity of which they are elements or differences. In this case the system of classification is its own end, an end in itself. On the other hand, an artificial classification may be described as an arrangement of things in a system which we construct for an ulterior end, for practical convenience in action or thought. " A farmer does not divide plants, like a botanist, into dicotyledonous and monocotyledonous, but into useful plants and weeds." [1] In natural classification we are finding the unity or system of things ; in artificial classification we are putting things into unities or systems. As instances of artificial classifications we may take catalogues of books, arranged as alphabetic lists of titles, and classifications like the Linnean in botany, the basis of which is the various numbers of stamens and pistils in flowering plants. " To think of them in that manner is of little use, since we seldom have anything to affirm in common of the plants which have a given number of stamens and pistils." [2] An artificial classification is usually based on some property, which may not be an essentially important one, but which is easily observed. The really important properties of things, on which a natural classification is based, are not usually the most manifest properties. Thus an artificial classification is mainly of value for purposes of identification or as a key to a natural classification. An alphabetical catalogue of books enables us to identify certain books as contained in a particular library or as the best books on a certain subject, etc. The Linnean classification enables us to identify a plant as having a certain place in a natural classification. In each case the artificial classification is a key either to a natural classification or to some other information about the things classified.

The distinction between a natural and an artificial classification is, however, not an absolute one. An artificial classification must always be based on some natural

[1] Mill, *System of Logic*, bk. iv. ch. vii., § 2. [2] Mill, *loc. cit.*

property, for the nature of the things classified determines the kinds of classification that are appropriate to them and the ends for which the classification is made. Again the properties of a thing are not mutually independent, but themselves form a system or unity, and thus when we find a large number of objects agreeing in some superficial quality, which may be taken as the basis of an artificial classification, there is a considerable probability that they agree also in some more important qualities which might form the basis of a natural classification. On the other hand, every natural classification is artificial to this extent, that it is not an absolute classification, but is limited by the end of the particular science for the sake of which it is made. The special sciences (*e.g.* physics, chemistry, etc.) investigate the same objects from different standpoints. The basis of a chemical would thus differ from that of a physical classification. In both cases the classification would be natural ; but in neither case would it be absolute, for the same objects would be expressed in different systems or unities. But with the progress of knowledge it may in time become possible to discover some deeper and more complex unity by which the various systems of the special sciences may be harmonised. In proportion as anything of this kind is achieved, the artificial element in a natural classification will, of course, disappear. On this account it is usually given as one of the tests of the value of a natural classification that we should be able to arrive at the same distinctions (or classes) from a variety of different considerations (or principles of classification).

There are no strictly logical rules of classification ; but two guiding principles are generally recognized. (*a*) The higher the group in the classification, the more important should be the characters by which it is determined. (*b*) The groups should be so arranged that those which have most affinity with one another may be nearest together.

Principle (*a*), which is sometimes described as the principle of the subordination of characters, suggests the

question : What is meant by an important character ? In an artificial classification (a classification for a special purpose) the importance of an attribute or character is determined by the special purpose of the classification. As regards natural classifications Mill says [1] that important attributes are " those which contribute most, either by themselves or by their effects, to render the things like one another and unlike other things ; which give to the class composed of them the most marked individuality ; which fill, as it were, the largest space in their existence, and would most impress the attention of a spectator who knew all their qualities but was not specially interested in any." Mill also suggests that we should select as important qualities those which are the causes of others. Other writers have, more generally, described important characters as those which furnish an invariable index to the possession of other characters, the degree of importance of the character depending on the number of other characters by which it is accompanied. More recently it has been pointed out by Darwin [2] that in biological, ethnological, philological and other classifications, the most important characters are those which indicate descent. Rudimentary organs in an animal may indicate its connection with far-off progenitors, who may apparently have little in common with the animal itself. On this view a biological classification would be analogous to a genealogical tree.

These considerations indicate that the importance of a character in any classification depends not merely on the number of other characters to which it is an index, but rather on its relation to the system of characters of which it is an element. The view which lays stress on the number of characters indicated is, explicitly or implicitly, based on the assumption that the characters are a sum or aggregate, rather than a system of interdependent attributes. In reality, as we have already seen (p. 47), qualities are always present in a whole or system ; they are always differences within a unity ; and the importance of a quality is deter-

[1] *Loc. cit.* [2] *Origin of Species*, ch. 13 (14[1]).

mined by its relations within the system. The system, of course, may be more or less thoroughly thought out, in proportion to the extent of scientific knowledge at any particular time. Thus different sets of characters may, at different stages of knowledge, be regarded as the most important and used as the basis of classification. The biological system which was the ground of classification before the time of Darwin was a static system—a system of fixed 'natural kinds.' Darwin thought out the system more thoroughly, treating it as a process of descent from generation to generation during vast periods of time and not as a fixed system, the origin of which was not matter of scientific inquiry. From this there naturally followed a change in the order of importance of the characters for purposes of classification.

EXERCISE XI

1. What did Aristotle mean by the Predicables? Under which of the Predicables would you place the predicates of the following propositions?
   (1) Birds are feathered.
   (2) Tom Jones is very ill.
   (3) A straight line is the shortest distance between two points.
   (4) Man is a laughing animal.

2. Explain and illustrate: proprium, differentia, *fundamentum divisionis*, cross-division, *circulus in definiendo*, *infima species*.

3. Explain the nature and discuss the logical value of definition *per genus et differentiam*.

4. What conditions must a satisfactory definition fulfil? Which of the following definitions are faulty, and in what way?
   (1) A university is an educational institution.
   (2) The base of a triangle is the side opposite the vertex.
   (3) A church is a building used for ecclesiastical purposes.
   (4) A triangle is a rectilinear figure contained by three straight lines and having its angles equal to two right angles.
   (5) Man is a human being.
   (6) Prayer is a sincere desire of the soul.
   (7) Time is the moving image of eternity.
   (8) A plumber is a man who is usually assisted by a plumber's boy.
   (9) A kitten is a playful creature which runs after balls of wool.
   (10) A blackleg is a workman who is not a member of a trade union.
   (11) A cow is a horned ruminant.
   (12) A disease is an unhealthy condition of the body or mind, having hurtful or fatal consequences.

5. Are there any indefinables? If so, in what sense are they indefinable?

6. Consider carefully the relation of definition to (*a*) description, and (*b*) division.

7. (*a*) Explain what is meant by a logical division; distinguish it from other kinds of division; and indicate the conditions which it should satisfy.

(*b*) Examine the following divisions:

(1) Pictures into sacred, historical, mythological, landscape and comic.
(2) Men into Mongolians, Africans, white and red.
(3) Living things into moral and immoral.
(4) Terms into collective, general, abstract and singular.
(5) Students into men, women, research, undergraduate.

8. (*a*) Distinguish between a natural and an artificial classification. (*b*) Discuss the statement: 'Different views of classification arise from different views as to the nature of things and their qualities.'

9. Draw up in as clear a form as possible a classification of the subjects which may be studied at a university. Illustrate from your answer the characteristics of a good classification.

10. Criticise the following as definitions:

(1) Noon is the time when the shadows of bodies are shortest.
(2) Virtue is acting virtuously.
(3) Life is the opposite of death.
(4) Music is an expensive noise.
(5) Man is a featherless biped.
(6) Virtue is that line of conduct which tends to produce happiness.
(7) A gentleman is a man who has no visible means of subsistence.
(8) Peace is the absence of war.
(9) Prudence is the ballast of the moral vessel. (Stock.)

CHAPTER XII

THE CATEGORICAL SYLLOGISM

HITHERTO we have been engaged in the consideration of terms and propositions, and of the logical distinctions and problems which they involve. But terms and propositions, as we have seen (pp. 17-18), are abstractions, incomplete elements of thought or reasoning which is the essential subject-matter of logic. Our inquiry, so far as it has gone, has been a confirmation of this. Thought or reasoning is universal, systematic. Every instance of thought or reasoning is an instance of connection and discrimination, an identity pervading differences, and wherever there is identity in difference there is system. In considering terms and their connotation and denotation, propositions and their kinds and import, the Laws of Thought, opposition and immediate inference, definition and classification, we found in every case that the logical distinctions and problems involved depended on the recognition of system in thought. We found it necessary at every stage to presuppose reasoning; we were, indeed, investigating reasoning in certain of its aspects.

All our knowledge presupposes system. When our conscious life begins, we find ourselves in a world apparently confused and chaotic, vague and indefinite. It is not, however, purely chaotic, for if it were, there would be no unity or connection within it and no difference or possibility of discrimination. So far as elements can be related, connected or discriminated, within an apparent chaos, it is not really a chaos, but a system. But undoubtedly we find identities and differences within our world; and all

our knowledge consists in this discovery. By the double-edged process of connecting and discriminating we find things and qualities; we proceed to find kinds of things and to distinguish between the qualities which any kind of things always possesses and those which it possesses occasionally or under certain conditions; we find kinds of kinds of things (genera of species) and thus we classify; we inquire into the conditions under which changes of quality take place and thus we develop the relations of things and qualities to one another, *e.g.* spatial and temporal relations, cause and effect, etc.; and from these relations we proceed to laws and principles. This is, of course, a very incomplete and general description of our thinking; but it may suffice for the purpose of illustration. Now throughout this process we are trying to define the system (or system of systems) in which we find ourselves. That is to say, we are trying to grasp more and more fully and comprehensively the unity or unities which pervade the system and which reveal themselves in all its differences, varieties, and changes. We are seeking to reveal universals, to make explicit the implicit universality of the system. The ideal of all thought or reasoning is to comprehend the system or world, which is the object of the thought or reasoning, as a self-determining universal, *i.e.* a unity which reveals itself in and through its varieties.

It is not easy at once to grasp and accept this account of reasoning. A full understanding of it can come only when we have considered the various forms of reasoning which have been recognized and examined by logicians. But one kind of difficulty which it presents may be briefly considered. We are apt to identify reasoning with argument. We do this because we are inclined to draw a very sharp distinction between facts and reasoning. Facts are often regarded as immediately certain, indubitable, while reasonings are inferences from the facts, constructions of our mind, which can never have the same certainty as the given facts. From the same facts entirely different conclusions may be drawn by different minds. In a law court, for instance,

the counsel on opposite sides will draw quite opposite inferences from the same facts. Each develops his own argument and seeks to overthrow his opponent's. Both cannot be right, and the business of the judge or jury is to determine between them. Now if you ask a really good judge what help he gets from the opposing counsel in coming to a just decision, he will tell you that their arguments, as arguments, are of little use to him, and that their speeches are of value only in so far as they help him to get a clear view of the whole situation presented by the case. His work as a judge is to make for himself a kind of mental model of the whole system of facts and principles involved in the case, so far as he is able to grasp it completely, and when this has been done his decision becomes immediately evident. The system, so far as it is known, determines the conclusion or decision. On the other hand, the opposing arguments of the counsel are reasonings, in so far as they are attempts to interpret the system, *i.e.* to exhibit the facts as expressions of a certain principle or universal, which we usually call a hypothesis or theory. But the arguments are not pure reasonings, disinterested scientific inquiries ; for the business of an advocate is to persuade the judge or jury to decide in favour of his client, and he therefore tends unduly to emphasise those elements in the situation which favour his theory and unduly to minimise or explain away whatever is opposed to it. The aim of the judge, on the other hand, is or ought to be the same as that of the man of science, namely, to take account of the whole situation or system and to reveal its meaning, *i.e.* to exhibit it as a universal pervading the facts or as a coherent whole. So far as this aim is achieved, there is valid reasoning. The facts undoubtedly determine the reasoning ; but they are not bare facts. The determining facts are the facts as a whole (or relatively whole) system, the facts as interpreted through an adequate theory or universal.

In inductive reasoning we endeavour to make explicit the universal implied in the system of facts, while in deductive reasoning we exhibit an explicit universal as it reveals itself

in subordinate universals and ultimately in the whole system which it pervades. There is no essential difference between these two kinds of reasoning ; but it is convenient in this way to discriminate between them. As will appear later (ch. xxiii.), induction and deduction are really two aspects of the same process. In the earliest stages of knowledge the inductive aspect of reasoning is the more prominent, and on the other hand it has often been shown that the ultimate aim of all sciences is to become deductive. It has, therefore, been suggested that inductive reasoning should be studied before deductive. There is undoubtedly something to be said for doing this ; but there are, on the whole, greater advantages in the traditional arrangement by which deduction is considered first.

The logic of deductive reasoning has as its basis Aristotle's doctrine of the syllogism.[1] But before Aristotle developed his systematic account of the subject there were important suggestions of logical method. The Greeks, like many other people, were fond of discussion, and they developed a method of orderly argument, which was described as Dialectic.[2] The method was one of question and answer, and it had considerable analogy with the examination and cross-examination of a witness in a law court. One of the disputants (the Respondent) maintains a thesis or proposition, *e.g.* that virtue can be taught, that the love of gain is not reprehensible, that education ought to be free, etc. The other disputant (the Questioner) tries to overthrow this thesis by leading the Respondent to draw absurd or contradictory consequences from it. The Questioner puts questions to which an answer ' Yes ' or ' No,' with perhaps a few words of explanation, may be given, and the Respondent must answer in this brief and definite way. As a rule the Questioner proceeds along various lines of inquiry,

[1] *An. Pr.*—Almost the whole of this work is devoted to the exposition of the doctrine of the syllogism.

[2] The word ' dialectic ' has been used in a variety of senses by logicians and philosophers in ancient and modern times.

in order to get the Respondent to admit two or more different propositions which, when combined together, yield a conclusion inconsistent with the original thesis.

Socrates made use of a method of this sort, not merely for controversial purposes, but in order to obtain definitions of things and conceptions, *e.g.* definitions of the virtues, of beauty, etc. A possible definition was suggested, *e.g.* 'Courage is remaining at one's post in battle,' or 'Temperance is quietness in behaviour.' This definition was tested by a process of question and answer, and if it was found unsatisfactory it was modified or a more likely definition substituted for it. The true definition was thus gradually approached by the elimination of false or imperfect definitions. But this negative process is very imperfect as a method. It is like trying to find where a thing is by finding where it is not. Some more positive method was required, and Plato suggested a method of definition by means of division. "The thing to be defined or classified is first referred to its genus, and then, by a series of dichotomies, the genus is divided into species and sub-species. At each division we ask to which of the species it gives us the thing to be defined belongs, and that is divided once more, the 'left-hand' species being left undivided as irrelevant to our purpose. The definition is found by adding together all the species 'on the right-hand' side."[1] For instance, we may take an ancient, but not necessarily a good, definition of man. Man comes under the genus being. Beings may be divided into incorporeal and corporeal. Man is corporeal. Corporeal beings may be divided into non-animal and animal. Man is an animal. Animals again may be divided into non-rational and rational. Man is rational. Thus we obtain the definition: 'Man is the rational, animal, corporeal being.' Each part of the definition is wider than the thing defined; but the whole definition must be exactly equivalent to it. This Platonic method introduces a considerable element of order into the process of definition; but it is still imperfect. It assumes much that requires

[1] Burnet, *Greek Philosophy*, Part i. p. 220.

explanation. It presupposes a fixed set of classes, universals, or genera and species, to which things must be referred, and its tendency is to pass over without explanation the connection or community between the genera and species, the way in which one passes into another. Thus it is not shown how or why beings come to be divided into corporeal and incorporeal, how or why animals are included in the class 'corporeal,' and how or why men are included in the class 'animal.' We pass, for instance, from 'Man is corporeal' to 'Man is animal'; but man is not animal *because* he is corporeal. It is quite true that man must be either animal or non-animal; but the same thing may be said with equal truth of 'shoes and ships and sealing-wax and cabbages and kings' or anything else we can speak about. To say that man is non-animal is little better than saying nothing, and if we are to say that he is animal we must have some reason for so doing. It is the ignoring of this reason that makes the Platonic method imperfect.

Aristotle's advance on the Platonic position consists essentially in taking this reason into account. If you know something about the meaning of man and something about the meaning of animal, but do not know whether or not man can rightly be called animal, you must, in your present state of knowledge, try to find some link between the two—something which man is and which at the same time involves animality. This link Aristotle calls the middle term. To find this middle term (in the case of man and animal) we must ask what are the characteristics which make anything an animal. One of these characteristics is that it is a living or at least organic being. But plants also have this characteristic. So we must add a differentia, *e.g.* 'having sensation and the power of independent locomotion,' or any other characteristic which may appear more satisfactory. This is the middle term, for we can say that all things which are organisms having sensation and the power of independent locomotion are animals, and also that all men are organisms having sensation, etc. Men are thus linked with animals, and we can conclude that all

men are animals. When this is carefully and formally stated, we have an instance of what Aristotle called syllogism :

All organisms having sensation, etc. are animals.
All men are organisms having sensation, etc.
∴ All men are animals.

The middle term in every syllogism is called by Aristotle the cause or ground[1] (αἰτία) of the conclusion. Syllogism is thus a form of mediate inference, *i.e.* inference by means of an explicit middle term.

In the syllogism we have just considered, two terms (man and animal) have been connected together by means of a third term (the middle term). The result of the inference is an affirmative proposition. But in order to discriminate between two terms, *i.e.* to establish a negative proposition, a middle term or ground is equally necessary. Thus, if we substitute 'plant' for 'man,' the same middle term as we have used in the previous syllogism enables us to discriminate between plants and animals, *i.e.* to show why plants are not animals. We get, in short, the following syllogism :

No organisms having sensation etc. are plants.
All animals are organisms having sensation etc.
∴ No animals are plants.

A syllogism may thus be provisionally[2] described as a formal statement of the way in which two things or terms are connected together or discriminated from one another by means of a third thing or term (the middle term). If the middle term is connected with both of the other terms, the conclusion asserts the connection of these terms (*i.e.* it is affirmative) ; if the middle term is connected with one term and discriminated from the other, the conclusion states the discrimination of the two terms (*i.e.* it is negative). If a suggested middle term is discriminated from both of the

[1] *An. Pr.* 78^b 4 ; *An. Post.* 89^b 36—90^a 34 ; 93^a 3-8.

[2] For a more precise description of the nature of syllogism, see pp. 226-7.

other terms, it is not really a middle term (because it is not connected with either of the other terms), and accordingly there is no conclusion, *e.g.*

No well-written books are worthless,
No fifth-rate novels are well-written,

from which we cannot infer either that fifth-rate novels are or that they are not worthless. We have thus two main forms of syllogism, one affirmative and the other negative. These may be represented as follows :

Affirmative.	*Negative.*
M is P	M is not P
S is M	S is M
∴ S is P	∴ S is not P

M stands for the middle term, and S and P for the terms to be connected or discriminated from one another. These forms require further definition, as will appear later.

It is evident that every syllogism consists of three propositions. In two of these the relation of the middle term to the other terms is stated, and these two are called the premises of the syllogism. The third proposition states the relation between the two terms other than the middle term, and it is called the conclusion. The predicate of the conclusion in each syllogism is called the *major term*, and the subject of the conclusion is called the *minor term*. The premise which contains the major term is called the *major premise*, and the premise which contains the minor term is called the *minor premise*. The propositions of the syllogism should be arranged so that the major premise comes first, the minor second and the conclusion last.

We have seen that if a suggested middle term is negatively related both to the major and to the minor terms, it is not a real middle term. In addition there is another important condition of the reality of a middle term. It must be distributed in at least one of the premises. If the term suggested as a middle term were undistributed in both premises, it would not have the characteristic of a middle

term, *i.e.* it would not be a common third term, a link between the major and minor terms. Some M, the term undistributed, is so vague that if it is taken in both premises as middle term, we cannot be sure that there is any definite common element between the some M in one premise and the some M in the other. If we say 'Some M is P' and 'Some M is S,' the some M which is P may be entirely different from the some M which is S. In such a case some M is really two distinct terms, and the propositions should be of the form 'X is P' and 'Y is S.' For instance, in the propositions (*a*) 'Some men are white' and (*b*) 'Some men are black,' the 'some men' of (*a*) are quite distinct from the 'some men' of (*b*). The middle term may be, and often is, distributed twice in the premises; but it is sufficient that it should be distributed once, because the whole (all M) includes the part (some M) and there is thus something in common between whole and part (between all M and some M). This common element is the link of connection between S and P.

The ruling principle of syllogism was expressed by Aristotle in the formula: "When one thing is predicated of another as subject, whatsoever things are said of the thing predicated will also be said of the subject."[1] Or, to put it more briefly: "Whatever can be predicated of a predicate" (affirmatively or negatively) "can be predicated" (in like manner) "of its subject".[2] *E.g.* (*a*) The King is a man, (*b*) man is mortal, ∴ (*c*) the King is mortal. This is a syllogism, (*a*) being the minor premise, and (*b*) the major premise, and to satisfy the formula it may be expressed in this way: 'Man' can be predicated affirmatively of 'the King,' and 'mortal' can be predicated affirmatively of the predicate 'man,' ∴ 'mortal' can be predicated affirmatively of the original subject, 'the King.' If in a proposition the predicate is regarded as expressing an attribute, characteristic or mark of the subject (the predicative view, p. 70), Aristotle's formula may be put

[1] *Cat.* 31^b 11. [2] *Cat.* 53^b 4.

thus: '*Nota notae est nota rei ipsius,*' or 'The mark of a mark is a mark of the thing itself.' (In negative syllogisms, the formula would be: '*Repugnans notae repugnat rei ipsi*' or 'What is denied of a mark is denied of the thing itself.') But after the time of Aristotle the 'class' view of the proposition became dominant in the traditional formal logic, which is often called Aristotelian, but is not really Aristotle's. The adoption of this view of the proposition led to a restatement of the principle of syllogism in what is called the *dictum de omni et nullo*. This Dictum may be expressed thus: 'What is true of all the members of a class (or things to which a term refers) is true of each separately, and what is true of none can be denied of each separately.' *E.g.* (*a*) 'mortal' is true of all members of the class 'man,' (*b*) 'the King' is a member of the class 'man,' (*c*) 'mortal' is true of 'the King.' (*a*) is the major premise, and (*b*) the minor premise. This formula explains the name *dictum de omni et nullo*; but the Dictum has been stated in many other ways, of which the most convenient for our purpose is that of Jevons[1]: "Whatever is predicated of a term distributed whether affirmatively or negatively, may be predicated in like manner of everything contained under it"; *e.g.* P can be predicated affirmatively or negatively of all M; S is contained under M; therefore P can be predicated affirmatively or negatively of S.

The principle of syllogism, or the Dictum, may be regarded either (*a*) as self-evident, or (*b*) as the direct consequence or expression of the Laws of Thought. (*a*) The Dictum, that what can be predicated of all the things to which a term refers can be predicated of each of them, is clearly self-evident. Similarly, in the case of Aristotle's formula, if we predicate anything of a subject, we predicate the whole thing, including its various qualities. Self-evidence is, however, a very imperfect ground of belief. Many propositions which have seemed self-evident have been found to be untrue. Therefore, if the Dictum is to be established as absolutely true, it is preferable to regard it (*b*) as directly

[1] *Op. cit.* p. 124.

dependent on the Laws of Thought. These laws, as we have seen (p. 116), are ultimately reducible to a principle, the denial of which makes it impossible to know or to prove anything. This principle is the postulate that thought and things are systematic, that all knowledge is concerned with universals, that the universe is not a mere flux or chaos of particulars. The formula that 'what is predicated of a predicate can be predicated of its subject' could be untrue only if both subject and predicate were so transitory, so merely particular, that there could be no proper predication regarding them. If anything can be predicated of a predicate, it means that the predicate has certain characteristics or qualities which are implied in it or at least can be connected with it. In other words, it means that the predicate has a permanent nature, that it is a universal or unity in difference. And the original subject must also be a universal, if the double process of predication is to be possible. Similarly, as regards the Dictum in Jevons's form, the 'term distributed' is a universal, *i.e.* something that is *one* in various forms, *e.g.* a term that is applicable in the same sense to a variety of objects at various times or to one and the same object at various times or under varying conditions. And to say that whatever can be predicated of the universal can be predicated of everything contained under it is just to say that it *is* the same in its various forms or in the various instances or occasions on which it appears. In short, the Laws of Thought postulate universals, and the Dictum makes clear what a universal is. The Dictum is thus bound up with the Laws of Thought.

In the Dictum, as stated by Jevons, the term distributed is the middle term M of the syllogism, that which is predicated of it is the major term P, and that which is contained under it is the minor term S. Thus the Dictum yields the two forms of syllogism shown on p. 170.

Evidently each of these forms may be analysed into two, one in which all the propositions are universal, while in the other the major premise is universal and the minor premise and the conclusion are particular. The four forms thus obtained

are said to constitute the first Figure of the Syllogism. All reasonings which can be expressed in one or other of these forms are valid. For if a reasoning in any one of these forms is invalid, the Dictum is invalid, and if the Dictum is invalid the Laws of Thought are invalid, and if the Laws of Thought are invalid all reasoning is impossible.

FORMS OF SYLLOGISM.

All M is P	No M is P
$\left.\begin{array}{c}\text{All}\\ \text{or}\\ \text{Some}\end{array}\right\}$ S is M	$\left.\begin{array}{c}\text{All}\\ \text{or}\\ \text{Some}\end{array}\right\}$ S is M
$\therefore$ $\left.\begin{array}{c}\text{All}\\ \text{or}\\ \text{Some}\end{array}\right\}$ S is P	$\therefore$ No S is P or Some S is not P

Aristotle's method of testing the validity of a deductive reasoning was to discover whether or not it can be stated in one or other of these four forms. But there are many deductive reasonings which do not immediately appear in any of these forms, and he devised for the testing of these a process of Reduction, which will afterwards be described (p. 183 ff.). Later logicians devised a different method of testing the validity of syllogisms. Instead of testing reasonings by the forms which follow directly from the Dictum, they framed a set of rules, which may be shown to follow from the Dictum. Every syllogism which conforms to these rules is valid, and no syllogism which fails to conform to them is valid. These Rules of the Syllogism are as follows:

I. Every syllogism contains three, and only three, terms.

II. Every syllogism consists of three, and only three, propositions.

III. The middle term must be distributed in at least one of the premises.

A breach of this rule is called undistributed middle.

IV. No term which is undistributed in the premises may be distributed in the conclusion.

This rule obviously applies only to the major and minor terms. A breach of it is called, in the case of the major term, an illicit process of the major, and in the case of the minor term, an illicit process of the minor. Of course the major or minor term may be *distributed* in its premise and *undistributed* in the conclusion, for what is true of the whole is true of a part, though what is true of a part is not necessarily true of the whole.

V. From two negative premises nothing can be inferred.

In this case, as we have seen (p 165), there is no real middle term. But in applying this rule to particular cases, we must be sure that the premises are really, and not merely apparently, negative. Often a proposition is expressed negatively for the sake of rhetorical effect or for some other reason, when in reality it admits of affirmative statement, *e g* 'No houses in this street are unoccupied' = 'All houses in this street are occupied.'[1]

VI. If one premise be negative, the conclusion must be negative, and if the conclusion be negative, one of the premises must be negative.

From these rules we get two further rules which should strictly be called corollaries, because the proof of them follows from the rules.

VII. From two particular premises nothing can be inferred.

Proof. (*a*) Both premises cannot be negative (rule V). (*b*) If both premises be affirmative,

[1] It should be noted that in Formal Logic two negative *statements* may always be converted into possible *premises* for a syllogism by obversion or contraposition No argument should, therefore, be rejected as invalid on the basis of rule V alone If an argument in the statement of which both premises appear to be negative is really invalid, the application of the processes of immediate inference to its premises will show that it breaks one of the rules regarding distribution. Similar considerations apply in the case of rule VI.

none of the terms is distributed, and consequently there is undistributed middle (a breach of rule III). (*c*) If one premise be affirmative and the other negative, only one term is distributed in the premises, and that term must be the middle term. Otherwise there would be undistributed middle. But the conclusion must be negative (rule VI), and therefore its predicate, the major term, must be distributed. But this involves illicit process of the major (rule IV).

VIII. If one premise be particular, the conclusion must be particular.

Proof. (*a*) If both premises be affirmative (one being particular), only one term is distributed in the premises. That must be the middle term, and therefore both terms in the conclusion must be undistributed, and the conclusion must be particular. (*b*) If one premise be affirmative and the other negative, two terms will be distributed in the premises, viz. the subject of the universal premise and the predicate of the negative premise, and two terms will be undistributed, viz. the subject of the particular premise and the predicate of the affirmative premise. One of the distributed terms must be the middle term and the other must be the major; for, as the conclusion is negative, its predicate, the major term, must be distributed in the conclusion and therefore in the major premise. The minor term must therefore be undistributed both in the minor premise and in the conclusion; and as the minor term is subject of the conclusion, the conclusion must be particular.

It should be noted that this rule does not state that if the *conclusion* be particular, one *premise* must be particular. Two universal premises may yield a particular conclusion.

EXERCISE XII

1. Explain and illustrate what is meant by saying that 'all reasoning presupposes system.'

2. What advance did Logic make among the Greeks before the time of Aristotle?

3. What is a syllogism? Explain Aristotle's view of the principle on which syllogistic reasoning depends? Is this principle susceptible of proof?

4. What is the Middle Term of a syllogism? Explain its function in reasoning. Why must it be distributed at least once?

5. What is the *dictum de omni et nullo*, and how is it related (*a*) to the Laws of Thought, and (*b*) to the rules of the syllogism?

6. Prove from the general rules of the syllogism, (*a*) that no conclusion follows from two particular premises; and (*b*) that a valid syllogism with a negative minor premise requires a universal major premise.

CHAPTER XIII

THE FIGURES AND MOODS OF THE SYLLOGISM, AND REDUCTION

WE have seen (p. 170) that all valid syllogistic reasonings may be reduced to one or other of four forms, which constitute the first Figure of the Syllogism. The figure of a syllogism is determined by the position of the middle term in the premises. If (as in the case of these four forms) the middle term is subject in one premise and predicate in the other, the syllogism, according to Aristotle, belongs to Figure I. Aristotle regarded this figure as the one perfect figure[1]; because it is the only figure which follows directly from the principle of the syllogism. But there are many syllogistic reasonings which do not immediately appear as syllogisms in Fig. I., though they may be valid reasonings. Thus Aristotle[2] recognized two other figures of the syllogism, that in which the middle term is predicate in both premises (Fig. II.), and that in which the middle term is subject in both premises (Fig. III.). It was observed, however, that Fig. I., as above defined, might conceivably have two forms, in one of which the middle term is subject in the major premise and predicate in the minor, while in the other it is predicate in the major premise and subject in the minor. The first of these is the true Fig. I., proceeding directly from the Dictum, and Aristotle, who did not distinguish between major and minor premises, regarded the second form as an imperfect and inverted form of the first. It can be converted into the first form by changing the

[1] *An. Pr.*, 1, 4, 26^b 29. [2] *An Pr.*, 1, 5-6, 26^b 34-29^a 18.

order of the premises. But by later logicians this second form of Fig. I. has been recognized as a separate figure, Fig. IV. It is a figure of comparatively small value. Very few reasonings naturally fall into it, and most of the reasonings which appear in it would be better expressed in one of the other figures. On the other hand, the recognition of it as separate enables us to define Fig. I. more exactly and gives us distinct figures for each of the possible positions of the middle term in the premises. We have thus the following symbolism for the figures of the syllogism :

Fig. I.	Fig. II.	Fig. III.	Fig. IV.
MP	PM	MP	PM
SM	SM	MS	MS
SP	SP	SP	SP

Each of the propositions contained in each of these figures may be an A, an E, an I, or an O proposition. Syllogisms thus vary according to the quantity and quality of the propositions which constitute them, one being AAA another AEE, and so on. This character of the propositions constituting a syllogism is called the Mood of the syllogism, and accordingly we speak of the mood AAA, the mood AEE, etc. As there are four letters (A, E, I, O) indicating propositions and there are three propositions in each syllogism, the total number of possible moods, in any one figure, is the third power of 4, *i.e.* 64. But of these 64 possible moods many are invalid in all the figures, and others are valid only in some figures and not in all. Leaving aside for the present Aristotle's method of determining which of the possible moods are valid (p. 183 ff.) we may adopt the method of later logicians, which is (*a*) to eliminate from the possible moods such as are manifestly inconsistent with the rules of the syllogism, and then (*b*) to test those which remain in order to discover in which of the figures they are valid. The process (*a*) will consist in eliminating all moods which have two negative premises, two particular premises, two affirmative premises with a negative conclusion, a negative premise with an affirmative

conclusion, or a particular premise with a universal conclusion. If this be done it will be found that twelve moods remain, viz. AAA, AAI, AEE, AEO, AII, AOO, EAE, EAO, EIO, IAI, IEO, and OAO. But one of these moods, IEO, is clearly a case of illicit process of the major; for the major term, being the predicate of a negative proposition, is distributed in the conclusion, and as the major premise is an I proposition, the major term is undistributed in its premise. The mood IEO must therefore be eliminated, and we have eleven moods possible, but not necessarily valid, in all the figures.

This completes process (*a*), and we turn to process (*b*). Here we may follow one of two methods. We may either test, by the rules of the syllogism, each of the eleven moods in each of the four figures and eliminate all moods in which we find undistributed middle or illicit process, or we may discover and prove special rules for each of the four figures, *i.e.* the conditions which any mood must fulfil if it is to be valid in a particular figure. The first of these methods may be usefully followed as an exercise; but the second is more valuable, as it indicates some important characteristics of the different figures. The discovery of the rules is made by examining each figure and inquiring what we can ascertain from its nature as to the quality and quantity of its premises and conclusion. For example it is evident in the case of Fig. II., that one of the premises must be negative, because the middle term is predicate in both premises and would therefore be undistributed, if both premises were affirmative. It follows that the conclusion is negative and also that the major premise must be universal, for the major term is subject in its premise and must therefore be distributed, in order to avoid the fallacy of illicit process. The rules of the other figures may be found in a similar way. It is, however, more convenient to state the rules and prove them, as follows:

FIGURE I. (1) The major premise must be universal. For if it were particular and the minor premise were affirmative, the middle term, being subject in the major

premise and predicate in the minor, would be undistributed in both. If the major premise were particular and the minor premise negative, the major term, being predicate of an affirmative premise, would be undistributed and as the conclusion is negative, there would be illicit process of the major.

(2) The minor premise must be affirmative. If it were negative, the major premise would be affirmative, and there would thus be illicit process of the major.

Figure II. (1) One of the premises must be negative in order to distribute the middle term. (The conclusion in this figure will therefore be negative).

(2) The major premise must be universal. Otherwise there would be illicit process of the major.

Figure III. (1) The minor premise must be affirmative. For if it were negative, there would be illicit process of the major, as in Fig. I.

(2) The conclusion must be particular. For the minor term, which is the subject of the conclusion, is the predicate of an affirmative premise.

Figure IV. (1) When the major premise is affirmative, the minor premise must be universal. Otherwise there would be undistributed middle.

(2) When the minor premise is affirmative, the conclusion must be particular. Otherwise there would be illicit process of the minor.

(3) In negative moods the major premise must be universal. For, as the conclusion is negative, the major term is distributed in the conclusion and must therefore be distributed in its premise, of which it is the subject.

Now if we test the eleven moods by these rules, we find that in each figure there are six valid moods, viz. :

Figure I.	AAA, EAE, AII, EIO, AAI, EAO.
Figure II.	EAE, AEE, EIO, AOO, EAO, AEO.
Figure III.	AAI, IAI, AII, EAO, OAO, EIO.
Figure IV.	AAI, AEE, IAI, EAO, EIO, AEO.

There are thus twenty-four valid moods of the syllogism. But of these some, though legitimate, are useless as forms of reasoning, for in their respective figures they draw particular conclusions, while their premises warrant universal conclusions. These are called weakened syllogisms or subaltern moods. The subaltern moods are : In Fig. I. AAI (subaltern to AAA) and EAO (subaltern to EAE) ; in Fig. II. EAO (subaltern to EAE) and AEO (subaltern to AEE) ; in Fig. III. none ; in Fig. IV. AEO (subaltern to AEE). There are no subaltern moods in Fig. III., because all the conclusions in that figure are particular. If we remove these subaltern moods as useless, we have nineteen valid moods remaining, four in Fig. I., four in Fig. II., six in Fig. III. and five in Fig. IV.

In order to facilitate the recollection of these valid moods the following set of mnemonic verses was invented in Scholastic times :

> *Barbara*, *Celarent*, *Darii*, *Ferio*que, prioris ;
> *Cesare*, *Camestres*, *Festino*, *Baroko*, secundae ;
> Tertia, *Darapti*, *Disamis*, *Datisi*, *Felapton*,
> *Bokardo*, *Ferison*, habet ; quarta insuper addit
> *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

The vowels of the words in italics indicate the various moods, *e.g.* *Barbara* stands for the mood AAA, *Celarent* for the mood EAE, etc. The moods of Fig. I. are given in the first line, those of Fig. II. in the second line, those of Fig. III. in the third and fourth lines, and those of Fig. IV. in the fifth line. It is usual, for convenience, to speak of ‘ the mood *Barbara*,’ instead of ‘ the mood AAA in Fig. I.’, ‘ the mood *Celarent* ’ instead of ‘ the mood EAE in Fig. I.’, ‘ the mood *Cesare* ’ instead of ‘ the mood EAE in Fig. II.’ etc. The mnemonic lines have also signification in connection with the process of Reduction, as will be afterwards explained.

Any concrete syllogistic reasoning may be tested by stating its propositions in logical form, discovering its figure and mood, and ascertaining whether or not it

belongs to one or other of the valid forms. If it should not belong to any of the valid forms, it will be found to break one or more of the rules of syllogism. Take, for example, the statement : 'The earth, being a planet, moves round the sun.' In order to express this (or any other reasoning) in syllogistic form we must first find and state logically its conclusion. In this case the conclusion is evidently 'The earth is a body which moves round the sun.' We have thus obtained the minor term, 'the earth,' and the major term, 'a body which moves round the sun.' The minor premise is clearly 'The earth is a planet.' Both the conclusion and the minor premise are A propositions, and if the syllogism is to be valid the major premise must also be an A proposition, consisting of the major and middle terms. The middle term is evidently 'planet,' and the major premise is thus 'All planets are bodies which move round the sun.' The syllogism accordingly is :

	A	All planets are bodies which move round the sun.
Fig. I.	A	The earth is a planet.
	A	∴ The earth is a body which moves round the sun.

The mood of the syllogism is *Barbara*, and the syllogism is therefore valid. As another instance we may take this : 'His cowardice might have been inferred from his cruelty ; for all cowards are cruel.' Here the conclusion is 'He is a coward.' The minor premise is 'He is cruel,' and the major premise is given 'All cowards are cruel.' Accordingly the syllogism is :

	A	All cowards are cruel.
Fig. II.	A	He is cruel.
	A	∴ He is a coward.

The syllogism is thus invalid, the fallacy being undistributed middle. If the reasoning had been merely : 'His cowardice might have been inferred from his cruelty,' we might have expressed this as a valid syllogism in *Barbara*,

by substituting 'All cruel men are cowards' for 'All cowards are cruel.' In cases like this, where the major or minor premise is omitted in the given statement of the reasoning and there are thus alternatives between which we may choose, we should always select an obviously true premise in preference to an obviously false one. If both alternatives seem equally true, equally probable or equally doubtful, we should select that which yields a valid syllogism in preference to that which yields an invalid one. In short, we should give the reasoning the benefit of the doubt. These difficulties arise mainly when we are asked to consider a briefly stated reasoning, cut off from its context. When the context is known or can be indirectly reconstructed (*e.g.* in the case of a quotation from a well-known book or passage), we can generally ascertain from the context which of the alternatives was intended to be used as the premise.

We have so far considered merely the formal differences between the figures, *i.e.* their differences as regards the position of the middle term in the premises. But these formal differences depend on real or essential differences in the nature of the reasonings which find appropriate expression in one or other of the important figures, viz. Fig. I., Fig. II. and Fig. III. We must now inquire into these special characteristics and uses of the figures.

I. Regarding Fig. I., Aristotle says[1]: "The scientific knowledge of the nature of a thing (literally, what a thing is) can be ascertained through the first Figure alone. For in the second Figure there is no affirmative syllogism, and the scientific knowledge of the nature of a thing is affirmative. In the third Figure there are affirmative conclusions, but they are not universal, and the nature of a thing is universal." The general principle underlying this statement may be put thus. The object of all scientific knowledge is definition, and a thing or species can be defined only through universals. In our earliest knowledge things are vague, blurred, indefinite, like things seen in a heavy fog.

[1] *An. Post.* i. 14, 79ᵃ 24.

Our knowledge progresses by defining things, *i.e.* by finding qualities in them and by discovering that they belong to this or that kind or species. But qualities or kinds are universal, and our definition may therefore be regarded as a universalizing of the given things to be defined. On the other hand we cannot universalize a thing without, by the same process, particularizing or individualizing it.

Suppose that you are walking in a fog, and a vague indefinite object looms out before you. You do not know what it is ; but as you advance it becomes more distinct, and presently you recognize that it is a tree. What you have done is to universalize the vague object which you first observed, by finding that it is a particular instance of a general type or kind. But in universalizing it you have also particularized or individualized it. As it appeared at first it might have been, so far as you knew, any one of a variety of things—a man, a tree, a lamp-post, a telegraph-post, a pillar, a chimney, and so on ; but now you have defined it as a tree, and you have thus made it, for your knowledge, much more particular and individual than it was. Now this process is characteristic of all knowledge and of all science. If you do not know much about ships, you can still distinguish a battleship from a torpedo-boat, or a liner from a tug. But a naval officer can tell you to what class a particular battleship belongs, and an expert in mercantile ships can recognize a particular liner as belonging to a particular company. The ordinary man's perception or knowledge of the ships is vaguer, more indefinite than that of the expert ; he sees less, and his knowledge is relatively foggy. Now the first Figure of the syllogism is the appropriate expression of scientific knowledge in this sense. It consists essentially in showing that the minor term S, being an instance or variety of a whole or unity M, may be more fully defined as having the characteristics P of that whole or unity and, in the case of the negative syllogism, as not having the characteristics which do not belong to that whole or unity. In other words, a syllogism in Fig. I. exhibits a unity as it appears in one or more of its

varieties or differences, and in so doing it further defines both the unity and its varieties, *i.e.* the whole universal.

It may be noted also that Fig. I. is the only figure in which valid conclusions may be drawn in all the four kinds of categorical propositions (A, E, I and O).

II. The second Figure is appropriate to a less advanced stage of knowledge or reasoning than that which appears in the first Figure. The chief characteristic of Fig. II. is that it represents a stage in the process of obtaining a positive or affirmative definition through a unity or universal. Reasoning in Fig. II. means negative definition, trying to find what a thing *is* by showing what it *is not*. The middle term M is affirmed of P and denied of S, or affirmed of S and denied of P, and consequently S and P are discriminated from one another, *i.e.* P cannot be a definition of S. Illustrations of this method may be found in Plato's Socratic dialogues. For instance, in the *Laches*, courage is hypothetically defined as endurance. But it is pointed out that courage is a good thing, while endurance may be hurtful, and on this account the suggested definition is rejected. This may be expressed as a syllogism in Fig. II. :

E Endurance is hurtful (*i.e.* not good).
A Courage is good.
E ∴ Courage is not endurance.

Similarly Aristotle in his *Ethics*, bk. vi. ch. 9, in considering the nature of deliberation, distinguishes it from scientific knowledge, opinion, conjecture, etc. This also may be expressed in a number of syllogisms in Fig. II., *e.g.* Scientific knowledge is M, deliberation is not M, ∴ deliberation is not scientific knowledge, etc.

III. The third Figure is appropriate to another incomplete stage in the process of definition. Its conclusions are all particular and thus suggest a partial or problematic connection or discrimination between the minor and the major terms. As the middle term is subject in both premises, the predicates of the premises are predicates of

the middle term. Consequently, if both premises are affirmative, it seems probable that there is some connection between their predicates, while if one premise is affirmative and the other negative, it is probable that there is no important connection between them. The conclusion of a reasoning in Fig. III. thus either suggests a possible definition (or universal affirmative statement) or, if it is negative, refutes such a suggestion, *e.g.*

A Gladstone, Beaconsfield, Balfour, Morley, Haldane, etc. are authors.
A Gladstone, Beaconsfield, Balfour, Morley, Haldane, etc. are statesmen.
I ∴ Some statesmen are authors.

This is an instance of *Darapti*. Taken by itself, it suggests the possibility that authorship may be one of the essential characteristics of statesmen. But this may be refuted by a syllogism in *Felapton* :

E Salisbury, Chamberlain, Lansdowne, Lloyd George, Harcourt, etc. are not authors.
A Salisbury, Chamberlain, Lansdowne, Lloyd George, Harcourt, etc. are statesmen.
O ∴ Some statesmen are not authors.

In general it may be said that an enumeration of instances, in any scientific or other inquiry, may suggest the likelihood of a strictly universal or generic proposition : but such a proposition cannot be proved in this way. Negative instances may, however, disprove the suggestion.

Aristotle described Fig. I. as the perfect figure, and the others as imperfect figures.[1] He tested syllogisms in the imperfect figures not, as was afterwards done, by means of rules of the syllogism, but by showing that they are equivalent to syllogisms in Fig. I. The method by which this is done is called Reduction, which may be defined as the process of altering the expression of a reasoning given in

[1] *An. Pr.*, 26b 29 ; 27a 16 ; 28a 4.

Fig. II., Fig. III. or Fig. IV., so that it may appear as a reasoning in Fig. I. As the figure of a syllogism is determined by the position of the middle term in the premises, alteration of the figure may evidently be made either by changing the order of the permises or by converting one of the premises or by both of these processes. For instance, in order to change *Cesare* from a mood of Fig. II. into a mood of Fig. I., all that is necessary is to convert the major premise.

Fig. II. *Cesare.*		Fig. I. *Celarent.*	
E	No P is M.	E	No M is P (conv.).
A	All S is M.	A	All S is M.
E	∴ No S is P.	E	∴ No S is P.

The reduction of such a mood as *Disamis*, Fig. III., is rather more elaborate.

Fig. III. *Disamis.*		Fig. I. *Darii.*	
I	Some M is P.	A	All M is S.
A	All M is S.	I	Some P is M (conv.).
I	∴ Some S is P.	I	∴ Some P is S.
			∴ Some S is P (conv.).

As the major premise of Fig. I. must be universal it is necessary to transpose the premises ; and the given major premise must be converted, in order that M may be the predicate of the minor premise in Fig. I. From these premises the conclusion is Some P is S, and this must be converted to get the given conclusion, Some S is P.

The mnemonic verses contain signs indicating what has to be done in order to reduce the moods of the imperfect figures to moods of Fig. I. The vowels stand for the moods. The letter 's' in the middle of an italicized word means 'convert simply the proposition indicated by the preceding vowel.' The letter 'p' in the middle of a word means 'convert the preceding proposition *per accidens*.' The letter 'm' means 'transpose the premises.' The letters 's' or 'p' at the end of a word mean 'convert simply or *per accidens* the conclusion of the first Figure syllogism obtained from the altered premises, in order to get the

original conclusion.' The initial letter of each italicized word indicates the mood in Fig. I. to which the other is to be reduced, *e.g.* B stands for *Barbara*, C for *Celarent*, etc. The letter 'k' has a meaning which will be explained presently. The other letters have no special meaning.

If we examine the various moods we shall find that all of them, except *Baroko*, Fig. II. and *Bokardo*, Fig. III., can be reduced to Fig. I. by the method of conversion and transposition of premises. In the cases of *Baroko* and *Bokardo*, however, transposition of premises alone will not effect Reduction, for the middle term in *Baroko* is predicate in both premises and the middle term in *Bokardo* is subject in both. And conversion will not help us, for if we convert the A proposition we get an I, the premises thus becoming IO, from which, as they are both particular, there is no conclusion. The O proposition, of course, is inconvertible. In order to reduce these two moods Aristotle had recourse to another method. The method of reduction by conversion and transposition of premises is called direct reduction, and the other method is called reduction *per impossibile* or indirect reduction. In indirect reduction we take as a hypothesis that the conclusion of *Baroko* or *Bokardo* is false, and then show, by means of a syllogism in *Barbara* (hence the initial 'B'), that our assumption leads to a conclusion which is inconsistent with the original premises.

All P is M.
Some S is not M. *Baroko.*
∴ Some S is not P.

The premises are given as true, and it is to be shown, by means of a syllogism in *Barbara*, that the conclusion must also be true. Suppose that the conclusion is false. Then its contradictory, All S is P, is true. Combine All S is P with the given major premise, All P is M, so that they may be premises of a syllogism in Fig. I.:

All P is M.
All S is P. *Barbara.*
∴ All S is M.

Now All S is M is the contradictory of the given minor premise, Some S is not M. But the given minor premise is true, therefore its contradictory, All S is M, is false. All S is M, however, is the conclusion of a valid syllogism in Fig. I., and therefore its falsity must be due to falsity in the premises of that syllogism. But the major premise, All P is M, is given as true, being a premise of the original syllogism. Therefore All S is P is false, and its contradictory, Some S is not P, is true. A similar proof may be given in the case of *Bokardo*, and all the moods of the imperfect figures may also be reduced in this way, by means of a syllogism in Fig. I. (not necessarily *Barbara*). The letter 'k' in *Baroko* and *Bokardo* indicates that they are to be reduced indirectly.

By using modern methods of immediate inference *Baroko* and *Bokardo* may be reduced directly to Fig. I. This is done by the use of obversion and contraposition, which enable us to overcome the difficulty due to the inconvertibility of the O proposition. Thus *Baroko* may be reduced to *Ferio* by substituting for the major premise its partial contrapositive and for the minor premise its obverse :

Baroko.	*Ferio.*
All P is M.	No not-M is P (partial contrapositive).
Some S is not M.	Some S is not-M (obverse).
∴ Some S is not P.	∴ Some S is not P.

Similarly *Bokardo* may be reduced to *Darii* by substituting for the major premise its partial contrapositive and transposing the premises :

Bokardo.	*Darii.*
Some M is not P.	All M is S.
All M is S.	Some not-P is M.
∴ Some S is not P.	∴ Some not-P is S.
	∴ Some S is not-P (converse).
	∴ Some S is not P (obverse).

The whole process of Reduction is mainly of historical interest, as the testing of concrete syllogistic reasonings may

be more conveniently done by means of the rules of the syllogism. And if, as we saw, the different figures have different characteristics which make each the appropriate expression of a particular kind of reasoning, reasonings fall naturally into one figure rather than another ; so far as the process of Reduction suggests that all reasonings can be put indifferently into any figure, or even into the first figure, it is misleading. Indirect Reduction, however, has interest as an instance of an ingenious method of proof.

In conclusion, we may note a simple method of testing the validity of arguments. If we assume rules I. and II. (which are simply definitions of what a syllogism is), and put an argument into logical form, in such a way as to guard against a breach of rules V or VI (*v.* p. 171 footnote), the reasoning is valid, provided that the middle term is distributed once at least and that no term which is undistributed in the premises is distributed in the conclusion. Accordingly, if we indicate as to each of the terms whether it is distributed or undistributed, the validity or invalidity of the syllogism will at once become apparent. Take for instance the argument : 'Only the ignorant despise knowledge ; hence this man cannot be ignorant, for he praises it.' Put in logical form, the reasoning appears thus :

All who despise knowledge are ignorant.
This man is not a person who despises knowledge.
∴ This man is not ignorant.

The distribution of the terms may be indicated in this way:

m d	*u*
d	*m d*
d	*d*

d stands for 'distributed,' *u* for 'undistributed,' and *m* for 'the middle term.' We see at a glance that this is a case

of illicit process of the major, and the reasoning is therefore invalid.[1]

EXERCISE XIII

1. Express the following arguments in the form of syllogisms, supplying any premises which are not explicit. If they are valid, name their figure and mood; if they are invalid, point out and name the fallacies they contain:
   (1) He is certainly guilty, for he behaved as a guilty man would.
   (2) I do not care for any of the books in this library, for all of them are mathematical.
   (3) Socrates must have been happy, for wise men alone are happy.
   (4) Seeing is believing; hence I refuse to believe in God.
   (5) It is possible to be a learned man and a fool. James I was both.
   (6) He need not fear madness, for he has no learning, and learning makes men mad.
   (7) The express train alone does not stop at this station; and as the last train did not stop it must have been the express.

2. What can be concluded from the following premises? Express the argumentation in strict form:
   (a) No one is a true poet unless he can stir the hearts of men.
   (b) Shakespeare wrote 'Hamlet.'
   (c) No writer who does not understand human nature can stir the hearts of men.
   (d) No one but a true poet could have written 'Hamlet.'

3. (a) State and prove the special rules of Figure II of the syllogism.
   (b) A valid syllogism with a universal conclusion has only two terms distributed in the premises. Determine its figure and mood.
   (c) Given that O is the major premise of a valid syllogism, determine, by the aid of the rules regarding distribution, its figure and mood.

4. (a) Why is the mood OAO excluded from the first and second figures?
   (b) How much can you tell of a valid syllogism in each of the following cases?
      (1) Given that the middle term alone is distributed.
      (2) Given that the middle and minor terms are distributed in the premises, but not the major.
      (3) Given that all three terms are distributed in the premises.
   (c) Show that an O proposition cannot stand as a premise in Fig. I., as a major in Fig. II., as a minor in Fig. III., or as a premise in Fig. IV.
   (d) Why are premises of the form IE inadmissible in any figure of the syllogism, while EI is a valid mood in all?

[1] I am indebted to Professor Leonard J. Russell for the suggestion of this method.

5. What is meant by Reduction? Construct a concrete syllogism in the mood AEE in the second figure and reduce it to the first figure both directly and indirectly.

6. Explain the characteristics of each of the figures of the syllogism and indicate the kind of reasoning which each is specially suited to express.

7. Express the following arguments in the form of syllogisms and examine their validity:

(1) No really musical person would applaud that, though some of the audience are applauding it.
(2) He who is most hungry eats most; he who eats least is most hungry; therefore he who eats least eats most.
(3) You say this man ought not to be punished. Am I to infer that you believe him to be innocent? For certainly an innocent man ought not to be punished.
(4) All trained logicians can detect fallacies; but few electors are trained logicians, so that few electors can detect fallacies.
(5) A man may be without a vote though he is not a minor; but from the fact that Mr. A. has a vote I am able to infer that he is not a minor.
(6) How can you maintain that anger is always an evil, and also admit that moral indignation is anger and yet may sometimes be good?
(7) What is the use of all this teaching? Every day one hears of a fraud or a forgery committed by some one who might have led an innocent life, if he had never learned to read and write.
(8) 'Direct action' by industrial organizations for political purposes is quite consistent with a Democratic form of Government. For a general strike could succeed only if the great mass of popular opinion supported its object; so that any decision of the Government against which 'direct action' was effective would not be an expression of the people's will.

8. (*a*) Prove by the general rules of the syllogism that no syllogism in Fig. IV. can be valid which has a particular negative premise or a universal affirmative conclusion.
(*b*) Construct a syllogism in the mood EAO in Fig. III. and reduce it to Fig. I. both directly and indirectly.
(*c*) Construct a concrete syllogism in the mood AEE in Fig. II. and consider whether you can get a conclusion of equal force, or any formally valid conclusion, (i) after obverting both premises; (ii) after obverting one premise and converting the other; (iii) from the contradictories of the original premises.
(*d*) Prove by the general rules of the syllogism:

(1) If the conclusion of a syllogism is universal, the middle term can be distributed only once.
(2) When the minor term is predicate in its premise, the conclusion cannot be an A proposition.
(3) Any mood which is valid in Fig. II. and Fig. III. is also valid in Figs. I. and IV.

(*e*) If it is (1) false that whenever x is present y is found with it, and (2) not less untrue that x is sometimes found without the accompaniment of z, are we justified in denying (3) that wherever z is found there you are sure of finding y? And however that may be, can we in these circumstances judge anything of y in terms of z?

9. Analyse the following arguments and examine their validity:

(1) Boswell: "After arguing the common plausible topics, I at last had recourse to the maxim *in vino veritas*—a man who is well warmed with wine will speak the truth."
Johnson: "Why, sir, this may be an argument for drink, if we suppose men in general to be liars."

(2) 'I argue thus: the world agrees
That he writes well, who writes with ease.
Then he, by sequence logical,
Writes best who never thinks at all.'

(3) It cannot be contended that honesty is the best policy, and at the same time that a man ought to pursue his own interests. For very often it is against one's interests to be honest.

(4) No man desires pain; and without pain your friend's cure is impossible. Therefore he will not desire to be cured.

(5) To be of any value the lesson should arouse the child's interest. But a child's interests are not the same as a grown-up's. Thus what interests the child does not interest the teacher. Hence to be of any value the lesson should not interest the teacher. (Russell.)

CHAPTER XIV

CONDITIONAL REASONINGS

THERE are three main forms of conditional reasoning: (1) the hypothetical syllogism; (2) the disjunctive syllogism; and (3) the dilemma.

I. A hypothetical syllogism is a reasoning which has a hypothetical major premise and a categorical minor premise and conclusion. In order that such a reasoning may be valid either the antecedent of the major premise must be affirmed in the minor premise and the consequent affirmed in the conclusion or the consequent of the major premise must be denied in the minor premise and the antecedent denied in the conclusion. This rule follows from the nature of the hypothetical proposition (*v.* p. 97 ff.). The hypothetical proposition asserts that the antecedent is a condition or ground (but not necessarily the only condition or ground) of the consequent. To affirm the antecedent is therefore to affirm the consequent; but to affirm the consequent is not necessarily to affirm the antecedent, for the consequent may have some other condition or ground. Similarly to deny the consequent is to deny the antecedent, for to deny what necessarily follows from a condition or ground involves denial of all its conditions or grounds; if premises necessarily result in a conclusion, denial of the conclusion must mean denial of the premises, *i.e.* if the conclusion is false the premises must be false. On the other hand, to affirm the consequent is not necessarily to affirm the antecedent, for the consequent

may follow from other conditions or grounds than those set forth in the antecedent.

There may be cases in which this rule of the denial of the consequent does not appear to hold, *e.g.* ' If the earth is round, the sea is shallow ; but the sea is not shallow, therefore the earth is not round.' But the discrepancy is only apparent. In every such case there is no real reasoning, no genuine dependence of the consequent upon the antecedent, of the conclusion upon the premises. The major premise is hypothetical only in form and is really meaningless. It is a statement like ' Abracadabra is a second intention ' or ' If potatoes are cheap, there will be an eclipse of the sun.' Aristotle says [1] that " if the conclusion is false, all or some of the premises must be false, but when the conclusion is true, it does not follow that any or all of the premises are true ; but although there be nothing true in the data of the reasoning, the conclusion may yet be true, but not necessarily," *i.e.* not as the conclusion from these premises. To meet the case we are considering this should be supplemented by the statement : Although the conclusion be false, all or some of the premises may yet be true, but not as premises of that conclusion. It may be true that the sea is not shallow, but it is not true as a ground of the conclusion that the earth is not round.

There are evidently two moods of the hypothetical syllogism, and two formal fallacies of hypothetical reasoning. The moods are the *modus ponens*, in which the antecedent is affirmed in the minor premise and the consequent is affirmed in the conclusion, and the *modus tollens*, in which the consequent is denied in the minor premise and the antecedent is denied in the conclusion. Formally these moods may be stated thus :

Modus ponens.	*Modus tollens.*
If A is B, C is D.	If A is B, C is D.
A is B.	C is not D.
∴ C is D.	∴ A is not B.

[1] *An. Pr.* ii. 4, 57^a 36.

The fallacies are the forms in which the antecedent is denied or the consequent affirmed in the minor premise :

If A is B, C is D.	If A is B, C is D.
A is not B.	C is D.
∴ C is not D.	∴ A is B.

If we state the hypothetical major premises in categorical form, we find that the two moods of the hypothetical syllogism correspond to two valid moods of the categorical syllogism, and that the two fallacies of hypothetical reasoning correspond to two invalid moods of the categorical syllogism. Thus the *modus ponens* becomes :

All cases of A being B are cases of C being D.
This is a case of A being B. *Barbara*, Fig. I.
∴ This is a case of C being D.

The *modus tollens* similarly becomes :

All A-B are C-D.
This is not C-D. *Camestres*, Fig. II.
∴ This is not A-B.

Similarly the fallacies correspond respectively to illicit process of the major and undistributed middle.

Some logicians describe the hypothetical syllogisms we have so far considered as mixed hypothetical syllogisms, because they consist of one hypothetical and two categorical propositions. In distinction from these they find a class of pure hypothetical syllogisms, in which all the propositions are hypothetical, *e.g.*

If C is D, E is F.	If C is D, E is F.
If A is B, C is D.	If A is B, E is not F.
∴ If A is B, E is F.	∴ If A is B, C is not D.

But it is simpler to regard these pure hypothetical syllogisms as cases in which the antecedent is affirmed or the consequent denied under a condition, which condition, of course, applies to the affirmation or denial in the conclusion.

It should be noted that, when the antecedent in the major premise of a hypothetical syllogism is qualified by the word 'only' or some equivalent expression, the syllogism is valid

whether the antecedent or the consequent is affirmed or denied in the minor premise. The only reason why the denial of the antecedent and the affirmation of the consequent are invalid is that the antecedent of an ordinary hypothetical proposition is not necessarily the only condition of the consequent. If the antecedent is known to be the only condition of the consequent, this reason disappears. Thus with the proposition, Only if A is B, C is D, as a major premise, we can argue validly in any of the following forms: A is B, ∴ C is D; A is not B, ∴ C is not D; C is D, ∴ A is B; C is not D, ∴ A is not B.

In some hypothetical propositions the subject of the antecedent or the subject of the consequent or both may be quantified, *e.g.* If all A is B, some C is D; If some A is B, some C is not D, etc. In such cases it is necessary to remember that, in denying the antecedent in the conclusion of the syllogism, we must do so by means of the contradictory and not the contrary of the antecedent, *e.g.* If all A is B, some C is D; but no C is D; ∴ some A is not B (not 'no A is B' or 'A is not B'). The following is an instance of a hypothetical syllogism, which is invalid through neglect of this distinction: 'If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it.'

II. A disjunctive syllogism is a reasoning which has a disjunctive major premise and a categorical minor which affirms or denies one member of the disjunction, while the conclusion denies or affirms the other member of the disjunction. It is insufficient to describe a disjunctive syllogism as 'a reasoning which has a disjunctive major premise and a categorical minor'; for in a disjunctive syllogism the reasoning must turn on the disjunction. For instance, the following syllogism, which has a disjunctive major premise and a categorical minor, is not a disjunctive, but a categorical syllogism:

M is either P or Q.
S is M.
∴ S is either P or Q.

In this case the reasoning does not turn on the disjunction. The whole disjunctive part reappears in the conclusion, and the syllogism is essentially the same as :

M is X.
S is M.
∴ S is X.

There are thus two moods of the disjunctive syllogism, (*a*) one in which either alternative is affirmed in the minor premise and as a consequence the other alternative is denied in the conclusion, and (*b*) one in which either alternative is denied in the minor premise and the other is consequently affirmed in the conclusion.

(*a*) is called the *modus ponendo tollens* :

Either A is B or C is D.		Either A is B or C is D.
A is B	or	C is D
∴ C is not D.		∴ A is not B.

(*b*) is called the *modus tollendo ponens* :

Either A is B or C is D.		Either A is B or C is D.
A is not B	or	C is not D.
∴ C is D.		∴ A is B.

In the disjunctive premise there may, of course, be more than two members, *e.g.* Either A is B or C is D or E is F. In that case one member is affirmed in the minor premise and the others conjunctively denied in the conclusion, or one member is denied in the minor premise and the others disjunctively affirmed in the conclusion. For instance :

Either A is B or C is D or E is F.
A is B.
∴ C is not D and E is not F.

Either A is B or C is D or E is F.
A is not B.
∴ Either C is D or E is F.

The disjunctive syllogism proceeds on the assumption that the disjunction in the major premise is exclusive and

exhaustive, *i.e.* that each alternative is completely exclusive of the others and that the alternatives taken together exhaust the possibilities of the subject.

Now, as we have already seen in considering the disjunctive proposition (p. 99 ff.), many logicians, including Mill, Jevons and Keynes, hold that we are entitled to infer only from the denial of one alternative to the affirmation of the other, and not from the affirmation of one alternative to the denial of the other. In other words, they maintain that the disjunctive syllogism has only one valid mood, the *modus tollendo ponens*, the other mood being invalid. On this view, the rule that the disjunction must be exclusive has no place. Whether or not the disjunction is exclusive, we can always infer from the denial of one alternative to the affirmation of the other. Now if we ignore all concrete reasonings and concern ourselves only with As and Bs, Ps and Qs, this view must be adopted. We can never tell whether or not the proposition, Either A is B or C is D, is a complete disjunction, in which the alternatives are exclusive and exhaustive. For no subject or universe of discourse is indicated by such a formula, except perhaps the various combinations of the letters of the alphabet in certain ways. If we confine ourselves to these forms (which, after all, are merely symbols), the only propositions in which the disjunction is complete would be: Either A is B or A is not B, Either P is Q or P is not Q, and so on (*v.* p. 100). But if we consider actual concrete reasonings, we find many cases in which we get a valid conclusion from the affirmation of one of the alternatives. Dr. Keynes [1] gives as an example: "He was either first or second in the race, he was second, therefore, he was not first." This is clearly a valid reasoning; but if it is regarded as a disjunctive syllogism, it must, on the view of Mill and Keynes, be regarded as invalid. But Dr. Keynes holds that the validity of the argument really depends not on the expressed major premise, but on the understood premise 'No one can be both first and second

[1] *Formal Logic* (Fourth Edition), p. 361.

in a race.' This understood major premise enables us to dispense with the disjunctive premise, which is therefore immaterial to the reasoning. But this contention means merely that we can express a disjunctive syllogism as a categorical, and that in doing so we must make the major premise quite unambiguous. 'No one can be both first and second in a race' is simply another way of saying that there is an exclusive disjunction between being first in a race and being second in that race. It is only 'when we are dealing with pure symbolic forms' that we are unable to say whether or not there is an exclusive disjunction between the alternatives, and in an actual concrete reasoning, unless we know this, the major premise is ambiguous (*v.* p. 101).

The same point may be put in another way. We have seen (p. 103) that, if the alternatives in a disjunctive proposition are not exclusive, the disjunctive proposition is reducible to one hypothetical proposition. The difference between them is only verbal. Disjunctive reasoning thus becomes indistinguishable from hypothetical reasoning. On the other hand, if the alternatives are exclusive, the disjunctive proposition is reducible to two hypothetical propositions, and the disjunctive syllogism thus expresses a kind of reasoning distinct from the hypothetical and appearing in many concrete instances. Hypothetical reasoning, in short, implies a system in which the affirmation of proposition Q follows from the affirmation of proposition P, but the affirmation of proposition P does not follow from the affirmation of proposition Q; and the denial of proposition P follows from the denial of proposition Q, but the denial of proposition Q does not follow from the denial of proposition P. Disjunctive reasoning implies a system in which the affirmation of either proposition follows from the denial of the other, and the denial of either from the affirmation of the other. And there are undoubtedly many actual systems of this kind. Dr. Keynes [1] refers to the 'paradoxical result' of this position, pointed out by Mr. G. R. T. Ross, [2] viz. that

[1] *Formal Logic* (Fourth Edition), p. 280, footnote.

[2] *Mind* 1903, p. 492.

" on the exclusive interpretation the disjunctives *A is either B or C* and *A is either not B or not C* are identical in their import ; for in each case the real alternants are *B but not C* and *C but not B*. Thus, to take an illustration borrowed from Mr. Ross, the two following propositions are (on the interpretation in question) identical in their import : 'Anyone who affirms that he has seen his own ghost is either not sane or not telling what he believes to be the truth.' 'Anyone who affirms that he has seen his own ghost is either sane or truthful.' " Mr. Ross is quite right, and the result seems paradoxical only because he does not, as he suggests, substitute *B but not C* for *B* and *C but not B* for *C*. If in the concrete instance we make this substitution, we get (*a*) 'Anyone who affirms, etc. is either not sane but truthful or not truthful but sane,' and (*b*) 'Anyone who affirms, etc. is either sane but not truthful or truthful but not sane.' These propositions are evidently identical, and they express what the given proposition really means. The paradox is therefore only verbal.

It may also be observed that, if our interpretation of the disjunctive proposition is correct, the disjunctive proposition is analogous to Hamilton's U proposition, All S is all P.[1] This proposition implies (*a*) All S is P ; (*b*) All not-S is not-P ; (*c*) All P is S ; (*d*) All not-P is not-S ; or (*a*) If S, then P ; (*b*) If not-S, then not-P ; (*c*) If P, then S ; (*d*) If not-P, then not-S. But these last four forms (' If S, then P ' etc.) are the interpretation of the disjunction ' Either S or not-P,' if this disjunction is complete. Take for example the U proposition 'The Lord Chancellor is Lord X.' From this we can infer that if a certain man is the Lord Chancellor, he is Lord X, that if he is not the Lord Chancellor he is not Lord X, and so on. And the content of the U proposition might be put in the form ' Either the Lord Chancellor or not Lord X.'

Now the U proposition is fundamentally and exactly equational, and the aim of physical science is to obtain laws which may be expressed equationally. For the expression

[1] *v.* above p. 73.

of such laws the U proposition is appropriate, and the fact that it is the same in character as the disjunctive proposition confirms the view that the disjunctive proposition, as we have interpreted it, represents the highest stage of scientific knowledge.

III. A dilemma is a syllogism, having a hypothetical major premise and a disjunctive minor premise. In the major premise two hypothetical propositions are conjunctively affirmed ; in the minor premise, (*a*) the antecedents of these propositions are disjunctively affirmed, or (*b*) their consequents are disjunctively denied ; and in the conclusion, (*a*) the consequents are affirmed, or (*b*) the antecedents are denied. The dilemma thus resembles the hypothetical syllogism in its type or general structure and the disjunctive syllogism in the disjunctive nature of the minor premise.

Dilemmas are usually classified as constructive and destructive, simple and complex. A dilemma is constructive when in the minor premise the antecedents of the major premise are disjunctively affirmed ; it is destructive when the consequents are disjunctively denied. It follows that in a constructive dilemma there must be two distinct antecedents in the major premise, while there may be either one or two consequents ; and that in a destructive dilemma there must be two distinct consequents in the major premise, while there may be either one or two antecedents. A dilemma in the major premise of which there is only one antecedent or only one consequent is a simple dilemma ; and a dilemma in the major premise of which there are two distinct antecedents and two distinct consequents is a complex dilemma.

There are thus four forms of dilemma :

(1) *Simple Constructive.*

If A is B, E is F ; and if C is D, E is F.
Either A is B or C is D.
∴ E is F.

(2) *Complex Constructive.*

If A is B, E is F ; and if C is D, G is H.
Either A is B or C is D.
∴ Either E is F or G is H.

(3) *Simple Destructive.*

If A is B, E is F ; and if A is B, G is H.
Either E is not F or G is not H.
∴ A is not B.

(4) *Complex Destructive.*

If A is B, E is F ; and if C is D, G is H.
Either E is not F or G is not H.
∴ Either A is not B or C is not D.

Whately, Mansel and Jevons maintain that every dilemma must have more than one antecedent in its major premise, and accordingly they say that there are only three forms of dilemma, omitting the simple Destructive form. Whately says[1] regarding this form that the disjunctive denial of several consequents "comes to the same thing as *wholly* denying them ; since if they be not *all* true, the *one antecedent* must equally fall to the ground ; and the syllogism will be equally simple." That is to say, he regards the simple destructive dilemma as merely a form of the ordinary hypothetical syllogism. But Whately's argument, if applied to the simple constructive dilemma, would reduce it also to a hypothetical syllogism. For, in the case of the simple constructive dilemma, the disjunctive affirmation of the antecedents comes to the same thing as wholly affirming them, since if they are not all false (*i.e.* if one is true) the consequent must equally follow. But the simple constructive is probably the most common form of dilemma and must therefore be recognized. Whately seems to have confused the function of the disjunctive premise in the dilemma with the function of the disjunctive premise in the disjunctive syllogism. These functions, however, are quite different. In the disjunctive syllogism the reasoning turns on the disjunction :

[1] *Elements of Logic*, Bk. ii. ch. iv. sect. 5.

e.g. one alternative is asserted in the conclusion *because of* the denial of the other. In the dilemma neither of the alternatives is asserted or denied in the conclusion. The alternatives in the minor premise are stated as the only alternatives, and the conclusion follows, in the case of the simple dilemma, from the assertion of either alternative, and in the case of the complex dilemma, from the disjunctive assertion of both alternatives. In criticism of the simple destructive dilemma Whately says [1]: "If the consequents be not *all* truc, the *one antecedent* must equally fall to the ground." This is just what makes the reasoning a dilemma. The minor premise asserts that the consequents *cannot* be all true, one or the other must be denied, and in the conclusion the antecedent is denied. The reasoning is similar in type to that of the hypothetical syllogism; but it is distinguished from it by the disjunctive element.

From what has been said it will be evident that if a dilemma is to be valid, if it is to yield a true conclusion, the disjunction in the minor premise must be exclusive and exhaustive, *i.e.* each alternative must be exclusive of the other and all the alternatives must be stated. The dilemma originally meant an argument in which mutually exclusive alternatives were offered, on either of which the same conclusion followed. If such an argument can establish a proposition, then that proposition must be absolutely and unconditionally true, for *ex hypothesi* it follows from all the possible conditions or alternatives. But as there are only two alternatives, the minor premise of the dilemma, if it is expressed in symbolic form, ought to appear as the disjunction between two contradictory propositions. Thus, *e.g.* the simple constructive dilemma ought to become:

If A is B, E is F and if A is not B, E is F.
Either A is B or A is not B.
∴ E is F.

In the ordinary symbolic statement of this dilemma there is nothing to indicate that the disjunction in the minor

[1] *Loc. cit.*

premise, Either A is B or C is D, is complete, and the reasoning does not involve (as in the disjunctive syllogism) the affirmation of one alternative as the consequence of denying the other or the denial of one alternative as the consequence of affirming the other. Now if, as in this new form, the alternatives are represented as contradictories, the reasoning becomes one in which a proposition follows equally from another proposition and from its contradictory, *i.e.* the conclusion follows equally from the truth and the falsehood of the same premise. But this means that the supposed reasoning is not a reasoning. The conclusion is not a conclusion from the given premise, but a proposition of which the ground or premise is not stated. Accordingly, if we confine our attention to the dilemma in its symbolic form and take no account of the matter which appears in concrete instances, the dilemma is either (*a*) not an inference, if the disjunction in the minor premise is complete, or (*b*) an invalid inference, if the disjunction is incomplete.

In actual concrete dilemmas, however, the alternatives are never really contradictory propositions, although they generally appear to be so. They are formally or materially contrary propositions, which do not cover all the possible conditions. The validity of the dilemma depends on the importance of the neglected conditions or possibilities. If these conditions, though formally possible, are actually of no consequence in relation to the particular argument, the dilemma may be regarded as practically valid. If they are real possibilities, affecting the particular argument, the dilemma is invalid. The alternatives offered are commonly called the *horns of the dilemma*, the idea being that the person to whom we offer the dilemma is tossed from one to the other and can get no satisfaction on either. But he may construct what is called a *rebutting dilemma*, on the horns of which we may find ourselves in turn. The rebutting dilemma is usually constructed by making use of the neglected possibilities, as if they were the only possibilities.

Accordingly, in order to test a dilemma, we must (1)

examine the disjunction in the minor premise, by interpreting it in relation to what is given in the major premise, in order to discover whether the disjunction is really, and not merely apparently, exhaustive, *i.e.* whether or not there are any neglected possibilities which may affect the argument. If the disjunction is not exhaustive, we must (2) inquire whether the neglected possibilities are of such a kind as to render the conclusion not proven.

One or two examples will make this clear. We may take first the ancient dilemmas of Protagoras and Euathlus. Euathlus, in order to be able to defend himself in the law courts of Athens, became a pupil of Protagoras, who taught Rhetoric. It was agreed between them that Euathlus should pay one-half of the fee at the beginning of his course of instruction, and the other half when he won his first case in the law courts. For a long time after completing his course, Euathlus was not engaged in any action at law. Protagoras brought an action against him to recover the second half of his fee and put his case in the following dilemma : " If the verdict of the Court is in favour of Euathlus, he must pay my fee, because he has won his first case ; and if the verdict is against him, he must pay it because of the decision of the Court ; but the verdict must be either in his favour or against him ; and therefore he must pay the fee." In answer to this Euathlus submitted a rebutting dilemma : " If the verdict is in my favour, I am absolved from payment by the decision of the Court ; and if the verdict is against me, I am absolved because I have not won my first case ; but the verdict must be either in my favour or against me ; and therefore I am absolved from payment." To test these dilemmas, we must in each case interpret the minor premise in terms of the major. The real meaning of the first hypothetical proposition in Protagoras's dilemma is that Euathlus must pay the fee, not because that is the decision of the Court, but because he has won his first case. The proposition ought therefore to be stated in some such form as this : If Euathlus wins his first case, he must pay my fee. The minor premise ought

accordingly to be : Either Euathlus wins his first case or the decision of the Court is against him. This is clearly not an exhaustive disjunction, for the exhaustive alternative to (*a*) winning his first case is (*b*) losing it ; and the exhaustive alternative to (*c*) the verdict of the Court being against him is (*d*) its being in his favour. It is the neglected alternatives (*b*) and (*d*) which become the basis of the rebutting dilemma. Thus one way of meeting a dilemma is to construct a rebutting one. There are two other ways in which a dilemma can be met. One way is to point out that the alternatives in the minor premise are not exhaustive and that therefore the conclusion does not follow. This course is described as *escaping between the horns of the dilemma*. The other way is to deny that the consequents stated in the major premise as following from the alternatives given in the minor premise do in fact so follow. This course is described as *taking the dilemma by the horns.*

Consider next the dilemmas between the crocodile which had stolen a child and the child's mother. The crocodile promised to return the child if the mother would guess correctly whether the crocodile is to eat it or not. The mother, arguing as follows, replies that the crocodile is going to eat it : " If I say the crocodile is going to eat it and am right, I shall get the child back because I am right ; and if I say the crocodile is going to eat it and am wrong, I shall get the child back, because it is not going to be eaten. But either I must be right or I must be wrong. Therefore I shall get the child back." To this the crocodile replies : " If you say I am going to eat it and are right, you will not get the child back because I shall eat it ; and if you say I am going to eat it and are wrong, you will not get the child back because you are wrong. But either you must be right or you must be wrong. Therefore you will not get the child back." Here the alternatives in the minor premises of both dilemmas seem to be between being right and being wrong, and so the disjunction seems exhaustive. But it is not really so. For in the first dilemma (the mother's argument) the alternatives are : (1) being right as to the intention of the

crocodile to eat the child, and (2) being wrong as to the fact of the child's being eaten ; while in the second dilemma (the crocodile's argument) the alternatives are : (1) being wrong as to the intention of the crocodile to eat the child, and (2) being right as to the fact of the child's being eaten. Thus there are four alternatives and each dilemma rebuts the other by insisting on the alternatives which the other had neglected. But neither can establish its own conclusion against the other.

Take again the Caliph Omar's dilemma to the Alexandrian librarians in 640 A.D. "If your books are in conformity to the Koran, they are superfluous ; and if they are at variance with it, they are pernicious. Either they are in conformity to it or they are at variance with it. Therefore they are either superfluous or pernicious ; and so they will be committed to the flames." Here we should note that there are two senses of the phrases 'in conformity to' and 'at variance with' ; (1) 'in conformity to or at variance with the letter (of the Koran),' (2) 'in conformity to or at variance with the spirit (of the Koran).' Now if we interpret the alternatives in the minor in the light of the major, *i.e.* as they must be interpreted if the major is to be true, we shall see that they are (*a*) in conformity to the letter of the Koran (otherwise they are not superfluous), and (*b*) at variance with the spirit of the Koran (otherwise they are not pernicious). So interpreted the disjunction in the minor is not exhaustive. There are other alternatives. The books may have been (*c*) in conformity to the spirit, (*d*) at variance with the letter, and (*e*), the really relevant alternative, in conformity to the spirit but at variance with the letter (of the Koran). In other words, they may have contained information which was not at variance with the principles contained in the Koran but which was supplementary to its teaching. This alternative could be regarded as practically negligible and the disjunction as practically exhaustive, only if the Koran contained the sum total of possible knowledge on all subjects.

These examples bring to light the most common fallacy

in dilemmas, namely, neglect of relevant alternatives. But even if the alternatives in the minor are exclusive and exhaustive, the dilemma may still be challenged on the ground that one or more of the consequents stated in the major as following from the antecedents do not in fact so follow, *i.e.* on the ground that the major is not true. And the dilemma is guilty of a formal fallacy, if the minor denies the antecedent or affirms the consequent of one or more of the hypothetical propositions in the major. The main reason, however, why so many dilemmas are fallacious is that the minor claims to be, while in fact it is not, an exclusive and exhaustive disjunction. It claims complete knowledge of the possibilities of the situation under consideration, exhaustive knowledge of the system of things or universe of discourse referred to. Such knowledge is always difficult and often impossible to attain, but we often assume that we have got it when we have not. There are, however, many situations in which the practically relevant alternatives are obvious enough. The other alternatives which are theoretically possible are not open to the person faced with the situation. People in such situations are literally in a dilemma: all the practical alternatives are unpleasant. The form of reasoning which represents the thoughts of such people is often a dilemma which is practically quite valid. Some examples will make this clear.

(1) The following dilemma represents the situation with which many a statesman, whose views on important questions of public policy have changed, finds himself confronted: 'If I see that my former views were wrong, and do not alter my course, I am guilty of deceit; and if I alter my course, I am open to the charge of inconsistency. Either I shall alter my course or I shall not. Therefore I shall be guilty of deceit or open to a charge of inconsistency.'

(2) A person who has only £1500 to spend wants to set up house. He is very anxious to get a particular house, but the house costs £1200, and it would require £500 to furnish it. He may argue thus: 'If I am to live in this house I must pay for the house; and if I am to live in this

house I must furnish it. Either I cannot pay for the house or I cannot furnish it. Therefore I cannot live in this house.'

Of course, there are other alternatives. He might get a loan or a bond on the house or get the furniture on the instalment system. Whether these are to be ruled out as practically irrelevant depends on the character of the person in question.

(3) A committee of which I am a member is to consider an important motion. Mr. A., a special friend of mine, is very strongly opposed to it, while I personally consider it as the embodiment of a highly desirable educational policy. My state of mind as the time of meeting approaches may be expressed : 'If I support the motion, I shall lose Mr. A.'s friendship ; and if I do not support it, I shall be untrue to my own sense of what is right. Either I must support it or not support it. Therefore I shall either lose Mr. A.'s friendship or be untrue to my own sense of what is right.' Here, too, there are many other alternatives. *E.g.* I need not attend the meeting, I may go but not vote, I may try to persuade my friend that the motion is desirable, I may try to get the motion modified, etc. But occasions do arise when all such alternatives may be regarded as practically irrelevant.

Such examples show us that the dilemma as a form of argument may be practically quite valid. But they should also bring home to us how difficult it is for us to be sure that we have got all the relevant alternatives. At a particular moment action may be necessary. The possible lines of action may appear, and may in fact be, very limited ; and we have to choose one of them. We are apt to say : 'The situation and the people's characters are what they are. Other alternatives might be possible if things were otherwise ; but they are not, and that settles the practical possibilities.' It is surprising, however, what a difference there is between the alternatives which different people see in the same situation. As the dilemma presupposes an exhaustive knowledge of the relevant conditions and practical alter-

natives, it should be used with great caution. No doubt, as a form of argument the dilemma is perfectly valid, provided the premises are true and the alternatives are exclusive and exhaustive. But it is difficult to satisfy these conditions and many fallacies are put into the form of dilemmas and made to appear plausible.

If instead of two hypothetical propositions we have three or four or more conjunctively affirmed in the major premise and so three or four or more alternatives disjoined in the minor, the argument is called a trilemma, tetralemma or polylemma, but the principle on which such arguments proceed is the same as that of the dilemma. The difference between them and the dilemma is one of complexity, not of kind, and so they do not require separate consideration.

EXERCISE XIV

1. Explain and justify the principle on which the hypothetical syllogism proceeds.

2. (*a*) If the hypothetical syllogism is expressed in categorical form, what fallacy is involved in the denial of the antecedent? (*b*) Is there any form of hypothetical syllogism in which the denial of the antecedent yields a valid conclusion?

3. Analyse the following arguments and examine their validity:
   (1) If he had left the house a few minutes earlier, he would no doubt have caught the train; but as he was delayed he must have lost it.
   (2) If there are sharpers in the company, we ought not to gamble; but there are no sharpers present, so gambling is quite legitimate.
   (3) If this measure is necessary for the successful prosecution of the war, it can be defended; and only then. But it is not necessary, and therefore it cannot be defended.
   (4) If man were not capable of progress, he would not differ from the brutes; but man does differ from the brutes; therefore he is capable of progress.
   (5) Fallacies would only be excusable if they were unavoidable; but they are avoidable, therefore they are inexcusable.
   (6) If Trade Union leaders really controlled their men, relatively few disputes would issue in strikes. Unfortunately they do not do so. Hence the recent crop of strikes.
   (7) If all philosophical theories were sound, some would be accepted by a majority of thinkers; but as none are accepted by a majority of thinkers, none are sound.

(8) If wages rise, prices rise. If prices rise, wages rise to overtake prices. There is therefore no prospect of any cessation in the advance of the cost of living.

4. Define a disjunctive reasoning and explain in accordance with the terms of your definition why a reasoning of the form :

A is B or C,
X is A,
Therefore X is B or C.

is not disjunctive.

5. Consider critically the doctrine that the alternatives in a disjunctive proposition cannot be regarded as exclusive. What effect would the truth of this doctrine have on disjunctive reasoning ?

6. Why are dilemmas so often fallacious ? What conditions must be satisfied before a dilemma can be regarded as valid ?

7. State the following arguments in logical form and examine their validity :

(1) He did not take Greek in his Degree course ; for all candidates must take either Latin or Greek, and he took Latin.

(2) If students are idle, examinations are unavailing ; and if they are industrious, examinations are superfluous. Accordingly, as all students are either idle or industrious, examinations are useless. (Stock.)

(3) If he were clever, he would see his mistake ; and if he were candid, he would acknowledge it. But either he does not see it or he will not acknowledge it ; therefore either he is not clever or he is not candid. (Stock.)

(4) Moral exhortations are useless. For good men don't need them, and bad men will pay no heed to them.

(5) The criminal who entered the room must either have had a skeleton key or else have obtained a key from one of the servants. Now there is no evidence of a skeleton key in the case, but the prisoner is known to have been 'walking out' with Maggie Findlater, who was the only servant in the house at the time. Nothing more than common sense therefore is needed to see who was the guilty party.

(6) Motor cycling is an unsatisfactory way of seeing the country. If the roads are good, you travel too fast to enjoy the scenery ; if they are bad, you are too uncomfortable to look at it.

(7) There is no harm in allowing boys to climb trees. If they are confident, they are perfectly safe ; if they are nervous, they will not climb high enough to run any risk.

(8) To speak of training people to be teachers is unmeaning. For either they have a natural gift for teaching and do not need to acquire it, or they lack the gift and no training can impart it.

8. (*a*) How would you rebut the following dilemma ?

If a man is single, he is unhappy because he has no one to take care of him ; and if he is married, he is unhappy because he has to take care

of a wife. As he must be either married or single, there is no escape from misery. (Creighton.)

(*b*) Consider the following argument with respect both to its matter and form :

If Logic deals with the matter of thought, it must either consider the whole of it, and then be identical with all science, or consider only a part of it, without being able to give a reason why it should choose one part rather than another. But both these alternatives are absurd. Therefore Logic does not consider the matter of thought.

CHAPTER XV

COMPOUND AND ABBREVIATED REASONINGS

In the reasonings which we meet in ordinary life and literature, syllogisms do not stand by themselves as isolated, self-contained units. They are connected together to form continuous arguments. In other words, syllogisms function not separately but in chains. In such a chain of reasonings we often have a series of syllogisms each of which depends on the one which precedes it : the conclusion of each becomes a premise of the next. A syllogism, the conclusion of which is a ground or premise of a succeeding syllogism, is called a *pro-syllogism*, and one which uses the conclusion of another as a premise is called an *epi-syllogism*. A chain of reasonings consisting of pro- and epi-syllogisms is described as a *poly-syllogism*. When a poly-syllogism is so expressed that the pro-syllogisms stand first and are followed by the epi-syllogisms, the reasoning is said to be *progressive*. If the epi-syllogisms stand first in the expression, it is said to be *regressive*.

Such a poly-syllogism is sometimes expressed in an abbreviated form in which is stated only one premise of each syllogism after the first and no conclusion before the final conclusion of the argument. Instances of such reasoning are to be found in the works of Aristotle, but he did not give it a special name. Later writers called an argument of this kind a *sorites*.[1] This name is used by

[1] The name sorites seems to have been also applied to a kind of fallacy, the fallacy of continuous questioning. This fallacy was defined by Ulpian as " a sophism, in which, by very small degrees, the disputant is brought from the evidently true to the evidently false. *E.g.* I ask, Does

Cicero. He calls it the Stoic reasoning, and gives as an instance of it : "What is good is desirable. What is desirable is to be sought for. What is to be sought for is praiseworthy. Therefore all that is good is praiseworthy."

We may define a progressive sorites as a chain of reasoning consisting of a series of propositions in which the predicate of each is the subject of the next and in the conclusion the last predicate is predicated of the first subject.

E.g. All A is B.
All B is C.
All C is D.
All D is E.
∴ All A is E.

Each sorites may be expanded into as many first figure syllogisms as there arc premises intervening between the first premise and the conclusion. The above sorites expands into :

(1) All B is C.	(2) All C is D.	(3) All D is E.
All A is B.	All A is C.	All A is D.
∴ All A is C.	∴ All A is D.	∴ All A is E.

In this form of sorites the conclusion of each of the component syllogisms becomes the minor premise of the next ; and in the sorites only the major premises are stated, except in the case of the first syllogism.

The rules of the progressive sorites are, (1) only the last premise can be negative, and (2) only the first premise can be particular. These rules follow from the special rules of Fig. I. (1) If any premise but the last were negative, we

one grain of corn make up a heap of corn ? My opponent answers, No. I then go on asking the same question of two, three, four and so on *ad infinitum*,—nor can the respondent find the number at which the grains begin to constitute a heap. On the other hand, if we depart from the answer that a thousand grains make a heap, the interrogation may be continued downwards to unity, and the answerer be unable to determine the limit where the grains cease to make up a heap" (Hamilton, *Lectures*, vol. iii. p. 465). Modern logicians, however, confine the term sorites to the sense in which it is used in the text : they mean by it a chain of reasoning, not a fallacy.

should have a negative conclusion in the syllogism of which it is a premise. This would mean a negative minor premise in the next syllogism, the conclusion of each syllogism being the minor premise of the next. But we cannot have a negative minor in the first figure. (2) Since all the premises but the last must be affirmative and all except the first are the major premises of their respective syllogisms, if any premise but the first were particular, we should have a particular major premise in a first figure syllogism, and this we cannot have.

Since the seventeenth century the above or progressive form of sorites has been called the Aristotelian in distinction from the Goclenian or regressive. The latter derives its name from Rudolf Goclenius (1547-1628), a professor at Marburg, who was the first to call attention to it, in an introduction to Aristotle's *Organon*, published in 1598. In the Goclenian sorites the subject of each proposition becomes the predicate of the next and the conclusion predicates the first predicate of the last subject.

E.g. All D is E.
All C is D.
All B is C.
All A is B.
∴ All A is E.

This form is just the reverse of the Aristotelian form and can be similarly expanded into a series of first figure syllogisms.

(1) All D is E.	(2) All C is E.	(3) All B is E.
All C is D.	All B is C.	All A is B.
∴ All C is E.	∴ All B is E.	∴ All A is E.

Here, it will be noted, only the minor premises of all syllogisms after the first are expressed in the sorites, and the conclusion of each becomes the major premise of the next. Hence the rules for this form are: (1) Only the first premise can be negative, otherwise we should have a negative minor premise in a first-figure syllogism. (2) Only the last premise can be particular, otherwise we should have

a particular conclusion to the syllogism of which it is a premise and so a particular major premise in the next syllogism. And in the first figure this is impossible.

We can get similar chains of reasonings the premises of which are all hypothetical. Here the consequent of each becomes the antecedent of the next.

E.g. If A is B, C is D.
If C is D, E is F.
If E is F, G is H.
∴ If A is B, G is H.
or A is B, ∴ G is H.
or G is not H, ∴ A is not B.

We can also get similar chains of non-syllogistic reasonings like :

A = B.	A is greater than B.	A is before B.
B = C.	B is greater than C.	B is before C.
C = D.	C is greater than D.	C is before D.
∴ A = D.	∴ A is greater than D.	∴ A is before D.

The character of such arguments will be considered later (v. ch. xvi).

The term *epicheirema* is used by scholastic and modern logicians to describe a syllogism to one or both of the premises of which a reason is attached.

E.g. All unnecessary duties on imports are impolitic, as they impede the trade of the country ; the American protective duties are unnecessary, as they support industries which are able to stand by themselves ; therefore the American protective tariff is impolitic. (Welton.)

When such a reasoning is fully expressed the reason (or reasons) attached takes the form of a pro-syllogism (or pro-syllogisms) on which the given syllogism, as an epi-syllogism, is founded, and the whole reasoning becomes a poly-syllogism. Such arguments may often be most suitably expressed as sorites. Thus the argument just given becomes :

All duties which impede the trade of the country are impolitic.

All unnecessary duties on imports are duties which impede the trade of the country.

All duties which support industries which are able to stand by themselves are unnecessary duties on imports.

The American protective duties are duties which support industries which are able to stand by themselves.

Therefore the American protective duties are impolitic.

Aristotle[1] used the term epicheirema in quite a different sense, to describe a kind of indirect verification. For him it means a process by which the truth or falsehood of a proposition is tested by drawing consequences from itself and its contradictory. If its consequences are clearly true and those of its contradictory clearly false, we may be fairly certain of its truth, though we cannot demonstrate it as the conclusion of accepted premises.

Another term which is used in different senses by Aristotle and modern logicians is *enthymeme*. The word ἐνθύμημα literally means 'a consideration,' as distinct from a logical demonstration. This is the sense in which Aristotle used it. He recognized that there are many arguments which we constantly employ and which are good enough for practical purposes, good enough within certain ill-defined limits, but which at once appear as invalid if we express them in strict syllogistic form. Syllogism he regarded as rather an ideal to which a perfect argument ought to conform than a statement of the form which every argument that has any worth must take. Arguments that are good enough so far as they go without being absolutely or unconditionally valid he calls rhetorical.[2] Such arguments are sufficient in an ordinary case to convince us, although they are not ideally perfect demonstrations. Those of the rhetorical arguments that take the syllogistic form Aristotle called enthymemes. He distinguished two main kinds[3]: (*a*) Arguments based on probable statements, *i.e.* statements true on the whole but not universally true,

[1] *Top.* 8. 2. 165^b 3; 162^a 16. [2] *An. Post.* 1. 1. 79^a 9.
[3] *An. Pr.* 2. 27. 70^a and 70^b.

like 'Envious people hate those they envy.' A probable statement of the general form 'Most M are P' or 'Ms as a rule are Ps' is the major premise of this kind of argument.

E.g. Most men work.
A B is a man.
Therefore A B probably works.

(*b*) Arguments based on a sign or symptom, *i.e.* a fact which indicates the existence of some other fact.

E.g. Ambition is a sign of generosity.
X is ambitious.
Therefore X is probably generous.

Such arguments may be used in any of the three figures, but they are not strictly demonstrative, and so the conclusion is not strictly proved. The form of argument used in them is syllogistic. The difference between the Aristotelian enthymeme and the syllogism strictly so-called is in the nature of the major premise rather than in the form of the argument.

Later logicians have used the term enthymeme to describe a syllogism which is 'incompletely stated.'[1] The incompleteness of the argument for Aristotle was one of proof or evidence. For the moderns it is merely one of expression or words. Jevons and others support this use of the term by a mistaken derivation of the word. They derive it from ἐν, in, and θυμός, the mind, and think that it indicates that one of the premises or the conclusion is kept 'in the mind,'[2] *i.e.* not expressed. Such syllogisms, in the expression of which one of the premises or the conclusion is omitted, have been divided into three orders according as the major or minor premise or the conclusion is unexpressed.

'Socrates is mortal, because he is a man' is an enthymeme of the first order, the major premise 'All men

[1] Jevons, *Elementary Lessons in Logic*, p. 153. Cf. Hibben, *Logic Deductive and Inductive*, p. 130.

[2] Jevons, *loc. cit.*

are mortal' being unexpressed. 'All men are mortal, therefore Socrates is mortal' is an enthymeme of the second order; and 'All men are mortal and Socrates is a man,' a form in which we are allowed to draw the conclusion for ourselves, is one of the third order. But we need not enter into further details, for the suggested derivation is mistaken and the distinction between enthymeme and syllogism is of no logical value. A syllogism with one of its premises or its conclusion unexpressed is simply an argument not in logical form. It is not strictly a syllogism at all. As far as its logical character is concerned, it differs in no way from other syllogisms. Indeed in most ordinary arguments one or other of the premises is not expressed, but that does not render them in principle different from those which are fully expressed. It is as unnecessary to give a special name to syllogisms which are imperfectly expressed or not in strict logical form as it is to give one to imperfectly expressed propositions, an error of principle which we noted in considering the so-called class of indesignate propositions (pp. 54-5). To exhibit the logical character of such reasonings we must put them into strict logical form, and in so doing we must make the implicit premises or conclusions explicit. Enthymemes, then, do not constitute a separate class of *logical* arguments and they require no special consideration. If arguments the formulation of which is defective were to be regarded as a class by themselves, most, if not all, ordinary arguments would have to be included in it. For we seldom, if ever, meet a strictly syllogistic argument outside the pages of logic text books. Many logicians recognize this. They point out that 'the enthymeme is the natural mode of reasoning,'[1] the form taken by the majority of 'the arguments of everyday life.'[2] But if so, why treat it as if it were logically a separate kind of argument? If the term enthymeme is to be used at all it seems better to use it in Aristotle's sense than in one in which it has no logical significance.

[1] Boyce-Gibson, *The Problem of Logic*, p. 231.

[2] Keynes, *Formal Logic*, Fourth Edition, p. 368.

EXERCISE XV

1. Explain and illustrate with concrete examples: prosyllogism, Goclenian sorites, rebutting dilemma, epicheirema.

2. State and prove the rules of the Aristotelian sorites.

3. 'Enthymemes do not constitute a separate class of logical arguments.' Explain and discuss.

4. Explain the nature of the following arguments and examine their validity:

(1) Athletic games are duties; for whatever is necessary to health is a duty, and exercise is necessary to health, and these games are exercise.

(2) To distrust a person is to treat him as a person who cannot be trusted. To treat him in this way is to encourage him to think of himself in this way. Distrust breeds dishonesty. We ought then always to trust people until we discover that they cannot be trusted.

(3) Wars injure trade and therefore impoverish the community. This, however, amounts to diminishing the means of happiness. Since therefore all that diminishes happiness is to be condemned, entry upon war is wholly reprehensible.

(4) None but the contented are happy; none but the virtuous are contented; none but the wise are virtuous; therefore none but the wise are happy.

(5) If thou wert never at Court, thou never sawest good manners; then thy manners must be wicked, and wickedness is sin, and sin is perdition: thou art in a parlous state, shepherd.

(6) If our intellectual part is common, the reason also in respect of which we are rational beings is common; if this is so, common also is the reason which commands what to do and what not to do; if this is so, there is a common law also; if this is so, we are fellow citizens; if this is so, we are members of the same political community; if this is so, the world is in a manner a state.

CHAPTER XVI

NON-SYLLOGISTIC MEDIATE INFERENCES

ALL the reasonings which we have hitherto considered were recognized by Aristotle and the traditional logic. The categorical mediate inferences admit of being put directly into syllogistic form. The conditional ones also can, though not without loss of meaning, be reduced to the categorical form ; and it is usual to call them syllogisms [1] even in their hypothetical and disjunctive forms. It was the conviction of Aristotle and of many of his successors that all exact mediate inferences can be put into the form of syllogisms ; and that indeed their real nature is not fully manifested till they are so expressed. Such arguments they called deductive, and they believed that they alone are strictly demonstrative. Thus, according to their view, syllogism and deduction are identical, and syllogism is not only *a* form but *the* form of strict or exact reasoning ; and the Dictum, the principle of the syllogism, is the principle according to which all exact inferences proceed.

There are, however, exact mediate inferences which, we shall find, proceed on principles different from the Dictum and which therefore do not admit of being expressed as syllogisms. They are sometimes called relational arguments. They were not entirely neglected by the traditional logic, but so far as it took account of them it regarded them as syllogisms and tried to express them in the strict syllogistic form. But the result of such treatment of them

[1] To avoid confusion we have ourselves used the traditional names and called them syllogisms.

is confusion, not clearness. It obscures rather than reveals the real nature of the arguments. A consideration of the way in which they have been syllogistically expressed will help to throw light, on the one hand, on the nature and limits of the syllogism and the principle on which it proceeds and, on the other, on the character of the relational arguments themselves. The principles on which the latter proceed are co-ordinate with, not subordinate forms of, the Dictum.

Let us take some examples :

(1) Most M is S ; most M is P ; therefore Some S is P.

If we put this into the ordinary syllogistic form, namely,

Some M is S
Somc M is P
Therefore Some S is P,

it is obviously invalid. Yet the argument as it stands is quite valid. The conclusion follows in the strictest and exactest sense from the premises. It has, therefore, been suggested that there is an implied major premise which is not stated in the argument and that, if it is made explicit, we shall see the validity of the argument and recognize that it is a syllogism. This implied major premise may be stated in some such form as ' Any two things which can each be positively predicated of more than one-half of the same thing can be partially positively predicated of one another ' ; or more shortly, ' Any two classes of which each includes more than half of a third class must partially overlap.' The two statements which appear to be the given premises are then combined and treated as the minor premise. So we get the form :

Any two classes of which each includes more than half of a third class must partially overlap

S and P are two classes of which each includes more than half of a third class

Therefore S and P must partially overlap

I.e. Some S is P.

It has been objected that such a formulation of the argument makes no reference to or use of M, which has a prominent place in the original argument. To meet this objection another form of the major premise is suggested—not the above principle in all its generality but one of its particular manifestations, namely, 'Any two classes of which each includes more than half of M must partially overlap.' With this, it is contended [1], we get a syllogism which uses all the data and guarantees the conclusion.

(2) A = B ; B = C ; therefore A = C.

This is a strictly demonstrative argument, and yet if we regard it as a syllogism, we have the fallacy of four terms or ambiguous middle. In the one premise the middle term is 'equal to B,' while in the other it is 'B.' Accordingly it is contended that here also we have an implied major premise and that the two given statements constitute the minor. So treated it becomes :

Things equal to the same thing are equal to one another
A and C are things equal to the same thing
Therefore A and C are equal to one another.

Or, if we wish to get B into the argument, we may take as our major not the axiom of equals but one of its expressions, namely, 'Things equal to B are equal to one another.'

The same difficulty arises in the case of all arguments expressing other relations,[2] *e.g.* of degree, of space, of time, etc. In all these, if stated directly and treated as syllogisms, we have the fallacy of four terms, and they are reduced to syllogistic form in the same way as the above.

(3) A is greater than B ; B is greater than C ; therefore A is greater than C.

Here we are given as major something like 'That which is greater than the greater of two is greater than the less.'

[1] Mellone, *Introductory text book of Logic* (ed. 12), pp. 235-6.

[2] *v.* Bradley, *Principles of Logic* (ed. 2), vol. i, bk. ii, part i, especially chaps. i. and iv.: and Joseph, *An Introduction to Logic* (ed. 2), p. 294 ff.

(4) A is before B; B is before C; therefore A is before C.

The implied premise here is 'That which is before the first of two is before the second.'

If the argument were 'A is 10 minutes before B; B is 20 minutes before C; C is two hours before D; therefore A is two and a half hours before D,' it would be more difficult to get the implied major (or majors), but no doubt ingenuity could devise one (or more). As the arguments become more complicated, it becomes increasingly difficult to discover the implied major premises necessary to put them into syllogistic form, and we do not get very far when the limits of even misguided ingenuity are reached. We shall give one or two more examples without attempting to express the implicit majors.

(5) B is 5 miles west of A; C is 5 miles north of B; therefore C is rather more than 7 miles northwest of A.

(6) A is the son of B; A is the husband of C; therefore B is the father-in-law (or mother-in-law) of C.

Now the ordinary man uses such arguments many times every day and he is perfectly certain that they are valid. He is convinced that the conclusion follows necessarily and with the greatest certainty from the given statements. But one wonders what he would say to the logician who should present to him the major premises required to turn them into syllogisms and tell him that these were premises from which he (the ordinary man) argued. He would no doubt protest, and quite rightly protest, that so far was he from using such premises that he never thought of them before, that it is with difficulty, if at all, that he recognizes them as true, and that in order to see their truth, if he sees it, he has to consider them not in their abstract form but in particular instances like the arguments as originally expressed. *E.g.*, we see the truth of the statement, 'What is before the first of two is before the second' in instances like 'A is before B; B is before C; therefore A is before C.' Unless we were sure of the latter we could not be sure of the

former. We see the universal principle in the particular case. Both have the same degree of certainty and stand or fall together. It is of the nature of truth to be universal, and so if the particular argument is sound the principle of which it is one instance is true universally. Thus the major premises given above must be true.

These principles then must be true if the arguments are valid, but are we therefore to regard them as premises of the arguments? Are we to regard *all* the principles that must be true if an argument is valid as its premises? If so, the principles above referred to as implicit major premises are premises. But they are not the only ones. There are other principles whose truth is implied in the above arguments, and so they also must be regarded as among their premises. *E.g.*, according to this view, the Laws of Thought, the truth of which is implied in every instance of reasoning, must be regarded as premises of these and all other reasonings. All arguments presuppose the truth of principles which do not find explicit expression in them, not only the truth of their own principles both in particular forms like 'Things equal to B are equal to one another,' and more general forms like 'Things equal to the same thing are equal to one another,' but also of still more general principles like the Laws of Thought. All these are grounds of the arguments in the sense, and only in the sense, that if they were not true the arguments would not be valid. But we must distinguish between such principles and the premises of particular arguments, *i.e.* the given data from which the conclusion follows.[1] It is a confusion between principles and premises that makes the syllogistic treatment of relational arguments so plausible. Behind such treatment there is real insight perverted by a mistaken tradition. The real insight recognizes the fact that arguments presuppose the truth of principles other than their given premises. The mistaken tradition insists that all arguments are syllogisms and therefore have major premises. Accordingly the principles whose truth the arguments presuppose are treated as major premises.

[1] *v.* Joseph, *op. cit.*, p. 295.

If, however, we distinguish, as clearness demands that we should, between the premises or data *from* which an argument proceeds and the principle or principles *on* which it proceeds and whose truth it presupposes, then the implicit majors given above are not premises or grounds of the arguments. They are rather the principles *according to* which the arguments proceed *from* the premises or grounds or data, principles *according to* which the conclusions are drawn from the premises, not premises *from* which they are drawn.

A comparison of such arguments with syllogisms strictly so-called will make the difference plain. Take the syllogism :

Man is mortal
Socrates is a man
Therefore Socrates is mortal.

In such an argument we have a principle *according to* which the argument proceeds, according to which the conclusion is drawn from the premises, namely, 'What can be predicated of a term distributed can be predicated of everything contained under it.' We have also data or premises *from* which the argument proceeds, namely, 'Man is mortal' and 'Socrates is a man.' Now if we wanted to treat this argument as the traditional logic treats relational arguments we could put the Dictum or one of its particular expressions as the major, and put all the given data as the minor ; and so get the conclusion. Then we should have the form :

What can be predicated of a term distributed can be predicated of everything contained under it

Man is a term distributed, mortality can be predicated of man and Socrates is contained under it [1]

Therefore mortality can be predicated of Socrates ;

or

What can be predicated of man (a term distributed) can be predicated of Socrates (a term contained under it)

[1] For reasons which will appear later we cannot get such a minor as this into one proposition and so our argument is not strictly syllogistic. See p. 238.

Mortality can be predicated of man
Therefore mortality can be predicated of Socrates.

Are we therefore to conclude that the Dictum, or one of its particular expressions, is the major premise of every syllogism? Is it not rather the principle *according to* which the syllogism proceeds, *according to* which the conclusion of the syllogism is drawn *from* its premises? The syllogism given above has its own major premise 'Man is mortal.' But that is not the principle on which the argument proceeds.

Now in the case of arguments like '$A=B$, $B=C$, $\therefore A=C$,' the alleged major premise is related to the conclusion '$A=C$' in the same way as the Dictum or one of its particular expressions is related to the conclusion 'Socrates is mortal.' It is not related to it as the major premise 'Man is mortal' is related to the conclusion 'Socrates is mortal.' There is in the relational arguments considered in this chapter nothing corresponding to the major premise of the ordinary syllogism; the type of argument is different. But such arguments, being valid, proceed on principles which are true and universal, the same for all the arguments which proceed according to them. And so, as it was believed that all valid arguments are syllogisms and as every syllogism has a major premise, premise and principle were confused, the principle according to which the argument proceeds was treated as its implied major premise and the argument was regarded as an instance of its principle. That was the only way in which relational arguments could be made to appear as syllogisms. But the fact that such arguments proceed according to a principle does not make their principle the *dictum de omni et nullo*, or the arguments themselves syllogisms.

Of the arguments given above (1) proceeds on the principle governing the relation of whole and part; (2) on the axiom of equals; (3) on the principle governing relations of degrees of quantity; (4) on the nature of time; (5) on the nature of space; (6) on the principle governing family relationships.

How then does the principle of the syllogism differ from these and other similar principles? The syllogism proceeds on the principle governing the relations of subject and predicate, substance and attribute, thing and quality, in the most general sense of these terms.[1] It is concerned with relations which are capable of being expressed by 'is' or 'are'[2] simply, as distinct from relations which require for their expression phrases like 'is equal to,' 'is greater than,' 'is west of,' etc. In each of the propositions contained in a syllogism the relation between its terms is one of predication merely, a relation which can be expressed by the present tense of the verb 'to be' without addition. Each of the premises expresses a relation of predication between the major or minor term and the middle term; and the conclusion expresses a relation of predication between the major and minor terms. In the relational arguments the relation between the other terms and the middle term is not one of predication but of equality, space, time, etc. No doubt, in the premises of relational arguments, there is a relation of predication asserted, but it is not asserted between the subject or predicate of the conclusion and the middle term. In the argument, '$A = B$, $B = C$, therefore $A = C$,' we predicate of A in the first premise not 'B' but 'equal to B' and in the second we predicate of B not 'C' but 'equal to C.' Therefore if we take account only of relations of predication and treat the argument as a syllogism, we have no middle term, we have the fallacy of four terms. On the other hand, in the case of a proper syllogism like, 'Man is mortal, Socrates is a man, therefore Socrates is mortal,' the relation is one of predication throughout. We predicate mortal of man, and man of Socrates.

Accordingly we may say that the syllogism is the formal expression of mediate inference, inference which takes place by means of a middle term, when the relation between the

[1] cf. Bradley, *op. cit.*, vol. i., p. 271; Joseph, *op. cit.*, p. 249.

[2] And 'is not' or 'are not.' For the sake of simplicity the negative forms are omitted throughout this argument.

middle term and the other terms is one of predication. This was the relation to which Aristotle paid special attention. Hence his predicative view of the import of propositions and his statement of the principle of the syllogism, namely, 'What can be predicated of a predicate can be predicated of its subject.' Now the relation of predication is the most general of all relations. It is a relation which finds expression in every categorical proposition. But it is not the only relation expressed in such propositions. When an argument proceeds on some other, more specific relation, it is not a syllogism in the strict sense, and nothing but confusion can arise from treating it as if it were.

All reasoning presupposes system and proceeds on the basis of it. A system consists of elements which mutually imply each other. The elements are so related that, some being given, the others follow. The Laws of Thought postulate the reality of such systems. Without them we could not infer at all. But there are various kinds of system, various principles of organization which constitute the elements of reality wholes. When we know the constitutive principle which connects the elements of a particular whole, we can infer from one set of elements to another within it. Now, as we have seen (p. 96), one very general relation or principle of organization according to which elements constitute a system is the relation between the predicates of a subject which makes it a subject of a particular kind. The principle governing this relation is expressed in the Dictum, and the inferences which proceed on the basis of it are syllogisms. The predicates of a subject of a particular kind, or the characteristics of a kind of thing, form a system on the basis of which we can argue that, wherever we get that kind of thing, all its qualities will be present; or that, wherever we get enough of its qualities to indicate the kind, the others will be there too. *E.g.*, if barking is a characteristic of the kind called 'dog' and a particular animal has other qualities that enable us to say that it is of the kind called 'dog,' we can predicate of it the

capacity to bark. Now such a relationship of predicates or qualities is quite different from that of points in space or instants in time or elements quantitatively related. But the latter relations also constitute their terms into systems and we can argue on the basis of them. And there are many other such principles of organization among the elements of reality. There are therefore many kinds of relation on the basis of which we can reason.[1]

The relation of predication is a very general one, and thus many inferences proceed on the basis of it. It was therefore assumed that all inferences proceed according to it or, at any rate, that it is an adequate expression of all other relations on which inferences proceed. As a matter of fact, however, it is just one relation among others and the principle governing it is only one of the principles of organization among the elements of reality. Inferences which proceed according to it are syllogisms, while inferences which proceed according to other principles of organization, like space or time or number, are not. The requirements of validity in the case of the latter are not the same as in the case of syllogism. What premises and conditions we require for valid inference depends on the nature of the system concerned.

We may put the matter in another way which will bring out the difference between the syllogism and the relational arguments we have been considering, and which will also show the plausibility, and yet the impossibility, of reducing such arguments to the form of syllogisms. In order to have inference we must have system. But we can look at a system in two ways, and we can use it as a basis for inference in each of them. We may consider the system as something which recurs or is repeated, and we can predicate of each new instance what is true of the system. In some systems, based on very general relations like predication, this may be the only course open to us. Whether it is the only

[1] For a more complete list of such relations see Bradley, *op. cit.*, vol. i., pp. 264-6. Bradley's list, however, does not profess to be complete.

one or not, it is one course open to us. When we adopt it, our argument is of the nature of syllogism. On the other hand, in the case of some systems, if not in all, we may take one instance of the system by itself and argue from the character of some of its elements to that of others. This is specially easy when the nature of the system is very simple and obvious, like some relations in space and time. In such cases our argument is of the relational kind which we are considering in this chapter. The one argument depends mainly on the recurrence of the system, the other on the inter-relation of elements within it. The one proceeds on the repetition of the plan, the other on its nature. In both cases there is system, mutual implication of elements, and the possibility, at least, of recurrence, but each type of argument emphasises a different aspect, and therefore has a different character.

Let us expand this. The categorical proposition, we saw, is the appropriate expression of the stage of knowledge at which reality is regarded as consisting of more or less isolated subjects with predicates, or things with qualities. Now many of these individual things are of the same kind. The same inter-relationship of elements, the same structural pattern, the same type or universal recurs in many individuals, *e.g.* the plan or pattern 'man' or 'dog' in many individual men or dogs. Now the major premise of a typical syllogism predicates a character of a kind or universal.[1] Its usual form, All S is P, emphasises the recurrence of the kind or type in various instances. And the syllogism is the formal expression of the process whereby we predicate of particular individuals what is true of the kind to which they belong. Now if we are thinking of the world as made up of individual, self-contained things, and if we regard natural and artificial kinds, like 'man,' 'horse,' 'chair,' as the only systems or universals, the syllogism becomes the formal expression of the process of bringing a particular under a universal or of applying a rule to a case or instance. This

[1] The relation between kind and instances is narrower than that of predication, but it is the most typical expression of predication.

process is called subsumption. The universal with which it is concerned is a kind with instances, or a rule with cases, but the instances or cases themselves do not constitute a system but an endless series. The plan or universal, man, dog, chair, is repeated indefinitely, but the individuals of one kind do not constitute a system, as the qualities of an individual dog or chair do. The syllogism emphasises this repetitive aspect of nature, and it predicates of each of the repeated individuals what is true of the plan or universal.

We may, however, in the case of some systems at any rate, base inferences not on the fact that the plan or system is repeated but on the nature of the plan, the mutual implication of elements within the system. *E.g.* 'B is ten miles west of A, C is ten miles north of B, therefore C is due north-west of A.' Or, 'In a right-angled triangle ABC, the angle at B being the right angle, AB is three inches and BC four inches, therefore AC is five inches.' Now the relationship on the basis of which these arguments proceed can be repeated indefinitely in space : it is true of any space. But the inferences are based not on the repetition but on the inter-relation of elements. The conclusions are drawn not *from* the nature of space and right-angled triangles, but *according* to these *from* the given data. In the same way when we argue, 'A is before B, B is before C, therefore A is before C,' the plan or system on the basis of which we argue is repeated in different periods of time, but our argument proceeds on the principle governing the inter-relation of elements within the plan, not on its repetition. This is even more obvious in the case of more comprehensive systems like that expressed by the principle of gravity. Here the universal or plan is repeated in its instances, *e.g.* in every bit of matter. But the bits of matter are elements in a system as well as instances of a kind. They do not constitute an endless series. They fit together into a whole, and the law of gravity is the principle of their inter-connection. They are more like the system of elements that constitute an individual than the series of individuals which are the instances of a kind.

It should, however, be noticed that the individual which is an instance of a kind is also an element in a system, usually in several systems (cf. p. 115). *E.g.*, a particular tree is not only one of a kind ; it is also related in many complex ways to its environment, and its individual characteristics, those which distinguish it from the rest of its kind, are largely determined by its relations to its environment, *e.g.* the kind of soil, the amount of sunshine and moisture, etc. In the same way, a man, as well as being an instance of the kind 'man,' is a member of a family, a trade or profession, a society, etc., and his distinguishing characteristics, the qualities which make him the particular individual that he is, are mainly, if not entirely, determined by the nature of the groups or systems to which he belongs and his relations to other members within them. Thus the same individual may be regarded as an instance of a kind or plan and as an element in a wider system. What we want at present to notice is that arguments can and do proceed on the basis of each of these ways of regarding it, and that the syllogism is adequate to express only one of them. The individual instance of a kind or plan is itself a system as well as an element in a wider system ; and unless the system, the structural pattern, remained the same in the different instances, we could not argue from one instance of it to another. Nevertheless the syllogism, while, like all inference, it presupposes system, lays its main emphasis on its repetition, rather than the inter-relation of its elements. On the other hand, the system or inter-relation of elements on which relational inferences are based may be, and usually is, repeated. The same pattern or plan recurs in different parts of space or periods of time, and what is true of one part or period, as such, is true of any other. But these relational inferences emphasise, and proceed on, the mutual implication of elements within the particular system concerned and not on the repetition of the system itself.

Relational inferences, then, proceed on the fact that everything is an element in a system and has its character-

istics determined by its relations to other elements within the system. Syllogistic inference proceeds on the fact that many individuals in nature are of the same kind and that what is true of the kind is true of each individual. From this follows another limitation of the syllogism. All it can predicate of the individual is what is true of the kind or universal of which it is an instance. It cannot show how the universal differentiates itself in the particular case. It predicates the same character of each of the individuals of a kind, and therefore it cannot determine the individual characteristics of any. While what is true of all the others is true of this one, it is not enough to distinguish it from any of the others. The knowledge of the individual that we get by syllogistic inference is thus very indefinite. If we want to infer the distinguishing characteristics of an individual, we must find a system of which it is an element, and infer them from its relations to other elements within the system. And then the argument by which we infer them is not a syllogism.

And there are other characteristics of the syllogism which are absent in relational inferences. (1) The latter have no major premise. The data necessary to determine the conclusion may be stated in two or three or more premises, but no one of them is any more a major than the others. (2) When such arguments are expressed as syllogisms, the minor premise which includes all the given data is not in any genuine sense a single proposition like the minor premise of a syllogism. *E.g.* in the premise 'A and C are equal to B,' 'A and C' is not a subject of which something is predicated, something which is qualified by a predicate. 'A and C' are two separate subjects, and the proposition is as much two propositions as 'A = B' and 'B = C' are. The difference is merely verbal. (3) The principle or major does not help to prove the conclusion or make it more certain. Unless the conclusion followed from the given premises, the so-called major itself would not be true. We do not first recognize the truth of the major, and then make the inference. The order is the other way. We see that the conclusion follows from the data given

in the minor and, in seeing this, we recognize the truth of the major. No doubt it is well, and indeed part of the work of logic, to exhibit the principles on which such arguments proceed; but such principles are no more premises of, or prove, particular arguments than the Dictum is a premise of, or proves, a particular syllogism. Like the syllogistic conclusions, other conclusions are proved by the premises *from* which they follow *according to* their principles.

Accordingly we conclude that inferences which are concerned with relations of predication, of which a kind with instances is the typical form, are syllogisms; but that those which proceed on other relations are not. To torture the latter into the form of syllogisms is to obscure the principles on which they proceed and to blur important distinctions of principle between them and the syllogism. All the arguments which we have considered in this chapter proceed on principles, but their principles differ from one another and all of them differ from that of the syllogism.

What conditions, then, must be satisfied before a valid conclusion follows from premises on the basis of relations other than those of predication? Obviously any two such premises do not yield a valid conclusion. We get no significant conclusion from such premises as 'A is greater than B' and 'B is west of C' or 'D is before F.' To yield a valid and significant conclusion, (1) the terms given in the premises must be related in the same way, *e.g.* in time or space or quantitatively, etc.; and (2) there must be a common or middle term in the premises, one which relates the other two.[1] But even this is not enough. We might, *e.g.*, have 'A is north of B and B is south of C' without being able to draw a conclusion as to the relative position of A and C; or 'A hates B and B hates C' without being able to infer the feelings with which A regards C. We must therefore have in the premises sufficient data, and data of such a kind as to enable us to construct a sub-system, within the wider system of space or time etc., in which we can see

[1] *v.* Bradley, *op. cit.*, vol. i., pp. 257-9.

the relation between the given elements. But there are innumerable such relations and systems, and so we cannot say in advance what is necessary to determine a system or sub-system. All we can say is that the data must be sufficient to determine the nature of the system or sub-system. Given the nature of such a sub-system, we can read off some of the characters and mutual relations of its elements from the given data. *E.g.*, two sides and an angle determine the nature of a triangle, and many other characters and relations of its elements can be read off once the triangle is constructed.

In conclusion, we may notice that the arguments dealt with in this chapter are not strictly deductive. They do not proceed from an explicit universal or system. In this they differ from the arguments we have hitherto considered. They refuse to fit into the classification of inferences into deductive and inductive. For the universal or principle on the basis of which they proceed is not explicit either in the premises or in the conclusion. They do not proceed *from* or *to* a universal though they proceed *according to* one. It is partly for this reason that the traditional logic with its clear-cut division into induction and deduction tended to ignore them. At the same time these relational arguments are quite general. When we say 'A is to the right of B, B is to the right of C, therefore A is to the right of C,' the argument is valid whatever the character of A, B, and C. The argument has a general form which holds in any given matter.

And these relational arguments are not the only ones that fail to fit into the classification into deductive and inductive. Proportional inferences, for example, fall outside it.

E.g., 'Sugar is 4d. a pound, therefore a stone of sugar costs 4/8.' Such an inference has no middle term in the strict sense. It is not a syllogism, but neither is it an induction. Its general form is A : B : : C : D. Three of the variables are given ; and the fourth follows. In the above argument, if x is the price of a stone of sugar, we are given

$$4\text{d.} : x : : 1 : 14.$$

$$\text{Therefore } x = 4 \times 14\text{d., } i.e.\ 4/8.$$

EXERCISE XVI

1. Consider critically the attempt to reduce all mediate inferences to the form of the syllogism.

2. (*a*) Distinguish between the premises of an argument and the principle on which it proceeds. (*b*) Explain and discuss the traditional logical doctrine to which neglect of this distinction gave rise.

3. Consider the logical character of the following inferences and explain the principle on which each proceeds :

   (1) A is taller than B, B is equal in height to C, therefore A is taller than C.
   (2) A is west of B, B is west of C, therefore A is west of C.
   (3) A is simultaneous with B, B is before C, therefore A is before C.
   (4) A has a larger salary than B, B has the same salary as C, therefore A has a larger salary than C.
   (5) A is the son of B, B is the son of C, therefore A is the grandson of C.

4. What truth, if any, is there in the assertion that wherever an inference is drawn a universal judgment is implied ? How far would the truth of this statement justify the claim of the syllogism to be the type of all reasoning ?

5. What conditions must a valid relational inference satisfy ?

6. Distinguish carefully between the principle on which syllogism and that on which relational inference proceeds.

CHAPTER XVII

SYLLOGISM AND DEDUCTION

We saw in the last chapter that, though the relation between a universal and its instances, or the application of a rule or principle to a particular case, can be expressed syllogistically, it does not follow that all inferences are syllogisms. For there are many inferences not concerned with the application of a principle or rule to a case or cases. We have now to consider whether the syllogism is the only, or even the typical, expression of the application of a rule or law or principle to instances or cases. In other words, does the syllogism cover the whole of deduction, or is it even its most appropriate expression? The inferences considered in the last chapter, while mediate and general or formal, are not strictly deductive, and so, though they are not syllogisms, syllogisms may still be identical with deduction as the traditional logic held.

Deduction is usually described as the application of an explicit universal or principle to a particular case or cases, or the process which explains a fact by a universal law. Now, we have seen that the syllogism entitles us to infer only those characteristics of an individual which it has in common with all others of the same kind, not those in which it differs from them. Accordingly, if by applying a rule to instances or a law to facts we mean trying to discover or explain the distinctive character or mode of behaviour of individuals, syllogistic inference cannot help us. *E.g.*, we can syllogistically infer from the law of gravitation expressed in the form, 'All material bodies gravitate,' that a par-

ticular body which we know to be material gravitates, but not the velocity with which or the direction in which it will do so. If we want to infer the latter we must infer them from its present position and mass and the positions and masses of other bodies in the same gravitating system. Similarly we can infer syllogistically from the principle that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides that this general relationship will hold in the case of a particular right-angled triangle, but not, *e.g.* the length of a particular side of that triangle. If we want to infer the latter, we must infer it from other characteristics of the triangle in question. *E.g.* if one side is six inches and the hypotenuse ten, we can infer that the remaining side is eight inches. In such inferences, we are applying a universal or law to a particular case or fact, in the sense of using our knowledge of the law to determine the individual characteristics of the fact, *e.g.* to determine where a particular planet will be on the 12th October, 1949. We cannot derive this knowledge from the law of gravitation alone, but we can derive it *from* the mass and position of various bodies in the gravitating system *according to* the law of gravitation. The law gives us the system, the mutual relations of the elements, but we must in addition know the individual characteristics of some of the elements in order to infer those of the others. Here the system or law accounts for the differences, as well as the resemblances, of its elements. It is in virtue of their differences as much as their identity that the elements or parts constitute the system. Each element has its own place in the system and without it the system would be different, just as the solar system would be different, and behave differently, without the presence of one of its constituent bodies.

If we accept the usual definition of deduction, it must now be obvious that we cannot identify it with syllogism. For much of the deduction that we get in the exact sciences (as *e.g.* when the momentum of a falling body is deduced from its initial position and the time during which it has been

falling ; or when the size of an angle is deduced from that of the other angles of a triangle) is not of the syllogistic kind and cannot be expressed in the form of syllogism. Many scientific deductions proceed on the same principle of the inter-relation of elements within a system as we found at the basis of relational inferences. The only difference between such non-syllogistic deductions and the relational inferences which we considered in the last chapter is that in the case of the latter the nature of the system, the principle on which the argument proceeds, *e.g.* the axiom of equals or the relation of whole and part, is considered so obvious as not to need expression. All that is given us as premises is certain elements within the system, and from these others are inferred. In the case of non-syllogistic deductions, on the other hand, the nature of the system, *e.g.* the scientific law, is not so obvious. Indeed it has itself to be established by an elaborate process of inference, and until it is so established it is only a hypothesis on trial. It is therefore usually stated, and has the appearance of being a premise. Nevertheless it is not so much a premise as a presupposition of the particular deductive inference. It is the principle according to which a conclusion is drawn from elements within the system whose inter-relationship it expresses. In this respect, it is like the Dictum which is the principle according to which all syllogistic inferences proceed but is not a premise of any of them. Indeed arguments which deduce particular facts from a scientific law, like the law of gravitation, are of the same kind as arguments which deduce a particular syllogistic conclusion from the Dictum. They are not strictly syllogistic at all ; when they are regarded as if they were, the Dictum or the law of gravitation is made the major premise and all the given data from which in fact the conclusion is drawn become the minor. But such a minor is not one statement, and so the argument is not a formal syllogism. This is the reason why we found it difficult (p. 224) to put the argument with the Dictum as major premise into strict syllogistic form.

Now, much of the deduction of the exact sciences is of this

kind, as we shall see more fully later (ch. xxiii). It proceeds on the basis of the inter-relation of elements within a system, what Bosanquet has called 'implication.'[1] The scientific law or hypothesis states the principle of organization according to which the elements are connected. The character of certain elements is then given and that of another or of others is inferred. The characteristic which the conclusion predicates of a particular element or fact is not true of any of the others. It is its individual character in virtue of its place in the system.

Many of the deductions of everyday life are also of this kind, *e.g.* the deductions of a detective who has pieced together a number of isolated facts into the theory that a particular person has committed the crime he is investigating, or those of a child who is fitting his blocks together into a pattern. Parts fit into such a whole in virtue of their differences from one another, in virtue of their individual characteristics. If the part is to fit into its place in the whole it must have the characteristics which enable it to do so, and very few systems can be made up of parts which are exactly alike. *E.g.* suppose a child has a number of blocks on each of which there is a part of a pattern. If the child fits together all the blocks save one, in such a way that the pattern is obvious, the child can infer what must be the character of the remaining block.

Now the child's inference is a very simple one, but it proceeds on the same principle as that of the scientist or the detective. When Harvey inferred from the circulation of the blood that there must be a passage by which it goes from the arteries to the veins, a passage which was not observed till after his death, he was inferring on the same principle as the child. And so were Adams and Leverrier when they deduced from the movements of Uranus and its place in the system of gravitating bodies that there must be a body which had not hitherto been observed exerting an influence on it. When the telescope was turned on the place which they indicated as the locality of this unknown body, Neptune

[1] *Implication and Linear Inference*, p. 8, *et passim*.

was discovered. In all these cases the system, the pattern on the blocks, the circulating system of the blood, the system of gravitating bodies as expressed in the law of gravitation, demanded a certain element or part to complete what was known of it. Accordingly either the system must be rejected or the inferred element accepted. *E.g.* if the observations and calculations of Adams and Leverrier are right, either the existence of such a body as Neptune, a body which will account for the movements observed in Uranus, must be accepted as proved, even if it cannot be observed, or the law of gravitation must be denied. In the same way, if a detective relying on circumstantial evidence can get all the facts of a particular case to fit together on the hypothesis, and only on the hypothesis, that a particular person committed the crime in question, his inference that this individual did so is a deduction on the basis of the hypothetical system which the facts force him to accept. If the facts are numerous, if the system which they form is complete except for this one link, and if the facts bear no other interpretation, the inference must be accepted. We see the same principle in operation when an economist infers from the mechanism of the foreign exchanges that *e.g.* 'if there is a steady and considerable excess of imports over exports, gold will flow out of the importing country.' [1] The inference is a necessary implication of the system and the laws according to which it operates. If you deny the inference, all that the economist can do is to point you to the system and show that you must accept his conclusion or interpret the facts of the system differently.

In all such cases we must accept the inferred conclusion or reject the system or law on which it is based. Such a system cannot usually be expressed in two or three propositions. It often requires quite a long description. But if the facts described are seen to constitute a system and bear no other interpretation than that they are the parts of such a system, the inference from them is incontrovertible. The inference stands as long as the system is accepted.

[1] Taussig, *Principles of Economics*, vol. i. p. 455.

We shall further explain and illustrate the nature of such inferences when we come to deal with the way in which systems or laws are established (ch. xxiii). At present we note that, whether the law which expresses the mutual implication of the elements of the system is clearly established or is merely a hypothesis on trial, the inference from it is deductive and does not admit of being expressed as a syllogism. For a syllogism predicates of an individual what is true of all other individuals of the kind, the common not the distinguishing characteristics, and so it is quite unsuitable for the expression of such inferences as deal with the latter. Syllogism, as we have seen, emphasises the repetition of the kind, the common character of the individuals. The deductions which we have been considering, like relational inferences, emphasize the inter-relation of elements and their individual differences. Accordingly syllogism cannot be identified with deduction.

Moreover, if the syllogism was rightly described by us in the last chapter as confined to inferences which proceed on the basis of the relation of predication (and they are the only kind to which the Dictum applies), conditional reasonings are not strictly syllogisms. We have already seen that, despite the practice of the traditional logic, hypothetical and disjunctive propositions are more adequate to express exact knowledge than categorical. Their distinctive function is to express a kind of knowledge which cannot be expressed in categorical propositions. They cannot, therefore, be reduced to the latter in reality, though they can in outward form.

Now the doctrine that the Dictum expresses the principle on which conditional reasonings proceed is based on the possibility of reducing the latter to the form of categorical syllogisms, and this in turn rests on the possibility of reducing conditional propositions to the form of categoricals. If we deny the possibility of this reduction, the Dictum can no longer be regarded as expressing the principle of conditional reasonings. No attempt has been made to show that, in

their hypothetical and disjunctive forms, the Dictum accounts for the validity of their procedure.

Hypothetical inferences proceed not on the principle governing the relation of subject and predicate by means of a middle term, but on the principle governing the relation between antecedent and consequent. In some of them, at least, the middle term, an essential requirement of all syllogism, is missing. *E.g.* in a reasoning of the form 'If A is B, C is D ; C is not D ; therefore A is not B,' where is the middle term ? The relation concerned is a relation of propositions, not of subject and predicate. Disjunctive reasoning is even further removed from the categorical syllogism. It proceeds on the basis of the relation between exclusive and exhaustive alternatives in a disjunction, a relation in which the universal or system on the basis of which the inference proceeds is more adequately grasped than in either hypothetical or syllogistic reasoning. Hypothetical and disjunctive reasonings have in common with the syllogism that they proceed from an explicit universal or system, but the grasp of system and the relations in which they express it are different in each. As we have now seen that syllogism does not cover the whole of deduction, there is no reason for calling conditional reasonings syllogisms because they are deductive.

And there is another difference between the syllogism and systematic inference, if we may so call inferences which are based on the mutual implication of elements within a system. It is due to the inadequate grasp of system which we have already seen to characterize the syllogism. In a syllogism, the relation between premises and conclusion is one-sided, not mutual, dependence. We can infer from premises to conclusion but not *vice versa*, not from *e.g.* the conclusion and the minor to the major. The premises are supposed to guarantee the truth of the conclusion, but the conclusion does not help to confirm the premises. But in a genuine system, *e.g.* space, the numerical system, the system of gravitating bodies, etc., or any sub-system of these

like a triangle or circle in space, the relation of elements is not one-sided but mutual. We can go from part to part indifferently, *e.g.* from the conclusion and either of the premises to the other premise, as well as from premises to conclusion. A system consists of elements which are reciprocally related, mutually interdependent. Its parts mutually support and sustain one another.

We have seen (ch. xii) that the presupposition of all thought and reasoning is the systematic character of reality, and that the work of thought or knowledge is progressively to reveal the nature of this system and the sub-systems within it. The development of our inquiry has forced this systematic character of knowledge more and more on our notice. We met it in dealing with propositions and definition; the Laws of Thought are its explicit expression; and all reasoning proceeds on the basis of it. This is true even of the syllogism. It is because the kind or universal on whose repetition in instances syllogistic inference is based is a system of elements, such that, given the presence of one or more, we can infer others, that we can argue on the basis of it at all. If the system of inter-related qualities which constitutes man did not remain identical, in spite of the variety of its manifestations, we could not argue validly on the basis of it. Nevertheless in the syllogism, as in the categorical propositions of which it is composed, the grasp of system is relatively external and imperfect. On the other hand, in non-syllogistic deduction, which comprises a large part of both scientific and ordinary reasoning, the emphasis is all on system.

Now, as our knowledge in any sphere becomes more complete and scientific, the elements in it are found to be more closely inter-related. Each new element whose place in the system we discover enables us to see the nature of the system as a whole more clearly. So that our insight into the nature of a whole and our insight into the character and interconnection of its parts proceed together. *E.g.* we discover more of the nature of the numerical system, as we understand more of the relations of the different numbers within it.

We see more clearly the kind of system the physical world is, as science brings to light the exact quantitative relations between its parts. We grasp the nature of the solar system, as astronomers exhibit the mutual relations of its elements in the way of gravitation, etc. And at the same time and by the other side of the same process, we see more clearly the character and mode of behaviour of the constituents or elements of these systems. Our knowledge of the system and our knowledge of its elements develop *pari passu*. In such systems where the elements are mutually interdependent, provided we have enough data to determine the nature of the system, our inference can begin at any point and proceed to any other. Once we see the nature of the system, we can infer from one set of elements to another within it. All that the syllogism can tell us is that a particular system is of such a kind or that an element belongs to a particular kind of system. If we want to infer the precise and distinctive character of a part we must do so, not from the nature of the system, but *according to* the nature of the system, *from* the character of the other parts. The nature of the system is the principle according to which the inference proceeds rather than a premise of it.

This suggests that the syllogism is so far from being the only form of exact or demonstrative reasoning that it is not even the most adequate form. If the most exact reasonings are to be found in the sciences, and if the deductive inferences that we find in the most exact sciences are mostly non-syllogistic, syllogism cannot be the typical form of inference. To put it otherwise, if reasoning presupposes system, and if system means mutual dependence of elements, while what we get in the syllogism is not mutual but one-sided dependence, syllogism cannot be a complete expression of the nature of reasoning.

But before we consider further how syllogism is related to other forms of inference and whether it is an adequate, or the most adequate, expression of the principle of inference, we have to ask whether, or in what sense, it is a form of inference at all. For it has been held that it is so far from

being the only form of exact inference that it is not a valid form of inference at all, that it can prove nothing and that, if it is treated as a proof of its conclusion, it is fallacious. We have disallowed its claim to be the only form of mediate inference or the whole of deduction. Can we justify its claim to validity in the sphere which we have left it, the sphere of arguments concerned with relations of predication, a very wide sphere which includes many of the arguments of everyday life? This inquiry will enable us to see more clearly the nature of syllogistic inference and its relative importance as an embodiment of the nature of complete knowledge.

EXERCISE XVII

1. Distinguish carefully between the deductive inferences which are syllogisms and those which are not. Give examples of each kind.

2. On what grounds has it been held that the *dictum de omni et nullo* is the ultimate major premise of all syllogisms?

3. In a detective story (*e.g.* one of the Sherlock Holmes series), analyse and test the value of the evidence leading to the conclusion. Discuss whether before the solution is given the reader is in possession of sufficient information to enable him to find the solution for himself. (Russell.)

4. Do hypothetical and disjunctive reasonings proceed according to the *dictum de omni et nullo*? If not, according to what principles do they proceed?

5. Distinguish between (*a*) relational inference, (*b*) syllogism, and (*c*) systematic inference. In what sense, if in any, is it true that they all presuppose system?

6. On what grounds, and with what justification, has it been held that the syllogism is not the typical form of deductive inference?

CHAPTER XVIII

THE VALIDITY OF THE SYLLOGISM

IN considering the canons of the syllogism we have seen (p. 169 ff.) that they follow from the Dictum, that the Dictum is the immediate expression of the Laws of Thought, and that the Laws of Thought, though not susceptible of direct proof in the sense of being deducible from anything more certain than themselves, are as certain as anything can be. They are the fundamental presuppositions of all thought and action, all theory and practice, and therefore, unless they are true, there can be no truth or certainty. We cannot deny them without denying the possibility of all knowledge and the validity of every reasoning. Now if the Laws of Thought guarantee the truth of the Dictum and the Dictum guarantees the validity of the canons of the syllogism, it would seem that the validity of the syllogism cannot be questioned without undermining the whole fabric of knowledge. But at what price has this strict proof of the validity of the syllogism been bought? Have we not, it is asked, in the process of connecting it with the Laws of Thought through the Dictum, emptied its forms of all meaning and reduced it to a meaningless repetition that does not satisfy the requirements of knowledge or inference? Is not the resulting syllogism a form of words in which the conclusion just repeats what is already contained in the premises, a form in which we do not reason but merely reassert? If this is so, when we treat this reassertion as a reasoning, this repetition in the conclusion of what we already know or have assumed in the premises as a proof of the truth of

the conclusion, are we not just deceiving ourselves with a show of inference where there is none, or begging the question by believing that we have proved what we have merely assumed? Does a syllogism, it is asked, ever add to our real knowledge; does it supply us with any new information? And if not, can we regard it as an inference?

There are two distinct points involved in this inquiry and we shall consider them separately. The first is the contention that in a valid syllogism the conclusion does not give us any information not already contained or assumed in the premises, and that, therefore, it does not satisfy the requirements of real inference. Real inference, it is held, requires that in the conclusion there should be new knowledge, fresh facts, something which we did not already know. On the basis of the knowledge which we already have, we should be able to infer something new, to forecast the future. If inference is a mere restatement of what we know, of what use is it? But such a restatement is all that syllogism can ever give us, for "it is universally allowed that a syllogism is vicious if there be anything more in the conclusion than was already assumed in the premises."[1] It is concluded, then, that if a syllogism is valid, it is mere barren repetition; and if it gives new knowledge it is invalid, because the conclusion is not contained in the premises.

The second criticism of the syllogism is that in a formally valid syllogism in the first figure, the perfect figure to which all the others can be reduced, the conclusion is required to establish the truth of the major premise, and that therefore, if the major premise is used to prove the conclusion, we are begging the question and our reasoning is invalid. Both these criticisms of the syllogism have been urged by Mill. The consideration of them will enable us to see more clearly the requirements of inference and the way in which the syllogism, and the Dictum as its principle, must be understood if it is to be vindicated against such criticisms and regarded as a valid and typical

[1] Mill, *System of Logic*, bk. ii. chap. iii. sect. 1.

inference. In justice to Mill we must admit that the syllogism and its principle have been so understood and interpreted, by some scholastic and modern formal logicians, as to justify his contentions ; but they need not be so interpreted, and they were not so understood by Aristotle.

We shall begin with the second criticism and we shall state it in Mill's own words.[1] "It must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii*. When we say,

All men are mortal,
Socrates is a man,
Therefore Socrates is mortal ;

it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition, Socrates is mortal, is presupposed in the more general assumption, All men are mortal : that we cannot be assured of the mortality of all men, unless we are already sure of the mortality of every individual man : that if it be still doubtful whether Socrates, or any other individual we choose to name, be mortal or not, the same degree of uncertainty must hang over the assertion, All men are mortal : that the general principle, instead of being given as evidence of the particular case, cannot itself be taken for true without exception, until every shadow of doubt which could affect any case comprised with it, is dispelled by evidence *aliunde* ; and then what remains for the syllogism to prove ? That, in short, no reasoning from generals to particulars can, as such, prove anything, since from a general principle we cannot infer any particulars, but those which the principle itself assumes as known."

Mill here accepts the traditional view that the syllogism is the typical expression of deduction, but that does not affect his argument and so we need not consider it. His argument here is that, according to the traditional doctrine of formal logic, every valid syllogism can be reduced to Fig. I. In that figure, the major premise is universal and

[1] *System of Logic*, bk. ii. chap. iii. sect. 2.

the minor affirmative, bringing a case or cases under a rule or universal. In such arguments, the major premise states universally what is to be proved in a particular case or cases in the conclusion. Hence the conclusion is assumed in the major premise and is required to establish the latter. Accordingly, if the conclusion is doubtful before we prove it, the major premise, by means of which we prove it, is equally doubtful, and therefore the conclusion is as doubtful after we have proved it syllogistically as it was before.

This contention is based on a particular interpretation of the major premise and the Dictum, in fact on a particular theory of the nature of universals. It assumes that all universals, and therefore all major premises of first figure syllogisms, are aggregates of particulars, and that the only way in which a universal statement can be established is by establishing separately each particular case that falls under it. For such a view of universals, and, in particular, of the major premise and the Dictum, there is a good deal of foundation in the writings of formal logicians. The class theory of the import of propositions, the traditional doctrine of the distribution of terms, the very name *dictum de omni et nullo*, and the usual form of the major premise, *e.g.* *All* men are mortal, all suggest such an enumerative view. And Mill was led to adopt it by his own general philosophical position. He held [1] that all knowledge begins with particular facts, with the observation of individual instances, and that the universal statement is just a shorthand way of recording the results of a large number of particular observations. It is the particular observations that entitle us to make the universal statement, and as long as any of the particulars are unobserved the statement is not quite certain. According to this view the only kind of universal that is possible is the enumerative, *i.e.* one established by separately investigating each case that comes under it, as *e.g.* 'All the books on that shelf are philosophical,' or 'All the chairs in this room are made of oak.'

Now if the major premise of every syllogism is an enumer-

[1] *Op. cit.* bk. ii. ch. iii. sect. 3.

ative judgment, an aggregate of particular statements, Mill's contention that every syllogism involves a *petitio principii* is quite sound. For the form of the syllogism then becomes

All M (*i.e.* A, B, C, . . . S, T, . . . X, Y, Z) are P,
S is M,
∴ S is P.

e.g. All the chairs in this room are made of oak
The chair on which you are sitting is a chair in this room
∴ The chair on which you are sitting is made of oak.

This obviously is a begging of the question. For if we had not considered the chair on which you are sitting and found that it was made of oak, we should not be justified in saying that all the chairs were made of oak. There is no necessary connection between being a chair in this room and being made of oak, and therefore, before we can make the statement contained in the major premise of the above syllogism, we must already have examined each chair separately and so known the conclusion that we profess to infer from it.

Grant Mill's premises (and they are premises which, as we have seen, have a good deal of justification in the doctrines of formal logic) and his conclusion follows. If all universals are enumerative, then the Dictum and the major premise of every first-figure syllogism must be an enumerative proposition, and, if so, every syllogism involves a *petitio principii.* On such a theory, however, we can get very few universal propositions : we can get them only in cases where the objects referred to constitute a definite limited number, each of which can be separately inspected. And, what is more important, the resulting universals are no more universal than the particulars which compose them. They are merely collections of such particulars, and a collection of particulars is no more universal than one particular. On such a view we should seldom, if ever, be justified in stating the major premise of an ordinary syllogism. We certainly could not get such a major as, 'All men are mortal,' in that way. Mill recognized this, and so he regarded such a

universal as made up of a number of observed, together with a further number of unobserved, but inferred instances.[1] On the basis of the individuals we have observed we infer that what is true of them is true also of those which have not been observed, but, as we shall see, we are entitled to infer the character of the unobserved cases only on the basis of the universal which we have discovered in the observed cases and which we state in the major premise of our syllogism. It is because the unobserved cases are of the same kind, instances of the same universal, as the observed cases, that we can predicate of them what we have found true of the former. Such a universal cannot be of the nature of an aggregate of particulars; it must be rather an inter-relationship of elements within a system. We have already seen that if thought or reasoning is to be possible there must be universals of quite another kind than aggregates of particulars, universals in the sense of necessary connection between subject and predicate, a connection based on an identical relationship of elements in all the individuals that come under the subject, in virtue of which the predicate applies to them.

Even the syllogism, as we have seen, proceeds on the basis of system, though system of a quite definite kind, the systematic relationship between the predicates of a subject that constitutes it an individual of a certain kind. No doubt syllogism emphasizes the repetition of the system or plan or relation of elements that constitutes a kind as well as, or even more than, the inter-relationship of elements within it, but unless the system was repeated and so present in each of the instances they would not be instances of the one kind, and so we could not infer from one to another. So that, even in the syllogism, it is our grasp of the connection between qualities or elements, rather than our enumeration of individuals, that makes our inference secure.

Now if there are universals in the sense of a necessary connection of elements, we need not separately investigate all individual instances of them in order to be able to make

[1] *Loc. cit.*

the universal statement—in most cases we could not do so even if we would; and if such universals do not exist all knowledge and inference become impossible. Such a genuine or generic universal is, as we have seen (p. 93), more clearly brought out by the hypothetical form, 'If S then P,' 'It is of the nature of S to be P,' 'S as such is P,' than by the categorical form, 'All S is P.' The latter, especially if it is interpreted 'All Ss are Ps' or 'All members of the class S are members of the class P,' as many formal logicians tend to interpret it, lends considerable support to Mill's criticism. This form lays all the stress on enumeration and ignores the basis of system or necessary connection without which all inference becomes impossible. This is also Mill's error.

If we take the enumerative view of the universal, we should not be justified in making a universal statement like 'All men are mortal' till all men had died, or like 'All matter gravitates' till we had examined every particular piece of matter. But if these statements are genuine universals, they mean that there is something in the nature of man as an organism that makes him mortal, and something in the nature of bodies as material that makes them gravitate. So that the real universal indicates, not a definite number of individuals which have been examined, but the individuals of a certain kind, the individuals constructed on a certain plan. We cannot get such a universal by mere enumeration, and none of the universal statements of science are got in that way. How we arrive at such universals we shall have to consider when we come to deal with induction, but unless they existed we could not reason at all. Now in all cases where the major premise of a syllogism is a genuine or generic universal, and this is the case in most significant syllogistic inferences, the conclusion is not required in order to establish the major premise, and therefore the charge of *petitio principii* does not hold.

And apart from cases where the major premise states a characteristic of a kind, there are many ordinary reasonings

which can be expressed syllogistically in which it is obvious that the conclusion is not required to establish the major premise or contained in it separately : it follows from both premises combined. This is so in the case where both premises are singular propositions, *e.g.* 'The writer of this exercise does not know much about logic, A.B. is the writer of this exercise, therefore A.B. does not know much about logic.' I may make the first statement without knowing the truth of the second, but when the second is made a conclusion follows which is quite different from either and which was not required to establish either. The same is true where the major premise is accepted on authority, like most moral and social principles and religious beliefs, or man-made, like legal enactments. *E.g.* a judge is called on to administer a law which he has not made and dare not alter. The law lays down that crimes of a particular kind are to be punished in a particular way. A man is brought before the judge, and it is proved that he has committed the crime in question ; and the conclusion follows that he should be punished in that particular way. In such a case the major premise, by itself, did not contain the conclusion and the latter was not required to establish it. In such cases the conclusion may be said to be in a sense contained in the major, but it requires the presence of the minor to make it explicit. It is potentially present in the major, in the sense in which heat is potentially present in coal but requires a match to make it actual.

We often use such universal statements which we accept from others without seeing the necessary connection which constitutes their universality and without being able to prove them for ourselves (as I may accept from a chemist that water consists of oxygen and hydrogen in certain proportions), or which we ourselves have proved on previous occasions, though we have now forgotten the proof (as I may have proved that the angle in a semi-circle is a right angle). We treat such statements as major premises and bring particular cases under them, and a conclusion follows. In such cases we do not strictly prove the statement made

in the conclusion. We do not now see the connection or we may never have seen it at all; and so far Mill is right when he says that the inference, in the sense of proof, is over when we have got the universal statement which constitutes the major premise. Nevertheless even in these cases there is inference: we infer the conclusion from knowledge which we already possess, though we may not grasp the full significance of the statements that we make.

And there may be inference even in cases where the major is an enumerative judgment;[1] *e.g.* I may have examined all the books on a particular shelf and found that they are philosophical. If I were to see one of these books elsewhere I might not be able, without re-examination of its contents, to say that it was philosophical, but if someone were to come into the room looking for a mathematical book and to take a book from that shelf I could say to him: 'That book is philosophical.' If he asked the reason why I thought so, I might say: 'Because all the books on that shelf are philosophical.' In such a case I am not proving the conclusion or giving the reason why the fact is so, but *I am inferring, from what I know or remember, that it is* so. Here the major premise is what Mill[2] regards every universal statement as being, 'a record of our past observations.' And it is not the record but the facts recorded that prove the conclusion. There is no such difference between the reason why the fact is so and the reason why we believe it to be so in the case of the generic universal: the nature of the kind and the fact that it belongs to the kind is the reason why it has, as well as the reason why we believe it to have, the character we attribute to it. In many, if not in most, of our ordinary reasonings the major is accepted without proof or remembered as having been proved, and so the conclusion is not strictly proved. Yet there is inference and the conclusion is not required in order to prove the major premise.

[1] Cf. Joseph, *op. cit.* p. 310.

[2] *Op. cit.* bk. ii. ch. iii. sect. 3.

We come now to Mill's second criticism of the syllogism, namely, that it is not an inference because the conclusion is already contained in the premises. The answer to this criticism is contained in what has been already said, but further consideration of it will bring to light an important feature of inference. For the criticism could be made against all valid inference in the same sense in which Mill urges it against the syllogism. In all valid inference the conclusion must in a sense be contained in the premises: otherwise we should have no ground for drawing it from them or asserting it. On the other hand, unless the conclusion goes beyond the premises, contains something new, something which was not *in the same sense* contained in the premises, we have no reasoning but merely re-assertion of what we already know. These two conditions constitute what has been called 'the paradox of inference.'[1] The paradox consists in the fact that inference must satisfy what appear to be two incompatible conditions: (*a*) The conclusion must contain something new, be a genuine advance, otherwise there is no inference; and (*b*) the conclusion must contain nothing new, nothing not already contained in the premises, otherwise the argument is invalid. This paradox may be expressed in the form of a dilemma:

If the conclusion of an inference does not contain something not given in the premises, the inference is useless; and if the conclusion does contain anything not given in the premises, the inference is invalid.

Either the conclusion does contain something not given in the premises or it does not.

Therefore inference is either useless or invalid.

In this dilemma, as in most faulty dilemmas, the alternatives given in the minor do not exhaust all the possibilities; nor are they, properly understood, exclusive of one another. For it is the premises taken separately that must not already contain the conclusion; whereas it is the premises taken together that must contain the conclusion. Accordingly if we interpret the minor in the light of the major and state

[1] Bosanquet, *Essentials of Logic*, p. 157.

it precisely, it becomes 'Either the conclusion must contain something which was not in either of the premises alone or it must contain nothing which was not in both premises together.' When it is so stated it is obvious that there is another alternative, namely, that the conclusion makes explicit something which is contained in both premises together but not in either separately. This gives us the solution of the paradox. In any valid inference the conclusion, being the real union of the premises, gives something new, like a chemical compound (*e.g.* oxygen and hydrogen giving us water), but it must contain no more than the result of the two premises together. It thus appears that it is the act of bringing the two premises together, the act of combining them or seeing the connection between them, that constitutes the real act of inference; and this act gives something new.

In dealing with the import of propositions, we noticed two opposed one-sided views, each of which insists on an element of truth which the other neglects. The class view so emphasised the identity between subject and predicate that the proposition became pure tautology; while the attributive view so insisted on the element of difference that all connection between subject and predicate disappeared. In the same way, and as the result of over-emphasis on the same elements, we get two one-sided views of inference. The one takes the major term of the syllogism in extension and treats the major premise as an enumerative judgment; and as a result the syllogism becomes a barren analysis. The identity between premises and conclusion is so stressed that we have mere repetition and no inference. Mill, on the other hand, lays such emphasis on the element of difference between premises and conclusion, the element of novelty in the conclusion, that all connection between premises and conclusion seems to have vanished. For him the fundamental form of inference is from particular to particular,[1] from one individual fact to another; and many of his statements leave the impression that these particulars

[1] *Op. cit.* bk. ii. ch. iii. sect. 3.

have no universal element in them. But, as we have seen, every valid inference must satisfy two conditions : (*a*) there must be necessary connection between premises and conclusion, and (*b*) there must be something in the conclusion not contained in the same sense in the premises. Mill lays so much stress on the second condition, and expresses it in such a way, as to leave no room for the first, which is even more important. We may get inferences which are quite valid and significant, in the conclusion of which there is no new element, in the usual sense of novelty. The conclusion may be a fact with which we are perfectly well acquainted : we look for reasons why the familiar fact should be so (*e.g.* why one kind of soil is more fertile than another, why the tides are higher at full and new moon), and when we discover the reasons, they become the premises from which the fact follows as conclusion. That is perhaps the normal procedure of thought : we discover the conclusion first and then look for the premises. This shows that the element of necessity is more important than that of novelty. At the same time the conclusion must be new in the sense that it must not be contained in either of the premises separately. Whether as a matter of fact we know it on other grounds or not does not alter its relation to its premises. Mill's criterion of real inference seems to demand such an element of novelty in the conclusion that the conclusion could not follow from the premises ; but such a criterion would render all inference impossible. If the view which Mill criticises would reduce inference to mere analysis and barren repetition, his own view would make it pure synthesis without any element of connection. As we found in the case of the proposition, so here in the case of reasoning we find that neither mere analysis nor mere synthesis can account for inference : inference requires both analysis and synthesis, both necessary connection and advance, both identity and difference between the premises and conclusion.

In order to make this point clearer and to bring to light more fully the requirements of real inference, we shall con-

sider the kind of inference which Mill regards as fundamental, the kind which he believes satisfies his requirement of novelty without being invalid, namely, the argument from particular to particular. According to Mill,[1] the fundamental form of inference is that from one individual case (or several individual cases) to another (or others). We begin by observing particular cases. From these we infer to other unobserved cases, and then we can make a general statement covering the observed and the unobserved cases. But the inference is over before we make the general statement and the latter adds nothing to the grounds for any particular case. It is because the major premise of the syllogism is a general statement of this kind that, as an attempt to prove the conclusion, the syllogism is invalid. No doubt, says Mill,[1] in going from one particular to another, it is an advantage to pass through a general proposition, for it reminds us of what we have to assume as proved before we are entitled to go to the new particular. Thus the general proposition acts as a check on hasty inference.

Nevertheless it is not necessary to pass through such a general proposition. "Not only *may* we reason from particular to particular without passing through generals, but we perpetually do so reason. All our earliest inferences are of this nature. From the first dawn of intelligence we draw inferences, but years elapse before we learn the use of general language. The child who, having burnt his fingers, avoids to thrust them again into the fire, has reasoned or inferred, though he has never thought of the general maxim, Fire burns. He knows from memory that he has been burnt, and on this evidence believes, when he sees a candle, that if he puts his finger into the flame he will be burnt again. He believes this in every case which happens to arise; but without looking in each instance beyond the present case. He is not generalizing; he is inferring a particular from particulars."[2] In the same way, we are told that animals, which are not regarded as having the power of generalization, modify their actions as

[1] *Loc. cit.* [2] Mill, *Loc. cit.*

the result of experience. Such actions and inferences on the basis of our knowledge or experience in particular cases Mill calls inferences from particular to particular: one or a few particular cases may be enough to enable us to reach a conclusion about any 'similar' or 'parallel' case.

Now it is quite true that we sometimes, indeed usually, reason and act without formulating a general proposition, and often without being consciously aware of any universal principle involved in our reasoning or action. But that need not, and does not, mean that there is no universal principle involved, that there is no identical element in, or necessary connection between, the cases, or that our inferences are from one bare particular to another. How as a matter of fact we do reason, and whether we go from one particular case to another without *formulating* a general proposition or universal principle, is a psychological question. The question of logical importance is, On what does our inference depend? What is needed to make it valid? What premises must be true to justify it? Logically we have to ask not, Can we go from particular to particular without formulating a universal? but, Can we validly infer from particular to particular except on the basis of a universal or principle of which both are instances? Can we have valid inference which does not involve the presence of a universal, whether that universal be formulated in a general proposition or not?

No doubt, both in argument and in practice, we proceed on principles which are not consciously present before our minds and argue from premises which we do not explicitly formulate. Just as we digest or breathe before we know anything about the laws according to which these processes take place, so our mental operations are carried on according to principles which only later analysis reveals: the principles operative in our experience are not always objects of knowledge to ourselves. But that does not mean that there are no such principles at work. All that Mill has shown is that the universal principle need not be consciously present before the mind or explicitly expressed in a general proposition. He has not shown, or even tried

to show, that it is not there, not operative. At any period, even the most reflective, in our lives there is more in the mind than is consciously before it; and the child, or even the adult, who shuns a fire because he has had experience of the burning properties of another, does not usually formulate the proposition, 'Fire burns'; but the universal, the connection of elements, required to justify the inference involved in his action, when explicitly formulated, takes the form, 'Fire burns.' Of course, the child may shun the hearth at which he was burnt, even when there is no fire there, but in that case his inference is unsound, and the premise which it presupposes is false.

The business of logic is to discover the principles underlying sound reasoning and the premises which particular reasonings imply, but it need not and does not hold that all who reason soundly are consciously aware of these principles or explicitly formulate these premises. It does, however, hold that if the universal is not present, if the implied premises are not true, the reasoning is not sound. Mill himself says [1] we argue from 'parallel' or 'similar' cases, for it is quite obvious that we cannot reason validly from any particular fact to any other. We cannot reason validly even from any particular case to any other *resembling* or *parallel* case. No doubt we often do reason on the basis of superficial resemblances, like the child who shuns the hearth where he was burnt, even when there is no fire there; but our inference is valid only when we reason on the basis of a universal of which the different cases are instances. Hence the particulars from which we argue, when our argument is sound, are not bare particulars but individuals of a certain kind, instances of a universal, and it is on the basis of the identity of kind that the argument proceeds. The universal or kind need not be expressed in a general proposition, but unless it is there our inference has no justification. Why can we argue from the fact that a particular fire burns to the fact that another fire will burn, and not also from the fact that one fire is in an open hearth to the

[1] *Loc. cit.*

fact that the next fire will be in an open hearth? Is it not because in the first case we are arguing from the nature of the kind, while in the second case we are not? Or again, why cannot we infer that, given two animals resembling one another in size and colour, if the first is fleet-footed or carnivorous, the other will be so too? Is it not because the resemblances are superficial, not qualities of the kind and so not indications of the other qualities of the kind?

Our valid inferences, then, are never from one bare particular to another, nor are they merely from one particular to another resembling particular; they are rather from one individual which is an instance of a universal or kind to another individual which is another instance of the same universal or kind. There are, as we have seen, universals which are not kinds, but laws governing the inter-relation of elements within a system. We are not concerned with such universals here, for in their case we cannot argue from one particular fact to another even if we know the law or system. We require at least two facts as premises, *e.g.* 'A is north of B,' and 'B is north of C': we can conclude nothing from one fact alone even on the basis of the system. But the arguments to which Mill refers are concerned with the universals which are kinds. In their case also, in order to get valid inference, we require two premises, one—the syllogistic minor—stating that the particular is of a kind, the other—the syllogistic major—stating a characteristic of the kind. No number of particular instances in which individuals, even individuals of a kind, have a certain characteristic entitle us to believe that it will be found in a new instance unless we have reason to believe that it belongs to the kind as such. Now, what the major premise of the syllogism does, in cases like those referred to by Mill, is to state that a character belongs to a kind or universal, as such. This is one of the premises of the argument, and it is the business of logic to make it explicit.

The class of arguments from particular to particular includes reasonings of very different kinds and of widely

varying degrees of certainty. They vary from syllogisms where the major premise is unexpressed, because it is regarded as too familiar or too obvious to need expression, through cases where the implied major is doubtful and where to express it explicitly would reveal the uncertainty or false character of the argument, to cases where the argument rests on some resemblance between one thing and another and we are not able to formulate the major premise or connection of elements which the argument presupposes. This last class is called arguments by analogy. Such arguments are of varying degrees of probability but none of them give us complete certainty (cf. ch. xxi). In some cases we are almost certain that the resemblance indicates an identity of kind, in others we have little reason to believe that the resemblance is more than superficial. As we shall see, the main value of analogy is that it suggests a connection of elements: it does not itself prove the connection. The connection can often be proved by evidence from other sources; but in some cases it cannot, and we have to rest content with the inference by analogy.

We regularly use all these different kinds of arguments from particular to particular. To argue that because Socrates is a man he is mortal is to infer from one fact to another. But as we have seen when considering enthymemes (pp. 215-7), the difference between this form and the strict syllogism is one of expression, not of logical principle. Again, to argue that because A and B are brothers their characters are alike, or they will behave in the same way in a given situation, is to argue from particular to particular, but the value of the argument is very doubtful and its logical character is very different from that of the one we have just considered. What we are concerned to point out at present is that, whatever degree of probability arguments by analogy may have, unless the resembling individuals are instances of a kind and the resembling characteristics qualities of the kind, the conclusion is not certain.

We conclude, then, that when arguments from particular to particular are valid, they are so because the particulars

are instances of a universal. We must not confuse the individual, which is both universal and particular, a particular instance of a universal, with a pure or bare particular. This is Mill's error. Now if the universal is a kind and not an aggregate of particulars, we can discover the nature of the kind without examining all its instances, and on the basis of it we may infer the characteristics of fresh instances. Accordingly, in a syllogism which predicates of an individual the characteristic of its kind the conclusion is not necessary to establish the major premise and the fallacy of *petitio principii* is not committed. The syllogism is a strictly limited form of inference, but within its own limits it is quite valid.

EXERCISE XVIII

1. "Syllogism in no way increases knowledge, if once the premises are given; which is in perfect harmony with the circumstance that no rational being thinks in syllogisms." Discuss.

2. Explain and examine the view that every syllogism involves the fallacy of *petitio principii*.

3. What function does Mill assign to the syllogism?

4. How far and in what cases does the syllogism seem to you to *prove* its conclusion? Can any syllogism which fails to *prove* its conclusion be regarded as inference?

5. What is meant by the 'paradox of inference'? Explain the conditions of inference to which the 'paradox' calls attention and indicate how they can be satisfied.

6. "We much oftener conclude from particular to particular directly than through the intermediate agency of general propositions." Expound and consider Mill's view of the nature of inference.

7. In what sense, if in any, must there be (*a*) novelty, and (*b*) necessity in the conclusion of a valid inference?

8. "Valid inference is impossible, since for inference we require novelty and for validity we must limit ourselves to what is given." Carefully examine this argument in respect both of matter and of form.

CHAPTER XIX

THE PROBLEM OF INDUCTION

ALL inference presupposes and proceeds on the basis of system. In the inferences which we have so far considered, the system is either expressly stated or considered so obvious as not to need explicit expression. Such inferences are therefore called deductive. A deductive inference proceeds from an explicit universal or system, and either brings cases under the universal or infers the nature of some elements from others on the basis of the system. Such inferences therefore presuppose a prior process by which the universal or system is arrived at or established. This process is called induction. Accordingly we may express the problem of induction in the question 'How do we get or establish the universals or systems without which deductive inference cannot begin?' Another way in which the distinction between deduction and induction has been put is this. In deduction we infer from given premises and we draw a conclusion consistent with the premises. Our inference therefore does not guarantee the truth of our conclusion but only its consistency with our premises. Deductive logic has therefore been called the logic of consistency. In induction we are concerned with the process whereby the truth of our premises is established. The branch of logic which investigates it has therefore been called the logic of truth or material logic. From the point of view of this distinction we may put the problem of induction in the form of the question 'How do we establish the premises of deductive inference?' We may express the distinction in still

another way. Deduction begins with a given whole or system and infers from it the character of its parts or elements. Induction begins with individuals, particulars, elements, and tries to show the nature of the whole or system which they constitute, the law governing their inter-relationships. In whichever of these ways we state the problem of induction, we shall see that it takes a somewhat different form according to the type of system which we are trying to establish, according to whether we are trying to get premises for syllogistic or non-syllogistic deduction.

The purpose of science is to explain, and we are said to explain a thing when we show that it is an instance of a law or laws, or an element in a system, the effect of a cause, the means to an end, etc. We cannot explain anything by itself. We have to connect it with other things within a system. Now the processes involved in such knowledge and explanation include both induction and deduction. Indeed, like analysis and synthesis, deduction and induction mutually imply one another. Neither is complete or a full account of our knowledge of any object without the other; and the work of the one is not over before the other begins. The process of acquiring knowledge is not one in which induction, first, completes the work of establishing the universal or system or law or premises and deduction, then, explains the particulars or elements in the light of the law or system or draws a conclusion from the premises. Deduction is used in the process of establishing the law and bringing to light the nature of the system. And induction, in showing that the particular fact is an instance of a law or connected with others as an element in a system, throws light on its nature and helps to explain it. System and elements, law and facts, universal and particulars, premises and conclusion, mutually involve one another, and therefore insight into the nature of any one of these complementary opposites involves insight into the nature of its correlative.

And not only do deduction and induction mutually involve one another: they also proceed on the same ultimate principle. They differ in their starting point, not

in their principle and in many cases we can use them indifferently to explain a fact or establish a law. Which of them we begin with or employ in a particular case depends on our knowledge or purpose. We cannot in the end explain anything fully without both (*v.* p. 358).

The term induction has been and, indeed, is still being used in several different senses. A brief reference to the history of logic will help to make these varying senses clear. We shall see later that so far as induction means a process of inference which generalizes from particulars or which establishes universal statements and scientific laws—and that is an element in all its different meanings—it involves an element of deduction.

The need for a process of inference prior to deduction was recognized by Aristotle. According to his doctrine every syllogism must have at least one universal premise ; without a universal to begin with, a syllogism can prove nothing. In some cases this universal premise may itself be deduced syllogistically from another universal, but there must be an end to this process at some stage. We must in the end get universals which are not syllogistically established. Now experience supplies us directly not with universals or laws but with individuals or particulars. We never meet such a fact as 'man,' 'all men,' 'if a man.' We just meet individual men. From such individuals the syllogism cannot generalize. It must have a generalization as a premise before it can infer anything.

Aristotle's account of the process by which the universal premises of syllogisms are arrived at is neither clear nor full nor free from difficulty, and it is bound up with difficult questions of interpretation into which we cannot enter here. A brief outline of his view is, however, desirable. He bases the distinction between induction (ἐπαγωγή) and strictly demonstrative knowledge (ἀπόδειξις) on a distinction between things as they are in themselves and things as they are for, or appear to, us.[1] According to his doctrine

[1] *An. Pr.* 68ᵇ 35 ; *An. Post.* 72ᵇ 29 ; *Top.* 105ᵃ 16.

syllogism, which he regards as the typical expression of deductive inference, expresses the perfect knowledge of things as they are in themselves, the knowledge which proceeds from universals or causes to particular cases or effects, from explicit grounds or reasons to consequents or conclusions. Induction expresses a less perfect form of knowledge. It begins with that which is prior or more clear to us, not with that which is more intelligible in its own nature. It proceeds from effects to causes, from observation of particular cases to universal laws, from consideration of the part to knowledge of the whole. This process is necessary to supply the major premises of syllogisms. Accordingly as far as the development of our knowledge is concerned, the inductive process must come first.

Aristotle represents the principle on which the inductive process proceeds as the reverse of the principle of the ordinary syllogism.[1] He expresses it : Where certain species have a common characteristic, whatever can be predicated of all the species can be predicated distributively of the genus which is marked by the common characteristic. This principle, he suggests, may be formulated in a syllogism, *e.g.*

All S (*i.e.* A, B, C, etc.) is P,
All S (*i.e.* A, B, C, etc.) is M,
Therefore All M is P.

Here S stands for the species, and M for the common characteristic which constitutes all S the genus. As the formula stands, it seems to be an invalid syllogism in the third figure, an illicit minor. But as M stands for the common characteristic which constitutes all S the genus, the minor premise 'All S is M' is simply convertible. Aristotle's own illustration is : [2]

Man, horse, mule, etc. are long-lived,
Man, horse, mule, etc. are gall-less,
Therefore all gall-less animals are long-lived.

Here the conclusion follows only if the second premise is

[1] *An. Pr.* ii. 23. [2] *Loc. cit.*

simply convertible, *i.e.* if man, horse, mule, etc. are all the gall-less animals.

Now in all this Aristotle is not thinking primarily of individuals, which could not be exhaustively enumerated, but of the species making up a genus. In his chapter on induction [1] he seems to take for granted that we already know that men are gall-less, that horses are gall-less, etc. But how this is established he gives no indication. He tells us elsewhere [2] that the function of our observation of particulars is to enable us to see the universal in them. He seems to take the position that the universal is derived from the particulars not in the sense that we infer it from them but in the sense that they enable us to recognize it. The particulars do not prove the universal. We recognize it by means of them, but we do not infer it from them. Thus the process of induction becomes not inference, but direct insight. This is not unlike the modern view, which we shall have to consider later, according to which our inspection of particular facts suggests a universal or law, which we proceed to test in other ways. Aristotle has very little to say about these tests, but they play a prominent part in modern inductive theory. He does not, however, hold that the universal is established by examining all the particulars that come under it.

While Aristotle was not thinking primarily of an exhaustive enumeration of individuals, his illustrations contain a suggestion of enumeration, and the enumerative aspect was emphasized by later writers. They divided induction into two kinds, perfect induction based on complete enumeration, and imperfect induction where the enumeration is not complete. By complete enumeration we get enumerative judgments, like 'All the books on that shelf are philosophical,' by incomplete enumeration general statements, like 'All men are mortal.' Such a theory presupposes

[1] *An. Pr.* ii. 23.

[2] *An. Post,* 88^a 13; 88^b 4. Cf. Ross, *Aristotle,* pp. 40-1, and the references there given.

that the universal is to be regarded as an aggregate of particulars. That is all that we can get by mere enumeration; for, as we have seen, enumeration as such cannot establish a necessary connection. A number of instances of a conjunction, however, may suggest a connection. And (as long as there are no contrary instances) they may suggest it equally well when the enumeration of instances is not complete.

The general tendency among post-Aristotelian writers was to make logic more formal, to develop the syllogistic side and neglect induction. The main logical interest of the Middle Ages was in deduction. They tended to accept their first principles, their major premises, from authority, and the work of reasoning was to deduce particular conclusions from these premises. For this the syllogism is quite well suited.

With the rise of modern science, there came a new interest in induction, the process of arriving at universals from the observation of particulars. And the universals of science are not got by enumeration, complete or incomplete. For enumeration cannot give necessity; and what science tries to establish is necessary connection, universal laws, causal relationships. Enumeration therefore cannot satisfy its requirements. With the development of the exact sciences, inductive logic came to mean the logical investigation of the processes of inference employed in the natural sciences. This inductive logic was opposed to the old deductive logic, as if the two had nothing in common and as if the natural sciences did not use deduction at all.

The rise of inductive logic, in this sense, is usually associated with the name of Francis Bacon (1561-1626), but it was partly developed before his time and he himself did not fully understand its nature. What he did do was to emphasize one aspect of it and to protest against the syllogism as a method of advancing knowledge. The syllogistic method seemed to him to consist in taking unexamined prejudices as the foundation of all knowledge. But the logic against which he protested was the traditional

scholastic logic and not the strictly Aristotelian system. As Adamson [1] tells us, he had not "the acquaintance with the genuine Aristotelian system requisite in order to distinguish the elements of permanent value from the worthless accretions under which these had been buried," and as a result his own views "have a far closer resemblance to the Aristotelian doctrine than might be imagined from his attitude of opposition to them."

Now if induction is to be regarded as equivalent to the whole of the processes of thought involved in scientific procedure, so far is it from being the exact opposite of deduction that it includes deduction within it. Bacon neglected the deductive element which forms an essential part of scientific procedure. But in so doing he was not entirely without excuse. For, according to the accepted tradition, syllogism is identical with deduction, and as we have seen most of the deductions of the exact sciences are not syllogistic. So that it was his recognition of the fact that syllogism plays very little part in the procedure of the sciences that prevented Bacon from recognizing the deductive element in science. Nevertheless the separation of deduction and induction and the treatment of them as if they needed two separate logics, which resulted from Bacon's opposition of the processes of inference involved in science and those dealt with by the traditional logic, led to considerable confusion in logical theory. For induction and deduction, as we have hinted already and shall see more fully later, are not mutually exclusive nor are they different kinds of inference; they are rather different aspects of the same process. Both are employed in all concrete thinking whether in ordinary life or science, and they mutually involve one another. For in genuine reasoning the relation between premises and conclusion is not one-sided, but mutual. The conclusion helps to establish the premises as well as *vice versa*. The law or universal arrived at inductively is confirmed by deductions drawn from it, while deduction cannot begin without a universal which it cannot itself provide but has to accept

[1] *A Short History of Logic*, p. 85.

from induction. From the time of Bacon, however, we get logic treated under two heads: deduction, which meant mainly the traditional formal logic, and induction, which was sometimes regarded as co-extensive with the logical investigation of scientific procedure and at other times confined to the process of generalization from particulars. One unfortunate effect of this which we have already noticed was a tendency to neglect inferences which fail to fit into this classification. When they were not neglected, they were distorted so as to fit into the scheme.

There are two main kinds of generalization which science seeks to establish: (1) causal connections between particular facts or kinds of fact, and (2) wider, more comprehensive laws or generalizations which unify a large number of facts into a system, like the law of gravitation or the theory of evolution. Both kinds are universal and natural laws, but there are differences between them which make it desirable for us to distinguish them. We do not see causal connection or any other kind of connection. We see conjunction or succession, two things together or coming one after another. But we say not merely that two things are compresent or successive, but that the one is the cause of the other; and not just that there is a causal connection between these two things but that there is one between all things of these kinds. *E.g.* we say that arsenic causes death, that fire causes heat, not simply this bit of arsenic or this fire. The particular causal connection recurs. In every case in which we get the one element (the cause) the other element (the effect) follows. Just as we found kinds to recur in large numbers of instances and to have the same character every time they recur, like man—mortal, so causal relations recur in different instances, like arsenic—causing death. There are many instances of both kinds of relation.

Now, though the relation of subject and predicate is not the same as that between cause and effect, they can both be stated in general propositions, like 'All fires burn.' But while the traditional syllogistic logic lays more stress on

relations of predication, science lays more stress on causal connections. The former tends to emphasize the instances, the latter the necessity of the connection. Aristotle did not clearly distinguish between the two, but while his formula, All S is P, was meant to cover both it was relations of predication which mainly interested him; and it was to them that the scholastics devoted most of their attention. Physical science, on the other hand, while many of its laws admit of expression in the subject-predicate relation, is interested not in such relations, but in the necessary connection which happens to be expressed by means of them. In other words, science tries to discover a causal connection underlying the general statement; *e.g.* it tries to discover the cause of mortality in man. Accordingly with the rise of modern science, the problem of induction tends to take the form 'How are causal relations established?' rather than 'How are the premises of syllogisms arrived at?'

The term 'cause' has, as we shall see, been used in different senses, but even if we take the widest of these senses, science is concerned to establish laws which express relations other than causal. It tries to systematize facts and particular causal connections, to show that they are elements in wider systems, instances of more general laws. The term 'law of nature' is usually confined to the simplest and most comprehensive of these laws. Such laws are expressions of the ways in which the constituent elements of very wide systems of facts are inter-related. The law or theory or principle organizes many particular facts and thereby explains them, makes them more intelligible. It introduces or rather reveals order and system in the sphere with which it is concerned. Less general and comprehensive laws do the same thing on a smaller scale. A detective gathers a number of isolated facts, and suddenly finds that light breaks on all of them on the hypothesis that a certain individual committed the crime he is investigating. In that light they fit into a connected whole, they become intelligible, they are explained. An astronomer observes a particular heavenly body in particular positions at given

times, and then realises that these and other facts about it can be explained or rendered intelligible on the hypothesis that it moves according to a certain law. A child with a heap of blocks finds that the design on each of them becomes intelligible as he arranges them in such a way that the design on each appears as part of one complete pattern. The same is true on a more extended scale of the law of universal gravitation or the theory of evolution. And just as a law explains facts, so a more general law explains a narrower. The best guarantee of the truth of a more restricted law is that it should be seen to be an instance of a wider law. The relation of such laws to those expressing particular causal connections will become clearer as we proceed. What we want at present to notice is that to discover or establish them is part of the work of induction. We see particular events, individual things behaving in certain ways. We cannot see the laws governing their relations or behaviour. These have to be inferred, and inductive logic investigates the processes by which they are established.

There are three ways, then, in which the problem of induction may be stated, or, it might be more accurate to say, three aspects of the problem which may be distinguished. (1) How do we establish the general propositions which supply the premises of ordinary syllogisms, like 'All men are mortal'? (2) How do we establish causal connections, necessary connections between particular events, like 'Arsenic causes death'? (3) How do we establish scientific hypotheses or laws of nature, laws which reveal order and unity and system pervading large numbers of facts, like the law of gravitation or evolution?

The general proposition, the causal connection, and the law of nature, are all universal and profess to state necessary connection, but there are differences between them which make it important to distinguish them. Causal connections and laws of nature are the main interest of modern inductive logic. For, though it is concerned with the processes involved in all science, it tends to mean by science

the physical or quantitative sciences, and they are not specially interested in kinds or classes or relations of predication. But the less developed sciences, like the biological and social sciences, have often to be content with generalizations about kinds, of the form ' All S is P,' where we do not see the causal or necessary connection that makes the S a P. The ideal of science, however, is to discover this connection and so to reduce the statement about the kind or type to one expressing causal connection.

In the same way, science tries to show that the particular causal connection is itself an instance of a wider, more general law of nature. So that we get a progressive realization of the ideal of scientific knowledge as we pass from statements about kinds, through causal connections, to laws of nature. The explanation of a fact may pass through all these stages. It may first be stated as a characteristic of a kind, a generalization based on enumeration or analogy which suggests a connection but does not prove it. Further investigation may bring to light the causal connection which justifies the universality of the statement. But it may still be a comparatively isolated piece of knowledge, calling for further explanation. Such explanation is forthcoming, if it can be shown to be an instance of a still wider law or an element in a more comprehensive system. At each stage, the explanation becomes more adequate and the fact more certain. The first stage may be regarded as a tentative generalization, a suggestion of connection. In the second, analysis reveals the nerve of the connection. There is at this stage not only a suggestion of some connection, but a suggestion that it is of a particular kind or between certain elements ; and there may be a good deal of confirmation for it. But it is difficult to be quite certain that we have got the connection, or explained the fact, till we see how it coheres with other facts within a wider system. When we have done so the tentative suggestion becomes established fact. The work of inductive logic is to follow this process from the first suggestion to its final establishment.

We see the same process in establishing laws which are not causal. Kepler, as the result of a long investigation, came to the conclusion that all the facts observed about the path of Mars can be explained on the hypothesis that its orbit is an ellipse. By analogy he extended the hypothesis to other planets and found that it was confirmed by their movements. Its applicability to all planets was a considerable confirmation of its truth in the case of Mars. Later, when Newton put forward the law of universal gravitation, he calculated what kind of path a planet must follow if the attraction of the sun varies with the square of the distances between it and the planets, and found that the path must be an ellipse. The path of Mars was now seen to be not an isolated phenomenon, but an instance of a law pervading the whole of the material universe.

The problem of inductive logic is usually stated in close connection with the procedure of the more exact sciences. It is in them that we get the best and fullest illustrations of the principles which inductive logic investigates. But it would be a mistake to think that induction is confined to them, or, indeed, that it is confined to science at all. For science just does with greater accuracy and carries a stage further operations in which the ordinary man is continually engaged. It does not introduce new principles or processes of thought. The scientist just uses the ordinary thinking of ordinary men, and he begins with ordinary observation and analysis. But in his thought processes, and more particularly in his observation and analysis, he is more precise and accurate. Nevertheless the principles that make the ordinary man's thought valid also apply to the scientist's, and *vice versa*. We have to remember therefore that, while we go to science for examples of exact inductions and generalizations, we get the same principles operative in ordinary thought and practice ; and that, if we would discover the degree of their validity, the results of the latter have to be judged by the same criteria as the former. No doubt, generalizations which are not true without exception

serve most of the purposes of the ordinary man, but the elements of uncertainty in his conclusions are more usually due to defective premises than to invalid reasoning.

Even within the sciences, the degree of certainty that can be got varies indefinitely from rough generalizations, suggestions that are more or less probable, through all degrees of probability, to the strictest inductive proof. The difference between ordinary and scientific inductions, like that between those of the more and the less exact sciences, is one of degree, not of kind. The degree of certainty attained is in all cases determined by the degree to which the same requirements of valid induction are satisfied. Few sciences have advanced far enough to overcome all the difficulties in the way of establishing exact generalizations, and ordinary thought has in some cases arrived at comparatively accurate ones. The degree to which it is possible to get exact generalizations in a science varies with the subject matter dealt with ; but we must not imagine, as the examples of inductive inference generally given would suggest, that the sciences which measure and weigh or use quantitative methods are the only sciences. The work of science is to make its laws as exact as possible, but even where great exactness is possible it is an ideal to be attained at the end rather than something given at the beginning. Thought and inference of the most rigorous kind are necessary to arrive at such an ideal. Whatever degree of certainty such inference attains and however far its conclusions may fall short of the highest ideal of knowledge, we have to consider the processes involved in it. And they are in principle the processes involved in the daily thinking of ordinary men.

We have seen, then, that induction may mean the perfect induction of complete enumeration, the imperfect induction of incomplete enumeration, and the processes of arriving at generalizations from analogies, as well as the methods of establishing causal connections and laws of nature. And, in all of these, we get varying degrees of precision and certainty. Inductive logic, therefore, has to investigate the

processes of inference involved in all these procedures, estimate the degree of certainty attained, and indicate the conditions of validity. We have, however, time and again pointed out that we cannot have inference at all without a universal or system. How then, it may be asked, can we infer from particulars *to* a universal or from elements *to* a system? The strict answer is that we cannot do so, that in all inference we must have at least an assumed universal or a hypothetical system, and that therefore all inference is essentially deductive. Observation of facts suggests a universal or law or system. We then infer or deduce other facts which must follow if our suggestion is correct, and we observe if the facts are so. If they are not, the suggestion or hypothesis is rejected. If they are, it is regarded as more probable but it is not fully established. In order to establish it fully, we must be able to show, not only that a particular hypothesis accounts for or agrees with the facts, but that it is the only one that does so. Until this is done it can only have a high degree of probability.

Thus the universals or laws which are inductively established begin as guesses, suggestions, hypotheses. As Aristotle realized, we get them in the first place not by a process of inference but by direct insight into the facts. If there is a process involved in arriving at them, it is one for which logic can lay down no rules and in which it can discover no principle. Moreover, the inferences involved in establishing them are all deductive. They consist in deducing the consequences to which the law or universal, if admitted, commits us.

Accordingly induction, regarded as the process of establishing universals or laws, involves (1) the presupposition that there are universals, that events are connected by laws: we go to the facts with this presupposition and the facts suggest which events are connected and what the law of interconnection is; (2) observation, analysis and experimental manipulation of the facts till they suggest a law or connection, or, it may be, several such between which we have to decide; (3) a process of testing whether any of the

suggested laws or connections holds and, if so, which of them. This last process is apt to be regarded as the whole of induction, but it cannot begin till a law or connection is suggested, and this takes place not by inference but by direct insight. In some cases we have to be content with a suggested hypothesis. We cannot get beyond enumeration or analogy. We have no means of testing the hypothesis. Where we are able to test it further, the inferences involved in so doing are deductive.

While the principle involved in the testing of hypotheses is the same in all cases, the details of procedure vary according as the suggested hypothesis states a causal connection or a more general law uniting facts into a system. It is usual to confine the terms 'law' and 'hypothesis' to the latter case. This difference in terminology must not blind us to the identity of principle in the two cases. When a causal connection is first suggested, it is of the same hypothetical character as the laws of motion or gravitation; the procedure by which it is tested is in essence deductive; and a causal law is a law of nature. But just as the term 'law of nature' is generally confined to more comprehensive laws than those which express particular causal connections, so the term 'method of hypothesis' or 'the inductive-deductive method' is confined to the process of establishing such laws.

Since the time of Mill the methods of establishing particular causal connections have been termed the inductive or experimental methods. The inductive methods, however, are only instances of the method of hypothesis, just as causal laws are instances of laws of nature. But the way in which deduction is used differs in the two cases. The difference depends on the nature of the phenomena in question. The inductive methods apply only in cases where the cause can be observed or the effect experimentally analysed into its conditions. And the degree of probability of a causal connection, established by the inductive methods, becomes much greater when that connection is seen to be an instance of a wider law, established by the method of hypothesis. In

some cases, where the conditions of the full applicability of the inductive methods are not present, causal connections may be established by the method of hypothesis. Neither method can give us absolute certainty. The suggested causal connection or law of nature is always in process of being proved. The wider the range of facts which a law organizes and explains, the more unlikely is it that there can be any other law which can explain the facts equally well, and so the more certain it becomes.

Induction, then, is concerned with the processes whereby, beginning with the observation of particular facts, universals, causal connections, laws of nature, are first suggested and then tested and established. It thus takes for granted that there are universals and laws, and that events are causally connected. It does not try to prove that there are laws and connections, but rather to show which events are connected with which, and what laws govern their connection. Unless there were such mutual dependence of facts, no inference would be possible : we could not argue that because one thing is, another must be. In deduction, as we have seen, the law or principle of interconnection, on the basis of which the inference proceeds, is regarded as known or given, whether it is formulated in a proposition or not. But in induction it is not given or explicit. Induction begins with individuals or events or groups of these. The individuals it believes to be instances of universals, elements in systems, expressions of laws ; but at first it does not know the nature of the universals or systems or laws. In other words, the individuals with which it begins are seen to be conjoined and believed to be connected, but the connection cannot be seen. There is, however, the firm conviction that it is there.

Induction thus rests on the same presupposition of the connected character of events, the same conviction of the orderly systematic character of reality, which we found at the basis of deduction and which is expressed in the Laws of Thought. The form of this general principle which is presupposed in induction is usually called the principle of the

reign of law, or the Uniformity of Nature or the principle of Universal Causation. The first of these perhaps conveys most adequately the presupposition of the method of hypothesis, while the last expresses most clearly the principle underlying the inductive methods.

In our account of induction, we shall begin with a more detailed consideration of this presupposition involved in it—its nature and justification. We shall then explain the processes necessary before what is usually called induction proper—the establishing of causal connections and laws of nature—can begin, namely, observation and analysis of phenomena; and the way in which hypotheses are suggested through enumeration and analogies. We shall next consider the methods of induction, the processes by which suggested causal connections between particular events are tested. Partly because of the generality of the causal relation, partly because such relations are prominent in the more exact and the earliest developed of the sciences, and partly because of accidents of historical development—the importance attached to their canons by Mill, who was the first thoroughly to formulate the logic of induction—the inductive methods are apt to be regarded as the most important part of induction, much in the same way and for much the same sort of reasons as syllogism came to be regarded as the typical, if not the only, form of deduction. After the canons of induction, we shall discuss the method of hypothesis (the method by which laws of nature in general are established), the characteristics of scientific hypotheses, and the nature of explanation. We shall then be in a position to bring together the various forms of inference which we have met in the course of our inquiry so as to consider the general nature of inference and the goal of knowledge.

EXERCISE XIX

1. Explain and discuss Aristotle's treatment of induction.

2. How does scientific differ from ordinary thinking? Explain and exemplify the use of induction in each.

3. 'The difference between deduction and induction is not one of principle but of starting point.' Consider.

4. Explain the different senses in which the term 'induction' has been used, and discuss their relation to one another.

5. 'All inference is essentially deductive.' Discuss this statement; if it is true, does it follow that there can be no such thing as induction?

6. Consider briefly Bacon's contributions to inductive theory.

CHAPTER XX

THE PRESUPPOSITION OF INDUCTION

"In natural science to explain means merely to discover interconnections."[1] It is such interconnections that we find in causal connections and laws of nature. The work of science is to discover such connections and to explain particular phenomena by means of them. Now when a scientist looks for connections between phenomena, he is trying to discover them, not to invent them. His procedure presupposes that they are there if only he could find them. Without such a conviction that the events which we experience are not isolated and self-contained, neither the work of science nor that of ordinary life could be carried on. If, when we failed to find a cause, we could assume that in this instance there was no cause; if, when unexpected variations occurred, we could rest content that there was no reason for them; if events varied their characteristics and causes and effects without any reason and according to no law, science and induction would be impossible, and truth a word without meaning.

This principle or presupposition of the interconnected character of events has been expressed in many different ways. One of these is that everything which has a beginning must also have a cause. This is called the Law of Universal Causation. It means that for everything which happens there is a set of conditions from which it necessarily follows. Without these conditions it will not happen. Given the conditions it will happen again. Whatever the event (the

[1] Whitehead, *Concept of Nature*, p. 97.

course of a star, the path of a bullet, the fall of a stone, the growth of a plant, the changes of the weather) it has its definite set of conditions without which it will not occur, and given which it will not fail to occur. There is no such thing as a pure accident in the sense of an uncaused event. There are unforeseen events, unexpected happenings, occurrences whose causes we do not know, but not uncaused events or undetermined happenings. From this point of view we may say that to explain a thing is to discover its cause, the set of conditions which determine it.

Another way of expressing the presupposition of induction is to say that every fact is an instance of a law, every individual an instance of a universal, every event an element in a system. This principle is referred to as the reign of law or the Uniformity of Nature. It is a rather wider principle than that of Universal Causation, for it covers all laws, whether they are causal or not. According to this principle, to explain a thing is to show that it is an instance of a law, or an element in a system, and that its characteristics are determined by its place in the system, or its relations to other things as expressed by the law. It may be a part of space, an instant of time, a means to an end, a body in a gravitating system, an element in a chemical compound, etc., as well as an effect of a cause. Its nature, however, is exactly determined by its place in the system to which it belongs, or the law of which it is an instance. As one writer puts it "objects have ragged edges which imply other existences from which they have been torn and without which they do not exist." [1]

This principle means that nature is self-consistent or intelligible, that facts do not contradict one another but connect together or dovetail into one another in such a way as to satisfy the demand of our minds for order and intelligibility. To think is to unify, to connect, or at any rate to attempt to unify and connect. That which we cannot connect with other things, that which we cannot see as an element in a system or an instance of a law remains

[1] Bradley, *Appearance and Reality*, Second Edition, p. 176.

to us unintelligible, inexplicable, a challenge to our intelligence, something in which our minds cannot rest except by ceasing to think.

This presupposition of the orderly, connected, consistent, systematic character of the world we experience has been not inaptly described [1] as a demand which we or the scientists make on nature, a demand with which we go to the facts rather than something which they suggest to us. In the face of nature's persistent suggestions to the contrary, we refuse to believe that facts contradict one another or happen without causes. We may not know the cause of an event, but we believe that it has one and try to discover it. On different occasions, different causes may seem to produce the same event but we are convinced that it is not so, that there is in each case the same set of conditions which is the real cause of the event, and that the rest of the apparent cause is irrelevant. Two facts may seem to be contradictory, but we refuse to accept the contradiction. We say that at least one of them is an appearance, a datum wrongly interpreted.

Such is the general nature of the presupposition of induction, the principle required to justify the procedure of science in its attempts to establish connections and explain phenomena by means of them. Before asking for its justification, let us look a little more closely at its nature and the adequacy of different expressions of it. In so doing, it seems desirable to distinguish between causal laws and laws of nature in general. If we do not draw such a distinction we must regard all laws of nature as causal. This is what some writers on induction do. They identify necessary connection with causal connection and law with causality. Now, no doubt, there is a cause, *i.e.* a definite set of conditions which produce it, not only for every event but also for every conjunction or collocation of elements, for the ways in which facts cohere in systems; but in the widest systems it is useless to look for such a cause.

[1] L. J. Russell, *Aristotelian Society Proceedings*, 1924-25, p. 66.

All we can discover about them is that, according to the best of our knowledge, the systems are such that their elements are connected according to certain laws, as *e.g.* that all material bodies attract each other directly as the product of their masses and inversely as the square of the distance between them.

And even in the case of more restricted systems, though we may be able to find a cause why their elements cohere as they do, the relation between their elements as expressed in the law is not one of causation, *e.g.* the relation between points in space or instants in time is necessary and systematic but not causal. Similarly though we sometimes loosely call the law of gravitation causal (as *e.g.* when we say that gravity is the cause of a particular event), it is not strictly so. It expresses the interconnection of bodies within a gravitating system, the behaviour of the members of the system. In virtue of the nature of the system, the relations of a particular body to others within the system determine its behaviour, as when we say that the cause of a star moving in a certain way is its mass and position relative to other bodies. But the relation between the different gravitating bodies, in virtue of which they constitute the system, is not a causal one but the quite specific one which is expressed in the law of gravitation.

Take again the relation between a chemical whole and its constituents, *e.g.* oxygen and hydrogen and water. Oxygen and hydrogen are sometimes said to be the cause of water, but they are not strictly so. They are its chemical constituents. The relation between them gives rise to relations which can be expressed causally. We may say that oxygen and hydrogen treated in a certain way, or oxygen and hydrogen and a spark, cause water. We could put it equally well that water treated in a certain way causes oxygen and hydrogen. But, though they give rise to causal relations, that does not make the relation between the elements themselves, or that between them and the system, a causal one, so long as by a cause we mean that set of conditions given which the effect follows. For we

may have water without oxygen and hydrogen following, and *vice versa.*

We see, then, that elements which are not causally connected with each other cohere in systems or are connected into wholes; and we get laws which are not causal expressing the behaviour of such wholes. What then is the precise difference between a causal law and law in general? The simplest way of stating the distinction is to say that causality always involves an element of succession or at least the possibility of a time relation between cause and effect. Both in the history of philosophy and in popular language relations between successive events have usually been regarded as the typical, if not the only, form of causality; and if we are to give the term any precision, we must confine it to cases into which an element of succession enters. No doubt many of the causes and effects which we find in the world around us are so complex and difficult to disentangle that it is difficult to discover the element of succession in them. There are cases in which the cause continues to operate alongside of its effect, like a fire causing heat. There are other cases where the phenomena concerned interact and are each in turn cause and effect, like poverty and crime.

Moreover, the course of nature does not consist of detached events which follow one another in such a way that the time distinction between cause and effect is clear and obvious. It consists rather of continuous processes, complex sets of circumstances gradually changing into others. It is often impossible therefore to point to the sequence of events and say 'Here is the cause, there the effect.' Though they come after one another in time, there is no interval of time between them. Sometimes the only way to get the cause is to analyse the effect into its elements. The cause is the elements together with their combination. Once we have the elements of a thing combined, we have, not the cause, but the thing itself. Thus cause and effect may be regarded as the same thing looked at from two points

of view, the effect being the whole into which the elements combine, and the cause the elements plus their combination.

We must not, however, on account of the difficulty of discovering the element of succession in causality, go to the opposite extreme and use the term 'cause' loosely to cover any kind of connection, the presence of any law or uniformity, even uniformities of co-existence and rational connections, *e.g.* the relation of a pillar and an arch, an organism and its elements, a logical ground and its consequent—in fact, in any case where the word 'because' is appropriate. *E.g.* 'Why does the arch not fall? Because the pillar supports it.' The pillar is therefore called the supporting cause.

Even if we restrict causality to cases where there is succession, it is still a relation of such generality that the exact sciences have found it necessary to go beyond it to something more precise. They try to get a more exact formulation of the relation between conditions and consequents than the mere discovery that there is some necessary connection between them. In this respect the relation of causality is like that of predication, so widespread and general that exact knowledge requires something more specific and definite. The more exact sciences require quantitative relations between the phenomena with which they deal and are never content to say merely that one event is the cause of another.

We shall, then, confine the term 'cause' to relations where there is at least the possibility of succession, *i.e.* to relations between a totality of conditions and the phenomenon which in their combination they produce or are. Other relations and connections, where there is no succession, we shall treat according to their specific kind as relations within a system. According to this use of the term 'cause,' the Law of Universal Causation is not co-extensive with the principle of the reign of law or the Uniformity of Nature, which applies to all connections of whatever kind between phenomena. It is rather the principle of uniformity applied to sequence.

It might, however, seem that we could have Universal Causation without uniformity of sequence, *i.e.* that every event might have a cause without its having the same cause on every occasion on which it happens. The Law of Universal Causation asserts that everything has a cause, that there are no uncaused events. But we seem to find that an event which to-day follows from one cause, follows to-morrow from another, *i.e.* that the same event does not always have the same cause. Death may to-day result from pneumonia, to-morrow from heart failure, the next day from poisoning, and the next again from drowning. Motion may be caused by heat, by gravitation, by impact, etc. Similarly, the same event may seem to have different effects on different occasions. A lighted match thrown into water makes a hissing noise; thrown into a haystack, a conflagration; thrown into a heap of dry gunpowder, an explosion. Such are the appearances, and they seem to suggest that every event may have a cause and an effect but that it may have different causes and effects on different occasions.[1]

Closer inspection, however, shows that if we are to speak intelligibly of an event having a cause it must have the same cause on every occasion on which it happens. In other words, there is no meaning in saying that every event has a cause and an effect unless the same event always has the same cause and the same effect. The Law of Universal Causation properly understood involves the law of uniformity of sequence. And unless this were so induction would be impossible. For if the same event (A) could be produced on different occasions by different causes (B, C, D, etc.), we could not assert causal connection between it and any of them. We could only say that on one occasion we had an event B followed by A, on another occasion C followed by A, etc. We could not say that B is the cause of A, or A the effect of B, any more than we could say that C

[1] The treatment in this and the following paragraphs is deeply indebted to Mr. Joseph's lucid account of the subject in his *Introduction to Logic*, p. 403 ff.

is the cause of A, or A the effect of C. When A occurred again, we could not say whether B or C or D or none of them caused it. In such circumstances we could not assert connection but only conjunction, not consequence but only sequence.

We cannot assert connection unless we have necessity, a connection which always holds. If one event to-day and another to-morrow produced the same effect, we could not say that there was any causal connection between events. We should have chaos, not cosmos. If events changed their causes and effects, it would be useless to look for causal connections, just as we could not make any assertions about things if they changed their characters from moment to moment. If, *e.g.* without any change of circumstances the same degree of frost were to cause flowers to wither to-night and to grow to-morrow night, we could not say that it was the cause of either. If without any other change of conditions food which nourished me to-day were to make me sick to-morrow and to cause my death the next day, we could not regard it as the cause of any of these things. We cannot, therefore, say that one event is the cause or effect of another unless they are so connected that the one always follows the other and the other is always preceded by the one. Now this means that the Law of Universal Causation implies as a consequence uniformity of sequence. For unless we have uniformity of sequence we cannot be said to have causation at all.

The appearances to the contrary are due to the ambiguity of the terms 'cause' and 'effect.' Nature, as we have seen, is not divided into distinct separate events. A large number of conditions have to be satisfied before an event takes place. We may say that pulling the trigger is the cause of the gun going off, or that applying a lighted match is the cause of the explosion. But we very soon discover that such a so-called cause is only one of a large number of conditions which must be satisfied before the event follows. *E.g.* the cartridge must be in the chamber and it must be of the right kind, the striker and the rest of the mechanism must

be in order, etc. When we pull the trigger and the gun does not go off, we realize that other conditions have to be satisfied and we look to see which of them are not ; *e.g.* we look to see if the cartridge is in the chamber. If it is, we try another, for perhaps it was a bad cartridge. If the gun still refuses to fire, we examine the striker and other parts of the mechanism, and so on. Thus in practice we are made to realize that a number of conditions have to be fulfilled before the event follows.

Now the cause, in the sense in which we have been using the term in discussing the meaning of the Law of Universal Causation, is the totality of the conditions required to produce the event. But when all the conditions save one are satisfied we are apt to call that one the cause. The last condition has a very great practical importance because the event follows immediately on its fulfilment. If the conditions necessary to produce an event are A, B, C, D, E, and A, B, C, D, are satisfied, we call E the cause. In the same way if B, C, D, E, are satisfied, we call A the cause. Such a cause is only the last of a series of conditions and is more properly called the occasion. Strictly speaking, the cause is the totality of conditions, and if they are all satisfied the event must follow.

If we were to push this view to its logical conclusion, we should be forced to conclude that the cause of an event is the total condition of the universe at the moment immediately preceding its occurrence, that the relative positions of the sun and earth, or the movements of the waves of the Pacific Ocean, are among the conditions, part of the cause, of a man falling off his bicycle or a gun going off. But the condition of the universe at the immediately preceding moment is not the cause of this or that particular event, but rather of the whole state of the universe at the succeeding instant. And even if we took no account of that part of the universe, or of those conditions which remain unaltered from moment to moment, we could never in practice exhaustively enumerate the remainder. Accordingly, for practical and scientific purposes, we isolate certain con-

ditions and treat the remainder of the universe as an unchanging background or as one whose changes may be regarded as irrelevant as far as the event under consideration is concerned. In so doing we may, even in science, treat as irrelevant conditions which are really relevant to the matter under consideration, *e.g.* the changes of temperature of a laboratory during certain experiments. What can be regarded as irrelevant in any particular case can be determined only by expert knowledge, and often as the result of a great deal of experimentation. And even what expert knowledge at one time regards as irrelevant may later be found not to be so. Many of the false hypotheses about causal relations asserted in the history of science were the result of such neglect of relevant conditions.

In the strict or scientific sense, then, the cause of an event is the totality of relevant conditions required to produce it, and the effect is the totality of relevant consequences which follow when the cause is operative. So regarded, the relation between cause and effect is reciprocal. Given the cause the effect must follow, and it never follows unless the cause is there. But to discover such causes and effects may require a great deal of investigation and analysis. For such causes and effects are by no means obvious. They need not be isolated separate events, or visible to the naked eye. They may be changes in things, qualities of things, elements in a complex. In certain cases we know that the cause is in a certain thing or complex, but we cannot distinguish it from the rest of the complex and, even if we can by analysis distinguish it, we cannot isolate it or get it by itself. So we call the whole complex the cause. Thus we are apt to include irrelevant conditions in our causes as well as to exclude relevant ones. The thing which we call the cause may be a whole including a great deal more than is necessary to produce the effect which we attribute to it, just as the event which proves the occasion of a happening may contain less than its cause. *E.g.* arsenic, drowning, etc. cause death, but they produce much more than mere death: they produce deaths of specific kinds, with specific

features, varying with the nature of the circumstances. The scientific cause of death is an element common to these different causes, and the effect of that cause is an element common to the different types of death.

Accordingly the causes of ordinary life and practice usually contain either more or less than the scientific cause, the set of conditions required to produce the effect. For when we are looking for causes in practical life, we are as a rule looking for separate events, usually events which we can produce or control. We are sometimes looking for an event which, added to existing conditions, will produce an effect, like pulling the trigger of a gun. At other times we are looking for a total situation which contains the cause as an element, like the cause of death. The complex set of conditions may contain much more than the scientific cause, but as long as it contains the latter we are satisfied. And so we call the whole situation the cause. It is in this sense of the term 'cause' that we get what is called a plurality of causes or non-reciprocating causes, *i.e.* many causes of the same event, *e.g.* many causes of death or motion. All that this really means is that we get different combinations of circumstances in which the same event is produced. It does not mean that in each of these combinations there is not a condition or set of conditions which is the same in all and which is the real cause of the event. In the same way, the effect itself may be complex and contain more than the event in question, as with the death due to drowning or poisoning. But it must contain at least the scientific effect, which is the same in all and which always follows when the cause is operative. Accordingly before scientific investigation and analysis such a plurality of causes and effects disappears, and we see that strictly speaking the same cause must have the same effect and *vice versa*: the relation between cause and effect is reciprocal. When his causes and effects do not satisfy these conditions, the scientist concludes that, from his point of view, they are not the real causes and effects, but include too much or too little.

The immediate purposes of the ordinary man and the scientist in looking for causes and effects are different, and so they use the terms in different senses. Once we realize this, we see that their views are different but that they need not be inconsistent. Nevertheless the use of the same term in different senses is apt to cause confusion, and the distinction between the practical and the scientific senses of 'cause' must be kept clearly in mind if such confusion is to be avoided. The practical man is not usually interested in scientific or reciprocating causes. They would not serve his purposes. Even if he knows them, he may still be asking for the cause in the practical sense. Accordingly, when he asks for the cause of an event, he is usually looking for something other than its scientific cause.

Sometimes in asking for a cause the practical man wants to know the means to produce something. But the cause, in the sense of that which will act as a means, usually contains a great deal that is scientifically irrelevant. However much more than the scientific cause it may include, the practical man is satisfied as long as it will serve his purpose. *E.g.* a farmer may want to exterminate vermin, moles, rats, rabbits, etc. When he asks what will effect his purpose, he is not satisfied by being told the scientific cause of death. He wants something practical, something that he can use ; and there may be several such things, *e.g.* rabbits may be shot or snared or trapped ; rats may be poisoned or trapped or caged ; moles may be trapped or poisoned, etc.

At other times, the practical man may be looking for some event which has happened in the past, the knowledge of which might help him to produce it or prevent it as he desires in future. He may ask what was the cause of an outbreak of smallpox or foot and mouth disease. Now when a medical officer of health or an official of the Ministry of Agriculture asks such a question, it is no answer to him to say that a certain bacillus is the cause of the disease. He knows that already. What he wants to know is how the bacilli happened to be carried to the infected man or

animal. His interest is to prevent its spread or its recurrence from the same or similar sources. In the same way, when a historian is looking for the cause of a past event, a war or a reform, he wants to discover the actual set of conditions that produced this event, the interplay of human passions and purposes at the time in question. But that cause produced much more than the particular event in question. It is a non-reciprocating, non-scientific cause.

Now, in all such cases, there is a reciprocating or exact scientific cause, a cause which is present in each of the so-called causes. And the effect as a total situation differs according to the difference in the practical causes. If we keep this difference of meaning between the practical and the scientific use of the term 'cause' clearly in mind, we shall understand in what sense there is a plurality of causes, and see that it is quite consistent with the scientific view of causality. We conclude then that when we use 'cause' and 'effect' strictly, the same cause always produces the same effect and the same effect is always produced by the same cause. The apparent exceptions are quite compatible with this; and if it were not so, there would be no use in trying to discover causal relations, nor could we say in any significant sense that every event has a cause. Thus the Law of Universal Causation does imply uniformity of sequence.

It has sometimes been suggested that the name 'Uniformity of Nature' is a misleading description of the reign of law or the presupposition of induction; that it tends to suggest monotonous repetition, an absence of variety in nature, or at any rate that it gives more prominence to repetition than to necessary connection. So far as it does so, it is a false description. No doubt there is repetition in nature. There are uniformities of sequence and of co-existence. The same cause produces the same effect again and again. The same plan of construction or inter-relation of elements recurs in different parts of space and time. The same kind is repeated in different dogs or horses or men.

But this does not mean an absence of variety in nature. The same connections and relations recur in the midst of variety, in a world which is always changing. And the presence of repetition in nature is a consequence rather than the essence of the Uniformity of Nature, regarded as the presupposition of induction. Its essence is rather the connected character of natural events, the presence of unvarying laws in the midst of variety, the reality of universals, the systematic character of nature. This reveals itself in the inter-relation of elements within systems, and the reality of these systems necessitates that wherever the same conditions recur the same effects should follow. If things have a definite nature and characteristics, that nature will reveal itself, and therefore, if like circumstances recur, the things will behave in the same way. The principle of the Uniformity of Nature, therefore, does not mean an absence of variety in nature. All it demands is that, so far as the same conditions recur, the same consequences should follow. It has been suggested [1] that the principle might be better expressed : All is relevant to all. 'The reign of law' is perhaps as good a name as any for it. The name is not of much importance so long as we understand what it stands for and use it consistently.

Such, then, is the presupposition of all science and induction. What is its justification ? Can we prove its truth ? If so, how ? Mill [2] held that it is an induction from experience, that it is itself proved inductively from the innumerable cases in which we have found it to hold, and the fact that we have found no exceptions to it. In other words, it is the result of simple enumeration, together with the conviction that if there were exceptions we should be sure to have come across them. But no process can prove the truth of the principle on which itself proceeds. If all induction presupposes the Uniformity of Nature, the Uniformity of Nature cannot be proved by induction. The

[1] Bosanquet, *Implication and Linear Inference*, p. 8 *et passim*.

[2] *Op. cit.* bk. iii. ch xxi. sect. 2.

fact that we have found a conjunction to hold in a large, even an innumerable, number of instances does not guarantee that it will always hold, that it has held in the cases which we have not observed, or that it will hold in future cases. Repetition of instances of the same conjunction, however frequent, produces a psychological not a logical effect.[1]

The problem of induction is, how we can generalize legitimately, how we can make a statement about all cases of a kind, how we can get a necessary connection. If we assume the principle of the Uniformity of Nature, provided we are sure that we have found a connection in one instance, we can generalize it, we can say that it will hold always; for truth is universal. But unless we do assume the Uniformity of Nature, no number of instances of a conjunction will enable us to arrive at a genuine universal, one that takes us beyond the cases we have observed. Accordingly, the Uniformity of Nature cannot be proved by induction because, unless it is true, induction can prove nothing. We cannot prove by induction that every event has a cause: unless the principle of Universal Causation is true, there is no reason why we should look for a cause where we cannot at first see one. We cannot prove by induction that there is law at work everywhere: unless we assume that there is, we have no reason to look for order and system where nature seems to present us with disorder and lawlessness.

No doubt science, going to work with the conviction that there is law and order in nature, has so far justified its conviction by its success in revealing laws according to which phenomena are connected. And this suggests the only kind of proof of which such a principle is capable. It cannot be proved, if by proving it we mean deducing or deriving it from something more certain than itself. It cannot be directly or positively proved, because there is nothing more certain from which it could be derived. That does not mean that it is not certain. As we saw in considering the Laws of Thought, which are just another expression of the

[1] Cf. Hibben, *Logic Deductive and Inductive*, p. 184.

principle manifested in the Uniformity of Nature, no ultimate truth can be proved directly. The proof of all ultimate principles is negative, indirect. It amounts to saying 'Deny them, and can you consistently assert anything?' The principle on which they rest has been called the 'this or nothing' principle.[1] If these principles are not true. there is no truth or certainty. If we take them away, nothing will stand. We cannot by thought and reasoning directly establish the principles presupposed in all thought and reasoning. If we try to do so, we shall find that we have to presuppose them and use them. For without them nothing can be proved. But if we deny them, experience becomes unintelligible, theory and practice impossible. They are so bound up with all our theory and practice that if they are taken away nothing will stand. Now we cannot deny all knowledge and all truth without self-contradiction, and therefore anything that all knowledge and truth presupposes is absolutely certain. If we try to deny it, we contradict ourselves, because we are holding as truth that there is no truth. Accordingly while we cannot directly prove such principles, no one can deny them without self-contradiction.

It is this same criterion that ultimately guarantees for us the truth and certainty of all principles, laws, and even facts, of whatever kind. These are not to be regarded as certain because they are in their isolation immediately self-evident [illegible] given. They become established by being seen to be so bound up by necessary links with other facts, laws, and principles, that if they are taken away whole systems of facts, whole departments of our experience collapse along with them and become chaotic, unintelligible. That it should be required in order to make the facts of our experience, or some section of them, intelligible and rational, a consistent orderly whole, is the ultimate test of the truth of any law or principle; and that it should form part of such a system and be connected with the other elements in it, in such a way that if it is denied or removed the others must go with

[1] Bosanquet, *Implication and Linear Inference*, p. 3.

it, is the ultimate test of the certainty of any fact. Such an ideal it is difficult, even impossible, in practice to realize in detail, in the case of particular facts and more restricted laws. But in the case of the more formal and pervasive principles like the Laws of Thought, the principle of the Uniformity of Nature or the Law of Universal Causation, we can easily see how much would disappear if they were denied. And as a particular science advances, the facts with which it is concerned become more and more linked together into systems, so that it becomes more and more difficult to deny any of them without the collapse of the whole system.

EXERCISE XX

1. What is the presupposition of induction ? In what different ways has it been formulated ? Which of them do you consider most adequate, and why ?

2. (*a*) Consider critically the doctrine that all Laws of Nature are causal laws. (*b*) If this doctrine is not true, what is the difference between causal laws and other natural laws ?

3. Examine the following argument both in respect of its form and its matter : 'How can we define a cause ? If we take it to be the sum of all the conditions, it is indistinguishable from the effect ; while if the cause be regarded as consisting of fewer conditions than the whole number, the cause might exist without the effect.'

4. What is the Uniformity of Nature ? Is its existence consistent with (*a*) catastrophic changes, (*b*) miracles, (*c*) magic ?

5. 'Plurality of causes is due to failure in analysis and disappears before scientific investigation.' Explain and discuss.

6. Can the presupposition of induction be proved ? If so, how ?

7. What is meant in science by the 'cause' of an event ? Explain the difference between the practical and the scientific senses of the term 'cause.'

8. Discuss the relation between universal causation and uniformity of sequence.

CHAPTER XXI

OBSERVATION, ENUMERATION, AND ANALOGY

I. OBSERVATION

Induction is the process of establishing universals or laws from particular facts or instances. It must, therefore, begin with the observation of concrete phenomena. This is the only ultimate way of getting facts. Now observation seems easy and simple and passive: we have, as it were, simply to open our eyes and we see certain things. But it is not quite so simple as it seems.

(1) In what is usually called observation there is involved a great deal of interpretation, as may be gathered from the fact that our observations may be and often are, mistaken. The objects presented to us in sense are usually complex, and no one sense, nor even all our senses, can give us the whole of such an object at once. But it is very difficult to distinguish between what is actually given in sense and what we believe to be there. When I say, 'There is Mr. X,' I may be seeing certain coloured shapes or I may be hearing certain sounds, the sounds of his voice or his footsteps. But strictly speaking, I do not see or hear Mr. X. I see colours and hear sounds. But as the result of various experiences, the colours or sounds come to mean for me Mr. X with all his qualities. The same process of interpretation can be seen in any of our observations, *e.g.* the observation of a horse or a house, a star or a drop of water. We have, therefore, to distinguish between different senses of the term 'fact.' It may mean (a) the datum or appearance given in sense; (b) what the datum means for us, our

interpretation of it; or (c) the object which appears or is actually there. The last is usually more than the datum or appearance, and it may be different from our interpretation of the datum, as, when we see a coloured shape at a distance and interpret it as a man, when it is really the stump of a tree. If we confuse these different senses of 'fact,' we are sure to get into difficulties. To get at the facts in the first sense, we have only to open our eyes; but to get at the facts, in the sense of discovering what is actually there, is not so simple a matter. In the one sense, we begin with the facts, the given appearances, which are capable of being interpreted in various ways. In the other sense, the business of science is to discover the facts, the correct interpretation of the appearances. In the one sense, facts are plastic, capable of many interpretations; in the other, they are stubborn and unyielding.

(2) Not only do we in our observations interpret what is given to us and so become liable to error, but we arrange the individuals observed and classify them into sorts or kinds according to various resemblances or identities. We form concepts like man, horse, chair, and classify individuals under them. In learning the language which we speak, we accept the classifications embodied in it. There is thus a great deal of organization of experience at the level of ordinary thought long before scientific observation begins. Unless we could classify things into kinds which we expect to behave in certain ways, our knowledge would not serve the purposes of practice. We could not infer from one thing to another, and our previous knowledge would not be of use to us in a new situation. In this again we are liable to error: we may classify things according to superficial resemblances rather than according to essential qualities.

(3) We do not observe anything and everything indiscriminately. We are practical beings with interests and purposes, and these determine what we shall notice or attend to. We often see just what we look for, and we look for what we have been trained to see, or for what interests us. Put a farmer, an artist, a botanist, and a geologist on a hill-

side on a summer evening and ask them what they see, and each will probably give a different answer. The farmer is likely to notice the nature of the grazing, the artist the beauty of the landscape, the botanist the kinds of grasses and plants, and the geologist the geological formation. In other words, the training and interests of each determine what he attends to or sees. Such selection is involved in all our observations. To observe all the facts in, all the aspects of, a concrete situation is impossible; for they are almost infinite. We therefore select, and in selecting we are liable to err, to neglect something relevant. How misleading such selection may be we can see if we compare the accounts given by different eyewitnesses of the same occurrence, or the description of a strike or an industrial dispute by a Conservative and a Labour newspaper respectively.

Thus before scientific observation begins, ordinary observation analyses the concrete given whole and selects certain parts of it for consideration. These it interprets as evidence of something not actually given in the observation, and it arranges the result of its interpretation into classes and kinds. Into each of these processes error is liable to enter. Now scientific observation goes a stage further, and performs more accurately and deliberately the processes involved in ordinary observation. Thus science finds the world of knowledge not quite uncharted but more or less mapped out. Its business is to discover deeper uniformities, more exact classifications, more comprehensive laws. This often involves correcting the analyses, classifications, and interpretations of ordinary thought. In so doing it is trying to get at the facts, the correct interpretations of the data given in sense, data which unscientific thought had misinterpreted. Our ordinary facts, as we have seen, involve theories or interpretations, and these may be mistaken. Getting at the facts in the sense of what is actually there is not an easy or a simple process, as the judge on the bench and the scientist in the laboratory know only too well.

In his observations the scientist is liable to, and has to guard against, the same sources of error as the ordinary man. The phenomena with which he deals are endlessly complex, and he is apt to neglect relevant conditions or include irrelevant ones. He goes to nature with a purpose, usually to prove or disprove a theory. He is thus an interested person and apt to be biased. "It is difficult to find persons who can with perfect fairness register facts for and against their own peculiar views. Among uncultivated observers the tendency to remark favourable and forget unfavourable events is so great, that no reliance can be placed upon their supposed observations. Thus arises the enduring fallacy that the changes of the weather coincide in some way with the phases of the moon, although exact and impartial registers give no countenance to the fact. The whole race of prophets and quacks live on the overwhelming effect of one success, compared with hundreds of failures which are unmentioned and forgotten. As Bacon says, 'Men mark when they hit, and never mark when they miss.' And we should do well to bear in mind the ancient story, quoted by Bacon, of one who in Pagan times was shown a temple with a picture of all the persons who had been saved from shipwreck, after paying their vows. When asked whether he did not now acknowledge the power of the gods, 'Ay,' he answered; 'but where are they painted that were drowned after their vows?'"[1]

And the scientist has another source of error to contend with. Not only is the mind apt to be biased, but the senses themselves are defective as instruments of observation. They cannot detect minute changes or differences, *e.g.* of temperature, pressure or size. And in certain cases "we actually see things different from what they are. A jet of water appears to be a continuous thread, when it is really a wonderfully organized succession of large and small drops, oscillating in form."[2] These limits and defects of our senses are in part overcome by the aid of more exact instruments of observation. The balance and the thermometer,

[1] Jevons, *Principles of Science*, p. 402. [2] *Ibid.* p. 406.

the telescope and the microscope, and many similar instruments considerably extend the range and the accuracy of the scientist's observations.

Moreover, the scientist experiments. He manipulates a fact in certain ways, introduces or removes conditions, to see what will happen. Experiment is just a kind of observation, but it is a kind that has very special advantages from the point of view of science. It is observation under conditions which we ourselves have prearranged. We can vary the conditions at will and produce them as often as we like. The line between unaided observation and experiment is difficult to draw, and it is drawn differently by different writers ; but in the main the distinction is clear enough. In observation we wait for an event to happen in the ordinary course of nature. When it happens, it is surrounded by a complicated set of circumstances, and it is impossible to say which of these are essential to its production. In experiment, we produce the event under conditions which we have ourselves prearranged and carefully selected. We vary the conditions one at a time so as to discover which of them are essential and which irrelevant. Thus we interfere with nature, and force it to answer our questions. The purpose of experiment is thus to simplify the conditions of observation by eliminating what is irrelevant.

The experimenter has his own sources of error to guard against. His ideal is to change one condition at a time, but it is very difficult to be sure that this ideal is realized in a particular case. What constitutes 'one condition'? If our analysis is not complete, our one condition may contain several elements, some of which may be irrelevant to the effect considered. Or again, in the complexity of conditions we may ignore either the presence of some conditions or the fact that changes are taking place in them. Thus we may ignore the presence of the air or of changes of temperature during an experiment ; and yet these may have a determining influence on the result. We shall further consider and illustrate such sources of error in experiment when we come to deal with the inductive methods, which are largely

concerned with experimental work. We can only experiment in fields where the phenomena are under our control, and this is not the case in many fields, *e.g.* in astronomy, geology, biology, the social sciences and all that relates to the past. In such cases we may partially arrange the conditions of our observations, and we may aid them with the use of instruments, but we can seldom realize the ideal of experiment, namely, to vary one condition at a time.

Even when the range of individual observation is extended by the use of instruments and supplemented by experiment, its limits are comparatively narrow. For much the greater part of our knowledge we have to rely on the testimony of other people. No doubt we could under proper conditions check for ourselves either by observation or experiment much of the information that we habitually accept on the authority of others. I might by experiment check the proposition that water consists of oxygen and hydrogen in certain proportions, instead of accepting it from people who have performed such experiments. I might go to Canada to see what kinds of fruit or wheat grow there, or even to prove that there is such a place. But life is short, and so even of the information that could be checked by ourselves we do and must accept much on the testimony of others. And much of the knowledge that we thus accept cannot be checked at all. This applies to all our knowledge about the past, the material of the historical sciences, and many of the facts on which the judge and jury base their decisions in criminal actions. In such cases the process of getting at the facts is even more difficult than in the experimental sciences. There are greater possibilities of error. Such facts rest ultimately on observation, and the observation of them is as fallible as any other kind; and there are additional sources of error. The results of the observations are not usually written down immediately, and the memory is deceptive. Even if the witness to them is a good observer and has a reliable memory, it may be to his interest to distort the facts, and so he may deliberately

deceive us. Moreover, many of the facts recorded about past events were not recorded by the people who observed them, and thus even if the recorder himself be accurate his sources of information may be unreliable. And in dealing with the remote past, the authenticity of the existing manuscripts is another source of difficulty. A great part of the work of the historian consists in criticising authorities to discover their credibility. In addition he has to be content with a selection of the facts about any particular period, and some of those relevant to the formation of a correct judgment may be missing.

In the same way, a great part of the work of the detective is to discover evidence relevant to a particular case and produce witnesses who know the facts of the case. And the greater part of the work of the judge and jury is to sift the available evidence and estimate its reliability. This difference between the ways in which the experimental sciences, on the one hand, and history and law, on the other, get their facts is brought out very clearly, though in a somewhat exaggerated form, in a conversation in one of Mr. Austin Freeman's [1] stories. A lawyer is conversing with a doctor :

Lawyer : " . . . The scientific outlook is radically different from the legal. The man of science relies on his own knowledge and observation and judgment, and disregards testimony. A man comes to you and tells you he is blind in one eye. Do you accept his statement ? Not in the least. You proceed to test his eyesight with some infernal apparatus of coloured glasses, and you find that he can see perfectly well with both eyes. Then you decide that he is not blind in one eye ; that is to say, you reject his testimony in favour of facts of your own ascertaining."

Doctor : " But surely that is the rational method of coming to a conclusion."

Lawyer : " In science, no doubt. Not in law. A court of law must decide according to the evidence before it ; and that evidence is of the nature of sworn testimony. If a

[1] *The Eye of Osiris*, chap. ix.

witness is prepared to swear that black is white and no evidence to the contrary is offered, the evidence before the court is that black is white, and the court must decide accordingly. The judge and the jury may think otherwise —they may have even private knowledge to the contrary— but they have to decide according to the evidence."

How, in detail, the historian and the lawyer get the material on which to base their judgments, *i.e.* how they establish their facts, we shall see later. What we want at present to note is the difficulties and the sources of error involved in the process.

Logic cannot lay down rules for observation or experiment or for deciding as to the credibility of testimony. It cannot say which facts are to be selected as relevant and which rejected as irrelevant, or what kind of testimony is to be accepted. These questions must be decided by the scientists and the historians, those who have expert knowledge in the fields in question. Whether they will do it well or ill in a particular case depends partly on their previous experience and training and the amount of relevant knowledge which they possess. But it also depends partly on personal qualities which no knowledge or training can impart—the eye and the imagination of the good observer, who must also be an active theorizer.

Having now discussed how science gets its facts, we have next to consider how it connects them. For no fact can be explained by itself. As has been already suggested, it is not the case that a scientist first gets all his facts, carefully registers and tabulates them and then looks for connections and laws. A few facts suggest a connection or a law, and that indicates a line of inquiry. The suggested connection or law guides further investigation. It determines what facts or kind of facts we are to attend to. We look for the facts that will prove or disprove it. Thus the scientist does not go to the facts or concrete phenomena with an empty mind, prepared to take them all in. No progress would be possible in that way. The mass of details is too over-

whelming. He requires a guiding thread that will enable him to neglect most of the phenomena as irrelevant and concentrate on relatively few. This is the function of suggested causal connections and scientific hypotheses. We have to inquire therefore how these are framed and tested. And we should note that their use is not confined to science. We get the same procedure in ordinary unscientific thought. The difference is one of accuracy and complexity, not of principle. But it is usual to confine the term ' hypothesis ' to suggestions put forward in the interests of science and even to the more general of these, as distinct from particular causal connections.

Logic can no more lay down rules for the guidance of the scientific imagination in the formation of hypotheses and the suggestion of connections than for the observation and analysis of facts. Here too a mind well acquainted with the character of the kind of facts in question and with the general nature of the department of knowledge to which they belong is necessary, but not sufficient. There is also required the genius of the great scientist. But though this work is, at its highest, the work of the scientific genius, it is also work which all of us in our everyday life regularly undertake. When we suggest a cause for the gas fire not burning properly, for the car stopping, for the flowers withering, for the government being defeated, etc., the difference between our procedure and that of a Newton or an Einstein is one of degree rather than of kind. The principle is the same, but the degree of complexity and exactness is vastly different. All that logic can say about this process of the formation of hypotheses is that the fruitfulness of the suggestion depends on the knowledge of the facts, and the degree of insight into them, from which it springs. The valuable hypothesis is usually the work of the penetrative imagination disciplined by regard for the relevant facts. Observation and analysis of the facts, experimental manipulation of them, taking account of their resemblances and differences, together with a knowledge of allied facts and departments of knowledge, are all useful ; but inborn qualities are also necessary.

The imagination, according to Tyndall, is "the divining rod of the man of science. Not, however, an imagination which catches its creations from the air, but one informed and inspired by facts."[1] . . . "Newton's passage from a falling apple to a falling moon was, at the outset, a leap of the imagination. When William Thomson tries to place the ultimate particles of matter between his compass points, and to apply to them a scale of millimetres, he is powerfully aided by this faculty. . . . In fact without this power our knowledge of nature would be a mere tabulation of co-existences and sequences. We should still believe in the succession of day and night, of summer and winter; but the soul of Force would be dislodged from our universe; causal relations would disappear, and with them that science which is now binding the parts of nature into an organic whole."[2]

To suggest connections and hypotheses is the main function of enumeration and analogy, with the character of which the remainder of this chapter is concerned. Enumeration and analogy are sometimes called methods of induction, but they do not establish connections or generalizations. The evidence they supply is not sufficient to justify the conclusions based on it. Before the connections which they suggest can be regarded as established, they have to be tested by other processes. In some instances such further testing is impossible. Neither the inductive methods nor the method of hypothesis will apply, and we have to be content with enumeration and analogy. Proof in the strict sense is then impossible.

II. ENUMERATION

Mere enumeration as such has no probative value. No doubt, complete enumeration of all the instances referred to in a proposition—perfect induction as it is sometimes called—enables us to make a general statement which is in fact true, like 'All the books on that shelf are philosophical.'

[1] *Use and Limits of the Imagination in Science*, p. 4. [2] *Ibid.* p. 16-17.

But such an enumerative judgment can scarcely be said to be inferred from the observation of instances. It merely asserts, in an abbreviated form, the result of the particular observations. Such a process is not without value. It is useful to have a brief record of a number of particulars. Whether we call such a process inference or not is largely a question of words. And if we allow that it is an inference, whether we call it induction or not depends on whether we allow that its conclusion is universal. The form of the reasoning is :

A is X, B is X, C is X, etc,
A, B, C, etc. are all the Ss,
Therefore All S is X.

The conclusion is a statement of fact or of a series of facts—a statement about the whole of a limited class. It is not a necessary truth, in the sense of stating a necessary connection between S and X. Complete enumeration thus gives us what is, not what must be ; and we must carefully distinguish such an enumerative judgment from one which states necessary connection between S and X. No doubt there is a reason why each of the Ss is X, *e.g.* why each of the books on that shelf is philosophical. But the reason may be human caprice or some other apparently accidental fact. It need not be anything in the nature of S and X themselves.

Moreover, it is very seldom that we get such a complete enumeration of instances, and we get it in no case of importance. We have usually to be content with a selection from all the instances. We observe a number of instances of a kind and find that each has a certain characteristic. If we meet no instance without that characteristic, we generalize the conjunction into a connection like, 'All swans are white,' 'All men are mortal,' 'All elements are volatile.' This process is called simple enumeration, sometimes imperfect induction or 'induction by incomplete enumeration.' But in such induction we never seem to rely entirely on the number of instances. If we did, as long as there were any

instances unobserved, no number of instances would justify the general statement. However many instances we met in which the character was present, we should not be justified in concluding that the next would be like them. The fact that we have not observed contrary instances does not prove that they are not there, and that they may not come to light at any time, as happened when black swans were found in Australia. Doubtless the larger the number of positive instances without any negative ones, the greater our *expectation* that the next one will be like them. But this expectation is psychological rather than logical. An accumulation of a large number of uncontradicted conjunctions produces a certain psychological impression, but as long as we rely on mere enumeration and the enumeration is not complete, such an impression and the expectation to which it gives rise are logically valueless.

The logical value of the number of instances, as far as it has logical value, depends on two things. (1) Multiplication of instances usually means added probability that, if there were contrary instances, we should be likely to meet them. Here, however, it is the *variety* rather than the *number* of instances that is important. (2) To think is to connect. We know that there is a cause for every conjunction: and therefore when we see a character repeated in many instances of a kind, we think there is something in the nature of the kind that accounts for the presence of the character. When we see and hear of many men who die and of no men who do not die, we conclude that man's mortality must follow from something in his nature. If it were due to some peculiarity in some men or class of men, we should be likely to hear of someone in whose case this peculiarity was absent. When not only the number but the variety of the instances increases, when we hear of men of all races, in all ages, and in all sorts of circumstances dying, and we hear of no instance of the contrary, the probability that there is a necessary connection increases.

This increased probability does not depend on mere number but rather on the suggestion of necessary connec-

tion, a connection which even complete enumeration cannot prove. Uncontradicted conjunction always suggests connection, and the force of the suggestion increases with the nature, rather than the number, of instances of the conjunction. But when we begin to look for and rely on something in the nature of the instances as the basis of their common character, we are going beyond mere enumeration. It is indeed questionable if we ever do in any case rely on mere enumeration. We certainly do not do so in many of the cases supposed to be proved by imperfect induction. From the outset we see that the instances have in common that which enables us to call them by the same name, *e.g.* man or horse. The possession of another character by many of them leads us to believe that there is something in their common nature which is the ground of this character. But as long as we have no suggestion of what this ground is, and rely mainly on number of instances, all that we get is an empirical generalization, a statement of fact where we can assert conjunction but cannot see connection. And incomplete enumeration cannot guarantee even this statement of fact. If we want proof by enumeration, the enumeration must be complete. But even then it cannot give us necessary connection or necessary truth. Incomplete enumeration as such gives neither necessity nor fact. It does, however, suggest necessary connection, and this suggestion may be followed up and tested in other ways.

III. ANALOGY

This suggestion of a necessary connection becomes stronger when we begin to analyse the instances of a conjunction and find resemblances between them. This is what we get in analogy. We may pass to analogy through enumeration, as happens when we begin to analyse a number of instances of a conjunction and find likenesses between them. But perhaps it is more usual to approach analogy directly by noting resemblances between individuals and arguing from the one to the other on the basis of the

resemblances between them. At any rate, we can get analogy where there cannot be enumeration because there are no instances to enumerate, except the two from the one to the other of which we argue. But whether there be many instances or few, the argument by analogy proceeds on the basis of some quality or relation common to them. The general form of the argument is :

> A which is M is X.
> Therefore B which is like A in being M is also X.

e.g. Lord Haldane and Lord Balfour who are statesmen are philosophers.
Mr. Baldwin and Mr. Lloyd George are statesmen.
Therefore Mr. Baldwin and Mr. Lloyd George are philosophers.

Or we may take an instance in which the resemblance is a relation rather than a quality :

e.g. A colony stands to the mother country as a child to its parent.
A child ought to obey its parent.
Therefore a colony ought to obey the mother country.

Such arguments are formally invalid and in many cases the conclusion is actually false. Indeed the above form reduces to an invalid third figure syllogism :

A is M.
A is X.
Therefore All M is X.

e.g. Lord Balfour and Lord Haldane are statesmen.
Lord Balfour and Lord Haldane are philosophers.
Therefore all statesmen are philosophers.

If the common quality (M) in which the analogous cases resemble one another is an accidental characteristic of them or one not connected with the quality (X) which is asserted on the basis of it, the argument from analogy has no logical

value and the conclusion no probability. On the other hand, if the common quality (M) is the actual cause of the asserted quality (X) or if there is a necessary connection between the two, the inference is sound and the conclusion holds. And there are many degrees of probability between these extremes. If we *know* that M is the cause of X or that there is a necessary connection between them, we are no longer arguing from analogy. Accordingly at best analogy gives only a high degree of probability, and at worst it not only has no logical value but is most misleading, as a study of the arguments of the popular orator, who relies largely on metaphors and analogies, will abundantly prove.

Whenever the argument from analogy is sound, the connection which it suggests may be generalized, and will stand when further tested. While enumeration merely suggests that there is some connection, analogy often suggests the nature of the connection and the basis of it. At times, however, the argument rests on a vague unanalysed resemblance between one fact or set of facts and another, as when a sailor judges that the wind will rise from a certain condition of the sky, a condition which he cannot precisely formulate but which he recognizes as resembling that which preceded a storm on other occasions. Even in such cases, if the argument is sound, there is a necessary connection, though at the stage of analogy it cannot be exactly formulated. Analogy, then, cannot establish a conclusion, but it is very suggestive and when properly used it has considerable value.

Many of the most important scientific laws were first suggested by analogy. The analogy of a falling apple suggested to Newton a falling moon, and so led to the formulation of the law of gravitation. The analogy of competition in the industrial world suggested to Darwin the principle of natural selection as the basis of evolution in the animal world. The analogy of light suggested to Huyghens in 1690 that heat is a form of motion, a theory which was finally established by Joule about a century and a half later. The resemblance between lightning and a spark suggested

to Franklin the essential identity of the two as forms of electrical energy.

Still more numerous, however, were the false connections and hypotheses suggested by analogies—even to the great scientists. But in the case of science such connections and hypotheses, even if they had in the end to be modified or rejected, served a useful purpose. They guided investigation and led to observations and experiments which furthered the progress of knowledge. From a logical point of view a study of the many false suggestions and wrong hypotheses that the great scientists tried and discarded, or the many false clues followed by the best detectives, would be very instructive. We are apt to imagine that all the suggestions, analogies and hypotheses of the great scientists were right : it is mainly of the correct ones that we hear. But this is far from being the case. Months and even years are often spent in formulating and testing a hypothesis which has in the end to be discarded, and the one that is suggested in its place may have to pass through the same process. We are told that Kepler tested and rejected nineteen wrong hypotheses before the right law of the orbit of the planets occurred to him ; and they were mostly the result of false analogies. Other scientists could tell the same story, and from a logical point of view it is a pity that we do not know more of the way in which such suggested hypotheses were worked out and the grounds on which they were rejected.

As a matter of fact we have many instances of hypotheses which were not only suggested and tried but which stood the test for many years or even centuries, yet had in the end to be discarded. The caloric theory of heat, based on the analogy of water, held the field for several hundred years till the beginning of the nineteenth century. So did the Ptolemaic theory in astronomy till the beginning of the sixteenth century, and Galen's theory of the movement (we can scarcely call it the circulation) of the blood till the seventeenth century. But though these theories had to be discarded, we must not imagine that they were valueless or

entirely false. They systematised the facts known at the time; they guided further investigation; they formed the basis of successful predictions; and deductions made from them had great practical value. But in the end new facts appeared which led to their supersession. We shall illustrate the inception, the growth and the final supersession of such theories more in detail later (p. 353 ff). Meantime we note that both true and false scientific hypotheses have been suggested by analogy; and that to suggest such hypotheses which have to be tested further is one of its main logical functions.

But analogy has another function. There are many departments of life, and even of science, where we cannot get beyond analogy. In such cases we cannot get proof in the strict sense, but we may get a high degree of probability. We have to ask, then, On what does the logical force of an argument from analogy, which cannot be further tested, depend? What conditions must a good argument from analogy satisfy? The principle on which analogy proceeds and which gives plausibility to its suggestions may be put thus: We know that every event has a cause. When we are looking for the cause of an event, it is natural to suppose that it will have the same cause as something else that resembles it. In the absence of any suggestions to the contrary, therefore, we conclude that the cause of the thing in question is the same as that of something which is like it and whose cause we know. Now, as we have seen, the logical value of this inference depends on whether the resemblances in question are essential and fundamental, or accidental and superficial.

What, then, are we to understand, in this connection, by 'fundamental' or 'essential' resemblances?[1] The number of qualities in which a resemblance occurs has been suggested as a criterion of the value of analogy. But here, as elsewhere in inference, number is not a safe guide. The

[1] Cf. pp. 155-7 on the allied question of ascertaining what qualities are important for classification.

phenomena in question may, so far as we can see, have only one quality in common, but that may be the essential one, and we may argue validly on the basis of it to another resemblance which has not been and cannot be observed. Other phenomena may have very much in common, and yet the resemblances may be superficial. It is not, then, the number of resemblances, but their relevance to the purpose of the argument, and the extent to which they can be regarded as an index of the nature of the thing, that matters in analogy. No doubt when two or more things resemble one another in most respects, it is unlikely that they are fundamentally different. But only expert knowledge in the field in question can say what resemblances are likely to be essential or relevant for the purpose of inferences by analogy.

Logic can only say in a general way that arguments from analogy have most force when they proceed on qualities which are an index of the kind of thing, qualities which are the cause of other qualities, qualities without which the thing would not be the kind of thing that it is. We may add a negative condition, that no argument from analogy is sound which attributes to a thing a quality inconsistent with one which it is already known to possess. Further, a connection suggested on the basis of analogy is more likely to be true if it harmonises with what expert knowledge in the field in question regards as antecedently probable. And it has still greater probability if it fits along with other known facts into a coherent whole. When we apply such a test, however, we are going beyond analogy to the method of hypothesis. Many of the inferences in history, biology, geology, and the mental, moral and social sciences, are based on analogy, and from the nature of the case cannot get much beyond it. Many of the conclusions thus arrived at are, however, rendered much more probable by the way in which they fit together, both among themselves and with other known facts, into a coherent whole. Such corroboration of them does not come from analogy in the strict sense.

We shall close this chapter with a few illustrations of arguments from analogy, and in so doing we shall confine ourselves to arguments which occur in the sciences as distinct from the loose analogies of ordinary life. We shall find that we need not go to the street corner orator for false analogical inferences : we get plenty of them in the history of philosophy and science. All arguments from the behaviour of people to their mental processes, their feelings, motives, purposes, etc., are based on analogy. We cannot directly observe the minds of other people, nor do we understand the connection between mind and body. But we know the relation between mental processes and behaviour in our own case. We see the behaviour of other people, and we infer the processes in their minds on the analogy of this known relation. It is on the same principle of analogy that we infer the mental processes of children, savages and animals. In the case of normal civilized adults this analogical inference can be corroborated and supported in various ways. But in other cases we are so liable to argue on the basis of inadequate resemblances that a special name, 'the psychologist's fallacy,' has been devised to describe the unsound arguments that result from this source ; and one of our leading psychologists [1] calls the liability to this fallacy 'the besetting snare of the psychologist.'

For example, "a little girl addresses her doll as if it was alive and capable of understanding speech." From this we may argue that the girl "believes the doll to be conscious ; for only on such terms can we conceive ourselves addressing reproofs and endearments to any object." If we so argue "our inference is unsound. It ignores the grave general difference between the child's attitude to her toys and the adult's attitude to the things in his world, and it overlooks the large part of the child's behaviour which is incompatible with a belief in the consciousness of the doll. Look at the child as she carries her doll by the foot, throws it into the corner, or even converts it into a seat. Imagine too the consternation of the child if the doll were to weep when the

[1] Stout, *Manual of Psychology*, Third Edition, p. 49.

child reproves it, or gurgled and cooed in response to endearments." [1] In other words, while there are resemblances in some respects there are also relevant differences which the argument neglects, and the neglect of which vitiates it.

There are, however, cases where the resemblances extend to all the relevant aspects, and where therefore the argument has such force that we regard the conclusion as quite certain. "We see two men talking. One of them makes a remark. The other stops short, gets red in the face, speaks incoherently, shakes his fist at the other, and perhaps strikes him violently. We infer that the second man is angry. How do we know? Because we ourselves would behave in that fashion only if we were angry," [2] and because others have told us that they were angry when they so behaved.

Take next an argument from anthropology based on the same analogical principle. Among some savage tribes, "when a man dies his personal belongings are often buried with him, and his hut may be destroyed. Why? Some say it must be because the dead man has gone from his old estate but has a new existence in which he requires his clothes, his weapons and his food as he used to. The practices are supposed to be a deduction by the savage from a belief in immortality. The anthropologists who say this can conceive themselves as acting in such a way only if they held that belief. They see that the virtue has not gone out of the dead man's possessions because he is dead, and they would not put them in the grave with the dead man, as the savage does, unless they thought that he had a use for them. But the savage may not so reason. He has not so clear an idea of a man's personality as the civilized anthropologist. For the savage the man is dead and has to be buried. He feels that in some way the dead man's intimate possessions are a part of himself, to be buried with him. . . . Other factors of the savage's behaviour have been overlooked. Why is the hut of the dead man destroyed, as it is among some of the Bantu? When the roof is crushed in, does that send it to the happy hunting grounds? To the

[1] Reyburn, *An Introduction to Psychology*, pp. 62-3. [2] *Ibid.* p. 59.

savage the hut is associated with its owner, it is his hut, part of his being. Where he has gone it should go too."[1] Such instances show the care with which analogical arguments have to be used.

We get many similar analogies, some of them unsound and misleading, others suggestive and fruitful, between conditions in the material world and those in the worlds of life and mind. To take but one instance. We often think of the process of acquiring knowledge after the analogy of erecting a building. We talk of laying the foundation of the edifice of knowledge. Here each part of our knowledge is regarded as isolated and self-contained like the stones in a building. The parts do not modify one another. But there are things which grow and develop in a different way from the erection of a building, *e.g.* a plant or an animal, which changes altogether if it changes at all. And there are things without a foundation, like the solar system, the parts of which mutually support and sustain one another. If these, as our whole argument seems to show, are more adequate analogies to the nature of knowledge, the analogy of a building is most misleading. But though an organism or a solar system is a better analogy, we should not forget that they too are analogies, and that if pressed too far they are apt to become misleading.

One further instance of the use of analogy by a great scientist, an instance which shows the value and at the same time the uncertainty of analogical inferences. Sir Issac Newton "had observed that certain 'fat, sulphureous, unctuous bodies,' such as camphor, oils, spirit of turpentine, amber, etc., have refractive powers two or three times greater than might be anticipated from their densities. He noticed also the unusually high refractive index of diamond, and from this resemblance, based on similarity in reference to one attribute only, he inferred that diamond also would prove to be combustible. His prediction in this regard was verified by the Florentine Academicians in 1694."[2] So far for the value of analogy. Now for the

[1] *Ibid.* pp. 63-4. [2] Hibben, *Logic Deductive and Inductive*, p. 187.

uncertainty. Brewster[1] points out that "if Newton had drawn a like analogy in reference to greenockite and octahedrite, as he did concerning diamond, inasmuch as they too have a very high refractive index, he would have been wholly incorrect."

EXERCISE XXI

1. Explain and illustrate the different senses of the term 'fact.' In what sense is it easy to 'get at the facts'?

2. Distinguish between ordinary and scientific observation, and indicate the main sources of error in each.

3. How would you distinguish between observation and experiment? What are the special advantages of the latter for scientific purposes?

4. Explain and exemplify the nature of the inference involved in 'perfect induction'.

5. On what grounds and with how much justification have enumeration and analogy been regarded as inductive methods?

6. (*a*) What is meant by simple enumeration and what is its logical value? (*b*) Bring out the importance of instances in induction.

7. Is an analogical argument ever conclusive? Explain and illustrate the conditions which a good argument from analogy must fulfil.

8. What is meant by a hypothesis, and what is its function in scientific investigation?

9. What inductive fallacy did David commit when he said in his haste that all men are liars?

10. Explain the logical character of the following arguments and examine their validity.

    (1) Communism cannot succeed in Great Britain. Look at the condition of Russia!

    (2) The prosperity of America is enough to prove the desirability of imposing an import tariff on manufactured articles entering this country.

    (3) I have gone carefully through the list of members and I find that they are all Irish.

    (4) That town must be unhealthy: I know three people who live there and not one of them is in good health.

    (5) *Counterfeit coin* supposes that there is such a thing in the world as good money, and no one would pretend outwardly to be virtuous unless some really were so. In the same manner false miracles suppose the existence of real ones; and the cheats that have been imposed upon the world, far from furnishing us with reasons for rejecting all miracles in general, are on the contrary a strong proof that some, of which they are imitations, have been genuine.

[1] Quoted by Hibben, *loc. cit.*

(6) Boswell : " I talked of the recent expulsion of six students from the University of Oxford, who were Methodists and would not desist from publicly praying and exhorting." Johnson : " Sir, that expulsion was extremely just and proper. What have they to do at a University who are not willing to be taught but presume to teach " ? Boswell : " But was it not hard, sir, to expel them, for I am told they were good beings " ? Johnson : " I believe they might be good beings ; but they were not fit to be in the University of Oxford. A cow is a very good animal in a field, but we turn her out of a garden."

(7) To the philosopher the state is a human organism, a human person ; but if so, the human spirit which lives in it must also have a human body, for spirit and body belong to one another and between them make up the person. In a body which is not organized and human, the spirit of man cannot truly live. The body politic must, therefore, imitate the body natural of man. The perfect state is, as it were, the visible body of humanity—(Bluntschli, *Theory of the State*. Quoted by Hibben.)

CHAPTER XXII

THE INDUCTIVE METHODS

Given the general principle that every event has a cause, we have now to consider the processes by which the causes and effects of particular events are discovered. We have the conviction that events are connected: we want to discover which are connected with which. The practical importance of this inquiry is obvious. All day and every day we are engaged in the processes concerned. An event happens. Our car stops; there is a railway accident or an explosion in a mine; our flowers wither, or they grow better in one soil than another; water boils or freezes. We are always asking for the causes of such events. We assume that definite sets of conditions were present to produce them, and that if we knew these conditions we should be in a better position to control the events. If we could produce the conditions the events would follow, and the first condition of producing them is that we should discover them.

We have seen that the term 'cause' is used in a variety of senses. It may mean the last, or the most prominent, of a series of conditions, or the one in which for some reason we happen to be interested. Or again, it may mean a complex set of circumstances which contains among other things the whole of the conditions, or some of the important conditions, for the production of an effect. But when in logic we speak of discovering or establishing causes, we are not using the term in any of these senses. We mean by it the scientific cause, the totality of conditions required to produce the effect, no more and no less. Doubtless

even in science we cannot always get the scientific cause, and we may have to be content with a non-reciprocating cause; but it is only in the case of scientific causes that we can lay down any rules for the discovery of causal connections. It is with them, therefore, that the inductive methods considered in this chapter are concerned

Now the problem which the inductive methods have to solve is this. We cannot see a causal connection. All we can see is one event or change or state of a thing and then another. We see sequence not consequence, conjunction not connection, one thing and then another, not one thing causing another. How then can we be sure that we have detected a causal connection?

This statement is however a simplification of the problem. For we never do see just one event or change and then another. No concrete situation consists of one event. At any moment or in any situation there are a large number of events taking place, and at the next moment there are again a large number—some being the same at both moments, others different. Now our question is which of those at the one moment are causally connected with which at the other. The whole at one moment causes the whole at the next, but which sets or pairs are connected?

We may put the matter symbolically in a way which ignores many of the difficulties but which is necessary before we begin to test causal connections or apply the inductive methods. At one moment we notice A, B, C, D, ... R^1. At the next moment we notice M, N, O, P, X, Y, ... R^2. Here R^1 and R^2 stand for the unanalysed residue of events at the moments in question, a residue which, sometimes rightly and sometimes wrongly, we regard as irrelevant to our problem. How, then, are we to discover the cause of X? It may be A or B or C or D or any combination or constituents of these, and there is the additional possibility that it may be found in R^1. A very great deal of observation and analysis and perhaps much experimentation has to take place before we can state the problem in this way.

First of all, we have to consider within what sphere we are to look for the cause; for the conditions under which the effect came about, or the events which immediately preceded it, are infinite. Let us take a simple instance. I come home on a wet day and put my bicycle in a shed. Next morning I find that there are spots of rust on it. What is the cause of the rust? If we try to enumerate the conditions under which the rust appeared, we find the task beyond our powers. There may have been a thunderstorm during the night and lime in the shed; the chain may have been newly oiled and we may find the back tyre flat; an eclipse may have taken place during the night and somebody may have left the door open. Thousands of things happened throughout the universe during the night in question; and on purely *a priori* grounds we have no right to regard any of these as unconnected with the appearance of the rust. We do not, however, as a matter of fact, regard all these conditions indifferently as possible causes of the rusting. Ordinary experience is not by any means completely unorganized. The observations of the ordinary man reduce considerably the sphere of possible causes, or at any rate they enable us to neglect the greater part of what is happening in the universe as irrelevant, though in so doing we are in danger of neglecting conditions without which the event would not happen, as we might ignore the presence of the air which is a necessary condition of the rusting. The special knowledge of the expert narrows the field of possible causes still further. But even with this narrowing of the field, a great deal of very difficult and important work has to be done in analysing and specifying the remaining conditions under which the event occurred.

Similarly, the event whose cause we are considering has to be isolated from the surrounding details. When we are looking for the cause of X we must be sure that we have got X and not just this X or that X, which may have peculiarities of its own, and which therefore is not simply X but X plus something else. The greater part of the work of science is concerned with the observation and analysis

of such details, work which often requires not only exact instruments but also repeated and careful experiments. The canons of induction or the experimental methods, as usually stated, take for granted that this work has been completed. But it is in this work that the cause of the greater part, if not of the whole, of the uncertainty of scientific results is to be found. In other words, the degree of certainty of the conclusions arrived at by the methods is determined by the degree to which what they take for granted has been done.

As a general rule we cannot take for granted that all the relevant antecedents have been exhaustively tabulated, so that the cause of X is to be found in A, B, C, D ; nor can we assume that X itself has been accurately described and differentiated from the mass of surrounding details. Sometimes, perhaps usually, this is not even attempted. One cause is suggested and we go on to test it ; or it may be two, and we have to decide between them. Whether one or two or more causes are suggested, and whether or not we have a guarantee that all possible causes are enumerated, the procedure of testing is in essence the same ; but the degree of certainty of the result may vary.

MILL'S EXPERIMENTAL METHODS

Mill's account of the experimental methods is very misleading. On the one hand, he regards them as beginning with particulars, concrete phenomena, and giving generalizations. He holds that all the discoveries that " were ever made by observation and experiment " were made " by processes reducible to one or other of these methods ".[1] On the other hand, he holds that they are " methods of proof,"[2] that they "provide rules and models (such as the Syllogism and its rules are for ratiocination) to which if inductive arguments conform those arguments are conclusive, and not otherwise."[2] The methods are important, but such claims on their behalf are exaggerated and tend to

[1] *System of Logic*, bk. iii. chap. ix. sect. 6. [2] *Op. cit.* bk. iii. chap. ii. sect. 5.

bring them into disrepute. They do not generalize from particulars or apply to concrete phenomena. The data of observation have to be analysed, and causes have to be suggested before they can be applied; just as premises have to be taken for granted and arguments put into strict logical form before the rules of the syllogism can be applied. When Mill tries to apply them to concrete phenomena the results are not very happy.[1] Moreover, even when this preliminary work has been completed, the methods do not *prove* causal connections. They can only give further confirmation or added probability to the connections already suggested. They narrow down the field of possible causes. And in some cases their main value is to suggest causes which can be further tested by other methods. This Mill himself elsewhere grants, when in more modest and accurate language he tells us that "the four methods have little more in their power than to supply premises for, and a verification of, our deductions."[2]

In other ways, also, Mill's account of the methods and his statement of their canons are by no means unexceptionable, though with that disregard for consistency and regard for facts which makes him so suggestive a writer he generally supplies the corrective to his own errors. He writes as if each of the four (or five) methods were more or less independent of the others, whereas they are all expressions or aspects of one principle. He seems to think that each inductive conclusion is established by the use of one method only, whereas several or all of them may be used to establish the one conclusion. And he states their canons in such a way that it is difficult to get any inductive arguments which naturally fall into them, though many can be tortured in such a way as to appear in one of the required

[1] See *op. cit.* bk. iii. chap. ix. sect. 6, where Mill gives 'Dogs bark' as 'a specimen of a truth ascertained by the Method of Agreement,' and 'Fire burns' as 'a truth made known by the Method of Difference.' On this Bradley, *Principles of Logic* (ed. 2), vol. i. p. 362, comments that the same way of arguing would prove 'that all dogs have the mange' and 'that every fire-place blisters.'

[2] *Op. cit.* bk. iii. chap. x. sect. 2.

forms. In our consideration of the methods, we shall first discuss the general principle underlying them; then consider each in detail, discuss Mill's formulation of their canons, and give illustrations.

The methods are all derived from the nature of causality. They are indeed a detailed statement of what scientific causality is. The nature of causality is such that wherever the cause is present, the effect follows; wherever it is absent, the effect is absent; wherever it varies, the effect varies, and their variations are proportional; and what is the cause of one thing is not the cause of a different thing.[1] And these relations between cause and effect are reciprocal. It is on these principles, which are statements of the different aspects or implications of causality, that the inductive methods are based; and so far as they establish causal connections, they do so not directly but by rejecting suggested causes which fail to satisfy some one of these conditions. Thus the principle on which the methods proceed is the elimination of the irrelevant or inoperative conditions, those which fail to fulfil the requirements of causality. It does not matter which one of these conditions a suggested cause fails to satisfy: non-conformity with any one is enough to disprove it.

Given A, B, C, D, E as possible causes of X, we may find one instance in which B is absent while X is present, another instance in which C is present while X is absent, another in which D is constant while X varies, and we may know or discover on other grounds that E is the cause of M which is different from X. This would not prove causal connection between A and X, but it would disprove causal connection between X and B or C or D or E. And so far as the probability of A being the cause of X is strengthened by the rejection of its competitors, it takes place not as the result of the violation of one condition of the causal relation but as the result of the violation of several. B is rejected on

[1] This formulation of the principle of what is called the Method of Residues is taken from Mr. Joseph, *op. cit.* p. 440.

the principle that whatever is absent while X is present cannot be its cause, C on the principle that whatever is present when X is absent cannot be its cause, D on the principle that whatever does not vary as X varies cannot be its cause, and E on the principle that whatever is known to be the cause of anything different from X cannot be the cause of X. These principles form the basis of the methods which Mill has called Agreement, Difference, Concomitant Variation, and Residues respectively.

THE METHOD OF AGREEMENT

In the Method of Agreement, we try to discover causal connection from agreement between instances, that is from what are called positive instances, instances in which the suggested cause and effect are both present. This method is a development of the methods of enumeration and analogy which we considered in the last chapter. It differs from the latter in making a more thorough analysis of the antecedent conditions. It proceeds on the principle that (1) there is a causal relation between the antecedents as a whole and the consequents as a whole; and that (2) whatever is not present on any occasion on which a particular consequent follows can be eliminated as not its cause. The more varied the instances in which A is present and X follows, the greater the presumption of causal connection between them. For there is a cause of X and it must be present wherever X is present. The cause must therefore be sought among the things that are common to the instances in which X is present. That in which they differ can be eliminated. Hence the more other things vary while A remains, the more likely is it that A is the cause.

This method brings to light the importance of instances in inductive inquiry, and the kind of instances that are likely to be helpful. If the instances are identical, they are valueless. If X is on each occasion preceded by A, B, C, and D, there is no possibility of deciding between these as its cause. One such instance is as good as a thousand. But if the

instances are varied, if X is met with under very different and widely varied conditions and on each occasion it is preceded by A, the variety of the instances helps to eliminate accidental factors. It is by their difference and variety, not by their sameness and pure repetition, that instances are helpful in establishing causal connections. But we seldom get pure repetition in instances, and therefore number of instances tends to eliminate the risk of mere coincidence.

But, however many and varied the instances in which A and X are found together, causal connection between them is not proved. The whole of A may not be required to produce X. Our analysis may be incomplete, and so we may not have isolated the cause which is in A, as when we say that impure water is the cause of typhoid, when impure milk would do equally well. The real cause is something in the water or milk. Again, A and X may be joint effects of a deeper cause and so always found together; or A may be a concomitant of X, as pleasure is of satisfaction. Or again, A and X may be always together, and we may not be able to say which is cause, as with pleasure and desire. Do we desire a thing because it gives us pleasure, or does the idea of it give us pleasure because we desire it? Yet again, A alone may not be the cause of X. There may be some common element in B and C and D, and one of these may have to be present with A to give the effect, or we may have neglected a condition common to all the instances and relegated it to the background which we regard as irrelevant, as when we say that damp is the cause of rust, while the presence of air is also a necessary condition.

Mill states the canon of the Method of Agreement thus:[1] "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon." But as long as we are confined to observation (and once we employ experiment we are getting beyond the Method of Agreement) it is not only difficult but even impossible to get phenomena

[1] *Op. cit.* bk. iii. ch. viii. sect. 1.

which agree only in one circumstance. And the circumstance that is the only apparently common one may not be the cause. "If a child were given the same medicine in a variety of jams, and always had a particular biscuit afterwards, it might very likely attribute the effect of the medicine to the biscuit. Suppose my apple crop fails four years in succession, and that each year it was 'overlooked' by a woman reputed to have the evil eye: were I to argue that the failure was not due to insufficient rain, since in the first year there was plenty—nor to late frosts, for in the last year there were none—nor to blight which only occurred once—nor to high winds since the third year was singularly quiet, I might at last attribute the failure of the crop to the 'witch-woman' overlooking it." [1]

Nature does not usually present us with phenomena which agree only in one respect, and the method will work even when the instances have more than one circumstance in common. We can say that the cause must be found among the common antecedents. And how difficult it is to know what antecedents are common, we may realize if we consider that medical science spends many years and hundreds of thousands of pounds trying to discover the antecedents of one disease like cancer. Moreover, a cause may be present and its effect not follow, because there is another cause at work counteracting it. If, then, in such a case we conclude that what can be present without the cause following is not its cause, we are rejecting the real cause.

These difficulties are more than warnings to be careful in the use of the method. They show that the conditions required by Mill's canon cannot in fact be realized by observing positive instances; and that therefore all that the Method of Agreement can give us is suggested causal relations which have to be further tested before they are finally accepted. With every increase in the variety of instances, and in the chances that our analysis of the conditions is final, the probability of the suggested connection becomes greater.

[1] Joseph, *Introduction to Logic*, p. 493.

THE METHOD OF DIFFERENCE

While positive instances, instances in which the effect occurs, are useful in the way of suggesting possible connections, still more important is the function of the negative instance, the instance where the effect, though expected, does not follow. It was indeed the element of negation, the instances of the absence of B, C, D, when X is present, that disproved connection in the Method of Agreement and so helped to narrow the field of possible causes. The other side of the same principle, namely, that whatever is present when X is absent is not the cause of X, forms the basis of the Method of Difference, and the two together give the basis of the Joint Method of Agreement and Difference, the method in which we have both positive and negative instances. In all of them we may note what has been called the power of the negative instance. A thousand positive instances, where A and X are conjoined, will not prove connection, but one negative instance, where A is present and X does not follow or *vice versa*, disproves connection. All the methods are methods of elimination, and it is the negative instances that make elimination possible.

Mill states the canon of the Method of Difference thus: [1] "If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." If we have two sequences, (1) ABCD followed by XYZ, and (2) BCD followed by YZ, we have two sets of conditions which differ only in one respect, the presence of A in the former and its absence in the latter: and the one is followed by X while the other is not. Accordingly A and X must be connected.

But the method is much wider than this formulation suggests. (1) It is not necessary that the two instances should

[1] *Op. cit.* bk. iii. ch. viii. sect. 2.

differ only in one respect. The method will yield a conclusion without that. If they differ in more than one respect the cause will be found among those in which they differ. (2) It is not necessary that there should be only two instances. We are seldom, if ever, content with two instances. No doubt the ideal of experiment is to get two instances which differ only in one respect; but usually many experiments have to be tried before we are satisfied that this condition is realized. The Method of Difference has been called the method of experiment and the Method of Agreement that of observation, and the difference between observation and experiment is a rough indication of the difference between the two methods. In observation, as we have seen, an event is presented to us amid an overwhelming mass of details and it is difficult, usually impossible, to tell which of the conditions are relevant to the production of the event. In experiment we change one condition at a time so as to see whether it is relevant or not. Leaving everything else unchanged, we try to introduce or remove a suspected cause. Thus experiment tries to realize the requirements of the Method of Difference, and its greater accuracy depends on the extent to which it does so.

The Method of Difference often supplies a further test of a connection suggested by the Method of Agreement. When we find a number of instances in which X follows A, a causal connection between the two is suggested, but on the strength of the Method of Agreement by itself we cannot be sure that A alone is the cause, or even that it is any part of the cause. But if we can remove A while the other conditions remain unchanged, and find that X disappears, the probability of the connection becomes much greater. If we introduce A into another context, and find that X follows, the probability is still further increased. Thus the Method of Difference has a much higher value than that of Agreement; nevertheless it cannot guarantee causal connection. Like the Method of Agreement, it proceeds by eliminating suggested causes which do not satisfy its conditions; but its conditions are more difficult to satisfy, and therefore that

which is not eliminated by it has a higher degree of probability.

If we were perfectly certain (as we never can be) that our analysis is complete and final and that we only vary one condition at a time, the Method of Difference would establish some connection between the condition changed and the resulting change, but it would not prove that the one was the cause of the other. If ABC gives XYZ and BC gives YZ, it follows that there is some connection between A and X, but not that A is the cause of X. A may not be the whole cause. The cause may be AB or AC or A and some element in B or C or both. Again, the whole of A may not be required. What do we mean by one condition? Is throwing a lighted match on a heap of gunpowder one condition? This condition is not only not the whole cause of the resulting explosion, but it contains an element that is not necessary. An electric spark would do equally well. Such a condition, therefore, is both more and less than the real cause. This shows how difficult it is for our analysis to be complete, and how necessary it is to have more than two instances. It is desirable to have several instances, both negative and positive and as varied among themselves as possible. This is what the Joint Method gives us.

THE JOINT METHOD OF AGREEMENT AND DIFFERENCE

The Joint Method of Agreement and Difference has advantages over each of these methods separately. (1) It supplements the positive instances of Agreement by negative instances. (2) It applies in many cases where the conditions of the Method of Difference cannot be realized, *e.g.*, where experiment is impossible because we cannot control the conditions or produce the event at will. (3) Even the Method of Difference, as its canon is formulated by Mill, cannot distinguish between the occasion, the popular cause, and the scientific cause, and therefore it only suggests and cannot prove connection. Even when we remove the

suggested cause and the effect does not follow, and introduce it and the effect follows, it is still far from being proved to be the real cause. But if we introduce it into a large number of different sets of circumstances which differ widely among themselves, and still the effect follows; and remove it from sets of circumstances as widely varied, and the effect always disappears, the probability of its being the cause becomes very great indeed. This is what we get in the Joint Method. It supplements the one method by the other, and covers cases where neither by itself will yield any degree of certainty.

These methods are not really three but one with two aspects, namely, positive and negative instances. Sometimes we have to be content with one aspect, because we cannot get the other. But we get the greatest certainty in cases where we employ both. And where we can get both, which we begin with is a matter of accident, depending on the state of our knowledge or the nature of our problem rather than on any question of principle. The method and principle of procedure is the same in each. First, we analyse the concrete given whole, the antecedent circumstances, to discover possible causes. Let us say we conclude that the cause of X is to be found in ABCD. Second, we conclude deductively from the nature of causality that the cause must satisfy certain conditions, which we have already enumerated. Third, we conclude from various instances, some in which X is present and some in which it is absent, that B and C and D do not satisfy these conditions and therefore are not the cause of X. Finally, we are forced to conclude that A, which is the only possible cause left standing, is the cause of X, or that A is part of the cause, or that the cause is in A. Often we are not able to decide between these alternatives, but as the instances, positive and negative, increase in range and variety the probability of the first increases. Sometimes only one cause is suggested, relevant knowledge excluding the others from the first as inadmissible. If this one cause satisfies the conditions and no other seems possible, we are obliged to accept it.

It is difficult to get any illustrations of the Method of Agreement by itself—a sure sign that it is not extensively used in science. Some of the illustrations which logicians, in their belief that it is used by science, have given of it serve equally well to illustrate the Method of Difference, and they seem to be more appropriately described as instances of the latter inasmuch as experiment is used in them. Here is one often given. "Brewster happened to take an impression of a piece of mother-of-pearl in beeswax and resin, and was surprised to see the colours (of mother-of-pearl) reproduced upon its surface. He then took a number of other impressions in balsam, gum-arabic, lead, etc., and found the iridescent colours repeated in every case. In this way he proved that the colours were caused by the form of the substance, and not by its chemical qualities or physical composition. The different substances, wax, balsam, lead, etc., in which the phenomenon of colour appeared had nothing in common except the form. This therefore, according to the method of agreement, was properly regarded as the cause of the phenomenon to be explained." [1] Now in each of these separate experiments, with balsam, lead, etc., we get an instance of the Method of Difference. For the balsam before the impression was made was the same as it was after it was made in all respects save one, namely, the presence of the impressed form after the impression; and before the impression the effect, the colour of mother-of-pearl, was absent and after the impression it was present. The same is true of the experiment with lead, etc. In each case the requirement of the Method of Difference is satisfied, namely, one condition is varied at a time. The impressed form is introduced, other conditions remaining the same, and the effect follows. This shows how futile it is to try to regard inductive inferences as complying with one method alone.

The above inference could equally well illustrate the

[1] Creighton, *An Introductory Logic*, pp. 241-2. See also Hibben, *Logic Deductive and Inductive*, p. 233; and Welton and Monahan, *Intermediate Logic*, p. 387.

Joint Method. We have positive instances, the wax, resin, lead, etc., with the colour of mother-of-pearl; and negative instances, the wax, resin, lead, etc., without the colour of mother-of-pearl. In the one set we have the form common to the instances, in the other it is absent. And both positive and negative instances are very diverse. Thus all the conditions required for a highly probable inference in the Joint Method are present.

And the Joint Method of Agreement and Difference is not the only joint method. We can get similar combinations of any or all of the inductive methods. We can have joint methods of agreement and concomitant variation, or of difference and residues, or of all these together. We shall illustrate such combinations when we have considered the remaining methods.

Mill states the canon of the Joint Method thus: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon."[1] This formulation, however, requires considerable qualification. It emphasizes, and rightly, (1) that we must have both positive and negative instances, and (2) that the instances must be very diverse. It does not, however, state that the negative instances must be as similar as possible to the positive. Any 'instances in which the phenomenon does not occur' will not do. They may be wholly irrelevant and throw no light on the matter. If we are investigating the cause of rust, the jumping of a horse over a hedge, the growth of flowers, a man falling off his bicycle, are 'instances in which the phenomenon does not occur' but they are not very helpful to us. Whether they 'have nothing in common save the absence of that circumstance' is, however, highly questionable. If we can

[1] *Op. cit.* bk. iii. ch. viii. sect. 4.

speak intelligibly of phenomena as having the absence of a circumstance in common, they must have the absence of many circumstances in common. The negative instances, then, as well as the positive, should be chosen from a definite field, the field which expert knowledge in the particular subject regards as the sphere of possible causes. The fact that they are called instances suggests that they are such that the phenomenon might be expected to follow.

The requirements of the Joint Method, then, are (1) a set of positive and a set of negative instances, (2) that both sets be drawn from the same field and be as like one another as possible, (3) that the instances making up each set be as diverse as possible. The method can be applied both in cases of observation and in those where experiment is possible, but experiment enables us to get the conditions required for success more easily and more surely. With every approach to these conditions the probability of our result increases, but it never becomes absolutely certain.

THE METHOD OF CONCOMITANT VARIATION

The Method of Concomitant Variation proceeds on the same general principle as the methods already considered. Causes are suggested, those which fail to satisfy the causal requirements are eliminated, and the cause is asserted as belonging to that which is left standing after elimination. The ground of elimination is, however, different. Mill expresses the canon of Concomitant Variation thus : " Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation." [1] This gives us a form relying on positive instances ; but here as in the other methods positive instances do not prove connection. The concomitant variation of two phenomena may be due to mere coincidence, but with every increase in the variety of

[1] *Op. cit.* bk. iii. ch. viii. sect. 6.

the instances this becomes less likely. The two phenomena, however, may not be related as cause and effect, they may be joint effects of some deeper cause. But we can say with certainty that whatever does not vary with X cannot be its cause. Here, also, it is the negative instance that is most powerful. It provides the ground of elimination.

This method has two main uses. (1) Where the variations are not quantitatively measurable or at any rate have not been measured, it can be used in exactly the same way as the methods already considered. It can be used either along with Agreement or Difference, or alone The variations may suggest causes, or eliminate causes which have been otherwise suggested. The method may be used in this way whether experiment is possible or not. There is a disease called cretinism which produces a stunted condition of body and mind. In cases where the symptoms of the disease are present, there is found to be an insufficient amount of secretion from the thyroid gland ; and the less the secretion, the more pronounced the symptoms. When treatment with a preparation of thyroid is tried, the symptoms gradually disappear. If the treatment is stopped, as it has been in some cases, the symptoms reappear. It is concluded therefore that insufficient secretion of the thyroid gland is the cause of cretinism.[1] In this inference we have the Methods of Agreement and Difference and Concomitant Variation supplementing and supporting one another. The two phenomena are present together (agreement), absent together (difference), and varying together (concomitant variation).

This method can also be used in cases where the Method of Difference cannot be applied, because the phenomenon in question cannot be eliminated or removed, though it can be varied. We cannot eliminate the atmospheric pressure, temperature, the influence of environment, but we can vary them or put ourselves into positions where we can perceive their variations, as when we climb a mountain and find the atmospheric pressure less. We can thus

[1] *v.* J. S. Huxley, *Essays in Popular Science*, p. 84.

observe the effects of variation where we cannot get absence.

(2) In cases where the variations are exactly measurable, the Method of Concomitant Variation gives very much more precise results than the other methods. In such cases it not only supports the other methods but it gives something which they cannot give. It then becomes a method not only of establishing causal relations between phenomena but also of determining the precise quantitative relations between them. The physical sciences always try to reduce qualitative relations to quantitative ones, and even in sciences where such exact measurement is not possible we sometimes get attempts at quantitative formulation, as when Fechner tried to formulate in mathematical terminology the variations in the intensity of sensation.

But whether or not such mathematical formulation of their laws be the goal of all sciences, the physical sciences are not content with anything short of it. They are not satisfied therefore with the discovery that two phenomena are related as cause and effect. They want something more precise and exact, and this Concomitant Variation can often give them. In their hands, the causal relation tends to be reduced to a quantitative formula. When the physicist says that heat is the cause of motion, he means that the amount of energy that we get in the form of heat is identical with the amount that we get in the resulting motion form, so that the relation between them is one of quantitative equivalence in the amount of energy together with a difference in its form. Any variation in the amount of energy in the heat form results in a similar variation in that in the motion form. To establish causal relations in this sense, we have to prove not only invariable sequence but quantitative equivalence. When we have arrived at such a degree of precision, the notion of causality in the usual sense seems to be superseded, to give place to something more exact, namely, mathematical formulae.

THE METHOD OF RESIDUES

Mill's last method, that of Residues, is applicable only after the work of establishing causal relations has made considerable progress. Mill expresses the canon of it: "Subduct from any phenomenon such part of it as is known by previous inductions to be the effects of certain antecedents, and the residue of the phenomenon is the result of the remaining antecedents."[1] If we are dealing with a complex set of phenomena and we already know the cause of some of them, we conclude that the cause of the remainder is to be found among the antecedents whose effects we do not already know. Here again the procedure is analysis followed by elimination; and the elimination is based on negation. The principle is: That which is the cause of one thing cannot be the cause of a different thing.

When the greater part of a phenomenon has been scientifically investigated and its cause is known, and there is very little other than the known cause and effect in the antecedents and consequents, this method gives a strong presumption in favour of a causal relation between the remaining antecedents and consequents. But even in such cases it is desirable, if possible, to test the suggested relation by one or more of the other methods. As a general rule, however, the Method of Residues sets a problem rather than solves it. In certain cases the effects of all the apparent antecedents are known, but they do not suffice to explain the facts before us. Hence we assume that there is some unsuspected cause which accounts for the residual phenomenon, as the unexplained remainder is called. The more exact our observations and analyses and the more precise the causal relations we have established, the more likely we are to notice such residual phenomena, and the more loudly they call for explanation. Thus the Method of Residues suggests the need for investigation to discover a cause more often than it suggests what the cause is. From this point of view it has been well described as "scarcely

[1] *Op. cit.* bk. iii. ch. viii. sect. 5.

more than a demand for complete and precise explanation."[1] Certainly to suggest the need for a search for hitherto unsuspected causes has been its main function in the history of science.

The researches that led to the discovery of Neptune as the result of the unexplained irregularities of the movements of Uranus are an oft-quoted instance of the value of this method. A similar instance of it is the researches that led to the discovery of the Companion of Sirius.[2] It was found that the path followed by Sirius could not be entirely explained by the influence exerted on it by known bodies according to known laws. Accordingly as the result of exact observations and a great deal of mathematical calculation, Bessel came to the conclusion, in 1844, that there must be an invisible star, of a particular mass and situated in a particular place, exerting an influence on Sirius. When the existence of this Companion of Sirius was first suggested or we might even say proved, probably no one expected that it would ever be seen. But in 1862 it was seen by Alvan Clark, and it can easily be seen with the aid of the more powerful modern telescopes. Here the unexplained residue of the movements of Sirius suggested a problem; exact observations and calculations based on them supplied a solution; and later discovery verified its accuracy.

Physicists are at the moment trying to solve a very interesting problem[3] suggested by this method. The atomic weights of all chemical elements are believed to be whole numbers. All variations from this law have been explained except in one case, that of hydrogen. Its atomic weight should be 1, but in reality it is 1.0077. Now, various experiments seem to show that the atoms of elements of higher atomic weight are made up of hydrogen atoms, *e.g.* that the atom of helium whose atomic weight is 4 consists of 4 hydrogen atoms. But if so what has happened to the residue of .0077 of each atom of hydrogen? That is a

[1] Creighton, *An Introductory Logic*, p. 263.

[2] See Eddington, *Stars and Atoms*, pp. 48-9.

[3] See Lodge, *Atoms and Rays*, chap. iv.

problem suggested by a residual phenomenon. Here is one solution which has been suggested. Each atom consists of a nucleus, *i.e.* a tightly pressed core or centre, and some free electrons; and almost the whole weight of the atom is in the nucleus. Accordingly, if the atom of helium consists of 4 hydrogen atoms they must be very tightly pressed together in its nucleus. It is suggested therefore that when the atoms are pressed together they diminish in weight, some of their matter being destroyed, as matter, and taking the form of energy.

And this hypothesis would not only explain the fate of the residue of the hydrogen atom; it would also solve another problem.[1] If even a very small bit of matter is destroyed, the resulting energy is very great. Hence the energy resulting from the packing of the hydrogen atoms into the nuclei of those of higher atomic weight would be almost unlimited. Such supplies of energy are needed to account for the extraordinary heat of many of the stars, *e.g.* the temperature of the surface of the sun is 6000° centigrade, but some of the stars are several thousand times hotter than this. And a hypothesis is always more probable when the consequences that follow from it are found to be true in fact. Though this hypothesis is far from being established, it is an interesting combination of induction and deduction; and it was suggested by the Method of Residues.

The reasoning involved in all the methods is quite simple. The difficult part of the work is to be found in the analysis, observation and experiments on which the elimination of suggested causes that will not satisfy the causal conditions rests. The principle common to all the methods is this: There must be a cause of every phenomenon, *e.g.* of X. It follows from the nature of the causal relation that the cause of X must satisfy certain conditions of presence, absence and variation. A particular phenomenon, *e.g.* A, satisfies them, and nothing else seems to do

[1] Lodge, *loc. cit.*

so. Therefore we are forced to conclude that A is the cause of X. To make this result certain, we must know not only that A satisfies the required conditions, but that nothing else does. Of this it is impossible to be sure, but with increasing knowledge of the field in question the probability that, if there were anything else that satisfied them, we should have come across it grows greater. Moreover, unless we are sure that our analysis is complete, we cannot say that the whole of A is required.

We may sum up our consideration of the inductive methods thus. (1) They apply only after a great deal of preliminary work in the way of observation, analysis, and suggestion of causes has been performed, and the conclusions at which they arrive are not more general than the premises from which they start. They eliminate or disprove part of the premises and leave the rest standing. (2) They give, not certainty, but varying degrees of probability. (3) They apply only where we can observe cause and effect, or where we can experimentally analyse the effect into its conditions. (4) They apply to the scientific, not to the non-reciprocating cause. Science itself has often to be content with a non-reciprocating cause, but as applied to such causes the methods give invalid results. (5) They deal with phenomena in comparative isolation, with particular recurring connections. They do not unify or organize phenomena into systems. Hence the connections which they force upon us are often comparatively unintelligible and call for further explanation.

We shall give one other instance [1] of the way in which the different methods combine in actual use, an instance in which the first result forced on us by the methods is an unintelligible conjunction which further use of the methods, or rather their application to another group of facts, renders more intelligible.

[1] Part of this illustration is used by Mr. Joseph in his *Introduction to Logic*, and some of the facts used in the text are taken from his account, pp. 445-6. The remaining facts are taken from Mr. J. S. Huxley, *Essays in Popular Science*, pp. 202-206.

"Frogs may be found of very various shades of colour, from a blotched yellow, through green and brown, almost to black ; . . . all this range of variety is within the capacity of a single individual. Put one frog in the dark with plenty of water, and another on a white background in the light in drier surroundings : at the end of a few hours the one will be nearly black, the other light greenish yellow." [1] Here is a fact or conjunction which, according to the Method of Agreement, suggests a connection. What is the cause of the conjunction, the explanation of the fact? Lord Lister showed that the colour of the background affects the frogs through the eyes and not, *e.g.* through the skin, which is another possible way. In arriving at this conclusion he used the Method of Difference. He found that when the eyes of a frog of one species were removed, it no longer changed colour with the change of its surroundings. The conclusion was further supported by the fact that when frogs of other species, lacking in the power of changing colour, were examined, they were discovered to be blind.

Here then is a fact. There is a change, and it takes place through the eyes. But by what mechanism does it take place? "Not for years merely, but for decades, the explanation of this trivial fact eluded science." [2] Scientists looked in the wrong direction for an explanation. The only known way of bringing about changes in animals was through the nervous-system, but researches along that line yielded little result. Comparatively recently, however, it was discovered that the blood of animals was affected by secretions from various glands. This suggested that the colour changes of the frog might be effected through glandular secretions carried by the blood ; and experiment confirmed the suggestion. In establishing this result the Methods of Agreement, Difference, and Concomitant Variation were all used. (1) Pituitary secretion taken from other frogs or other animals was injected into frogs and it was found that they became darker (agreement). (2) The more pituitary secretion was injected the darker they became (con-

[1] Huxley, *loc. cit.* p. 202. [2] Huxley, *loc. cit.* p. 202.

comitant variation). (3) The pituitary gland was cut out of some frogs and so the source of the pituitary secretion removed, and then the frogs became lighter (difference). But the explanation is not yet complete. Without following the detail of the experiments and inferences we may note the final result. The skin of frogs contains special pigment cells. These have nuclei of dark pigment. When the pituitary secretion, circulating in the blood, reaches these cells, it causes the dark pigment to stream out from the nuclei to the surface of the cells. This makes the animal darker. On the other hand, when the animal is on a light background, the stimulation of the eyes causes the pituitary gland to stop secreting, and thus the animal gets lighter in colour. Thus one fact is connected with another, and all of them become intelligible. But such systematisation belongs to the methods we are to consider in the next chapter: the inductive methods give us rather the particular connections, the relatively isolated facts.

EXERCISE XXII

1. Explain the problem which the Inductive Methods are intended to solve, and discuss their success in solving it.

2. 'The principles on which the Inductive Methods proceed are just a detailed statement of the nature of scientific causality.' Explain and discuss.

3. Explain and illustrate the Method of Agreement. What are its advantages and defects? How does it differ from induction by simple enumeration?

4. What is a negative instance? Bring out by concrete examples its importance in induction.

5. On what grounds is the Method of Difference called the method of experiment? Under what conditions can it be applied?

6. What is the Joint Method of Agreement and Difference? What advantages does it possess over the Methods of Agreement and Difference separately? Are there any other joint methods?

7. Construct an inductive argument to prove that some article of food or some habit is beneficial or injurious to you; and analyse your reasoning showing the method or methods you have employed.

8. Examine the argument of the man who, being charged with theft on the evidence of three witnesses who had seen him steal the articles in

question, proposed to bring in his defence thirty witnesses who had not seen him stealing it.

9. What is the Method of Concomitant Variation, and what are its main uses?

10. What conditions must be satisfied before we can legitimately assert a causal connection between two events?

11. 'Mill's Inductive Methods are all reducible to one principle—the elimination of the inessential.' Explain and discuss.

12. Explain the principle on which the Method of Residues proceeds, and illustrate its importance in the history of science.

13. Explain the logical character of the following arguments; indicate the methods which are used in them; and examine their validity.

(1) "I took an earthen vessel, in which I put 200 pounds of earth that had been dried in a furnace, which I moistened with rain water, and I planted therein the trunk or stem of a willow tree, weighing 5 pounds; and at length, five years having finished, the tree sprung from thence did weigh 169 pounds and about three ounces. But I moistened the earthen vessel with rain water, or distilled water (always when there was need) and it was large and implanted into the earth, and lest the dust that flew about should be commingled with the earth, I covered the lip or mouth of the vessel with an iron plate covered with tin, and easily passable with many holes. I computed not the leaves that fell off in the four Autumns. At length I again dried the earth of the vessel, and there were found the same 200 pounds, wanting about 2 ounces. Therefore 164 pounds of wood, bark and roots arose out of water only."

(2) The air must be the cause of sound: for (i) when you ring a bell in a vacuum there is no sound; (ii) if a little air is allowed to enter into the vacuum, a faint sound is heard; and (iii) as more air is allowed to enter the sound increases.

(3) We observe very frequently that very poor handwriting characterizes the manuscripts of able men, while the best handwriting is as frequent with those who do little mental work, when compared with those whose penmanship is poor. We may, therefore, infer that poor penmanship is caused by the influence of severe mental labour. (Hyslop, quoted by Creighton.)

(4) In decerebrate animals (or in man when as the result of injury, or disease, or anaesthesis the cord is cut off from the higher centres) certain responses may take place through the spinal cord alone. Thus the knee-jerk, the scratch reflex, and certain postural responses may continue, and the possibility of these reflexes is taken to prove the integrity of the cord at this or the other level.

(5) Overdriven cattle, if killed before recovery from their fatigue, become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death: cocks killed during or shortly after a fight, and soldiers slain in the field of battle. These various cases agree in

no circumstance directly connected with the muscles, except that these have just been subjected to exhausting exercise. (Creighton.)

(6) Worms do not possess any sense of hearing. They took not the least notice of the shrill notes of a metal whistle, which was repeatedly sounded near them ; nor did they of the deepest and loudest notes of a bassoon. They were indifferent to shouts if care were taken that the breath did not strike them. When placed on a table close to the keys of a piano, which was played as loudly as possible, they remained perfectly calm.

CHAPTER XXIII

THE METHOD OF HYPOTHESIS

We saw that the work of science is not only to observe and describe facts but also to connect and explain them. Now, since connections cannot be perceived directly, they are put forward as suggestions or hypotheses which have to be tested before they are accepted. The process of testing consists in asking, What will follow if the suggested hypothesis is correct? and then observing if these consequences are found to be true in fact. In other words, the process consists in making deductions from the hypothesis and appealing to facts for confirmation or rejection.

In the last chapter we considered one way of testing suggested connections. The methods there dealt with apply, as we have seen, only to causal connections, and not even to all of them, but only to those in which the cause can be observed or the effect experimentally analysed into its conditions. But there are causes which are not of this kind; and there are laws governing the relations between facts, which are not causal, in the strict sense. We have now to discuss the method of establishing these. It is usually called the Method of Hypothesis. In addition to performing the two functions just mentioned, this method is used to confirm and explain the connections established by the inductive methods. Strictly speaking, the inductive methods themselves are instances of the use of this method. The causal connections with which they are concerned are first put forward as suggestions or hypotheses. We then deduce what must be true if they are causal relations, and

we observe or experiment to discover if the facts agree with the results of our deductions. It is exactly the same procedure that is followed by the method of hypothesis. But it is not usual to apply the term 'hypothesis' to suggested causal connections which are tested by the inductive methods. The distinction between such connections and those which we are now to consider is enough to warrant a difference of name.

The first class of cases we have to deal with, then, is those causal connections in which only the effect, not the cause, can be observed. To this class belong all facts relating to the past, historical facts, crimes, biological changes, geological formations, etc. In such cases we may suggest a cause or several conjoint causes as an explanation of the phenomenon in question. There is no one method by which such causes are suggested. How we get them is a matter of indifference: what is important is the way in which they are tested. As one writer [1] remarks: "How slight a thing determines a discovery! An accident, a suggestion, an analogy, a phrase, a word, even a word misunderstood!" But however the cause or hypothesis is suggested, once we have it, it performs the same function: it guides the investigation, it decides our line of procedure. We look for facts which will confirm or disprove it.

Suppose you come home and find nobody in the house. On entering one of the rooms you see drawers and cupboards open, articles of various kinds littered about the floor, the furniture disarranged, etc. You ask yourself what caused this condition of affairs. A burglar may have broken into the house. That is one possible cause, one hypothesis that would explain the facts so far as you have observed them. You proceed to test it. If a burglar has been in the house valuables are likely to be missing, it is likely that a window or the lock of one of the doors has been tampered with, other rooms will probably have been visited, etc. If you find that several of these things are so, you conclude that the suggested hypothesis is correct. If you

[1] Singer, *Discovery of the Circulation of the Blood*, p. 66.

find that they are not so, you think of another possible explanation. The children may have been in this room before they went out, or somebody may be playing a practical joke. These hypotheses lead to other and different lines of enquiry.

A geologist finds in a particular valley a large mass of rock, which differs in kind from any of those in the neighbourhood. How did it get there? Glacial action may be suggested as a cause. It may have been carried by ice. If so, other evidences of glacial action are likely to be found in the district. Again, are there rocks of that kind, from which it could be carried down by glaciers?

Again, a historian is looking for the cause of a particular historical event. A desire for a personal revenge on the part of the principal actor is suggested as a cause. If this is the cause, there must have been a motive for revenge, the rest of the man's actions should show him to be the kind of man who would act from such a motive, etc.

Thus a suggested hypothesis always indicates a line of enquiry. Consequences are deduced from the hypothesis, and observation follows. The more consequences of a suggested hypothesis are found to be true, the greater the probability that the hypothesis is correct. There must, however, be no fact inconsistent with the hypothesis. It must be able to account for all the observed facts which are relevant to the matter concerned. But that is not enough to establish it. There may be other hypotheses which will also account for all the facts. Now, it is very difficult to prove about any hypothesis that it is the only one which will explain the facts. But until this is proved, we cannot be quite certain that the hypothesis is the correct one. Taken by itself, almost any fact is capable of several explanations or is consistent with several different hypotheses. But the wider the range and the more varied the nature of the facts to be explained, the less likely is it that different hypotheses will be found that will account for all of them.

Take the facts relevant to a particular crime, where no

evidence from eye-witnesses is available. Each of them separately may be consistent with different theories of the identity of the criminal. But as more and more facts come to light, they are less and less likely to be all consistent with different theories. A few facts may supply a clue to the identity of the criminal, and once the hypothesis that a particular person committed the crime is suggested it guides the investigation. Other facts consistent with his being the criminal, and likely to be true if he is, are looked for. If they are found, the whole of the facts may in the end prove such a consistent system, capable of only one interpretation, that the identity of the criminal is regarded as conclusively proved. Here is the summing up in a typical example of arriving at the identity of a criminal by circumstantial evidence. "And now let us review this mass of evidence. You will see that it consists of a multitude of items, each trivial or speculative. Up to the time of the actual discovery, I had not a single crucial fact. . . . But slight as the individual points of evidence were they pointed with impressive unanimity to one person " [1] ; and they led to the discovery of the crucial fact, the missing link that completed the chain of evidence. The procedure of the detective, whether in real life or in detective stories, provides excellent examples of the use of the method of hypothesis. And even eminent scientists regard their own work as in principle of the same kind as that of the detective. Professor Eddington [2] uses the language of the detective story to describe some of the recent investigations in astronomy.

When we turn from the discovery of causes where the inductive methods do not apply to the establishment of laws which are not causal, we find exactly the same principles and procedure. A certain number of facts are observed. They suggest a law or principle governing their inter-relationship, and it indicates a line of enquiry. Deductions are made from the law, consequences which

[1] Freeman, *The Eye of Osiris*, p. 300

[2] *Stars and Atoms*, p. 42 ff.

must hold if the law is true. If the facts are found on observation to be consistent with the deductions from the law, it is regarded as more certain. If they are found to be inconsistent with it, it is rejected or modified, and another is suggested in its place and tested in the same way. We must, however, be sure that the inconsistency is real and not apparent only. Sometimes apparent exceptions turn out not only to be compatible with a law, but to provide considerable confirmation of it.

The laws established by this method are of all degrees of comprehensiveness: from those expressing the behaviour of comparatively small and restricted groups of facts to principles and laws as comprehensive as those of gravitation and evolution. For the work of science is not merely to observe particular facts and discover causal connections between them, nor even to show the laws governing relatively restricted groups of facts; it tries also to unify these into comprehensive systems, so that whole departments of knowledge will appear as interconnected wholes, different manifestations of one fundamental law. The restricted law becomes more certain when it is seen to be an instance of a more comprehensive one. And the same is true of causal connections, whether established by the inductive methods, considered in the last chapter, or by the method of hypothesis which we are considering in this chapter. Not only are such causal connections and restricted laws difficult to establish securely (for there may be other hypotheses that would explain the facts equally well), but even if they are established they require further explanation. By itself, as we have seen, the particular causal connection is comparatively isolated and unintelligible, a brute fact calling for an explanation. Such an explanation, and the increased intelligibility which it brings with it, can be got only by connecting the particular connection with other facts as an element in a system or an instance of a wider law.

We may discover by the use of the methods we have considered that the movements of the tides are connected

with the phases of the moon, that frogs change colour with changes of the colour of their surroundings, that the planets move in elliptical orbits, etc. Very often we are content, and sometimes we have to be content, with such facts and restricted laws and the explanations they afford. Nevertheless, as long as such connections and laws remain empirical generalisations, more or less unconnected with the other facts of our experience, they are a challenge to further enquiry, hindrances to the freedom of our thought. For the impulse to think is the impulse to unify, to connect, to relate, to see things together. Thus the very impulse that drove us to look for the causal connection or restricted law prevents us from resting content with it. But we cannot unify or systematise where we cannot see connections or laws. There are no terms on which the facts can be made to live together. It is as such *modi vivendi* that the more general laws are put forward. The test of their adequacy or truth is their capacity to systematise the facts and thus explain them consistently with one another.

The suggested law or hypothesis has thus a twofold function : (1) it guides the enquiry, and (2) it systematises and explains the known facts. And it may perform both these functions even if in the end it has to be modified or rejected. We should note then that, on the one hand, the fact that a law systematises and accounts for the known facts does not prove that it is finally true ; and, on the other, the fact that in the end it is superseded does not prove it scientifically valueless.

We may take the caloric theory of heat[1] as an instance of a hypothesis which served a useful scientific purpose, but had in the end to be rejected. It shows us the functions performed by a hypothesis and the kind of evidence required to establish it, as well as the great difficulty of being sure that a particular hypothesis is the only one that will explain the facts. According to this theory heat is ' an imponderable fluid ' held in the pores of substances. As an object

[1] This illustration is taken from Wood, *Joule and the Study of Energy*.

gets colder the fluid flows out of its pores, and as it gets warmer it flows into them. *E.g.* " when two bodies at different temperatures are placed in contact, the hot one gets colder and the cold one hotter. This is due to the caloric fluid passing from the hot body to the cold one. The two bodies finally attain the same temperature. Temperature may be regarded as the 'level' of the caloric, and just as water in two vessels connected by a tube will flow from one to the other until the levels in the two are equal, so the caloric flows until the temperatures are equal." [1] Now this theory accounted for all the facts known about the nature of heat till about the end of the eighteenth century, and it led to predictions which were found to be in fact true.

Even in the seventeenth century, however, another theory of the nature of heat, 'born of the desire for simplicity,' was suggested, namely, that heat is a form of motion. This theory was suggested by the analogy of light, which was seen to involve motion. This theory would also account for the known facts, but it seemed to the scientists of the day that the caloric theory 'more simply and adequately fitted all the facts,' and therefore they continued to believe in it. In 1798, however, Count Rumford performed experiments, the results of which were more difficult to reconcile with the caloric theory, and shortly afterwards Sir Humphry Davy performed one which provided a crucial test. " He showed that if two pieces of ice were protected from all sources of heat and kept rubbing together, then the ice melted to form water. Now the ice must have combined with caloric to produce water. Where did this caloric come from ? " [2] To this the caloric theory had no answer. So here was a fact which apparently could not be accounted for by the old theory. On the other hand, it could easily be explained by the rival theory. For motion was involved. Davy, however, did not realize the full significance of this crucial experiment.

[1] *Joule and the Study of Energy*, p. 30-1. [2] *Ibid.* p. 39.

Accordingly, science still adhered to the caloric theory ; and, as late as the forties of the nineteenth century, Lord Kelvin made predictions on the basis of it which he was able to verify by experiment. It remained for Joule, by a long series of experiments, which we cannot describe here, to show the superiority of the motion theory and the inadequacy of the caloric. By these experiments he established what is now called the first law of thermo-dynamics, namely, that there is a definite numerical equivalent between heat and work, and that they are mutually convertible. With this the caloric theory disappeared.

The same story could be told about other theories which have been discarded. The Ptolemaic theory in astronomy illustrates the process very clearly. It systematised and explained all the astronomical facts known in ancient times and led to successful prediction of eclipses and other events. As more and more new facts came to light, the process of accommodating it to them made the theory increasingly complicated, with cycles and epicycles, etc., until in the end it was rejected in favour of the Copernican, which introduced a new simplicity into the observed facts. This process is typical of the progress of science. A law is suggested. It makes the facts appear simple, orderly, intelligible, and thus it explains them. New facts are observed, and the law has to be modified to account for them. It thus becomes increasingly complex, and, even so, it may not be able to account for all the facts. And then another law is suggested which introduces a new simplicity and intelligibility. Thus the hypothesis which has in the end to be discarded systematises the facts which it was put forward to explain, and it directs the course of further enquiries and so forwards the progress of that knowledge which at last rejects it.

Now, when particular observed facts, causal relations, and restricted laws, are seen to be instances of wider laws and elements in more comprehensive systems, they become not only more intelligible but also more secure. They are not only explained but confirmed. This supplies for

them more and more of the certainty which, as we have seen, it is difficult for them ever to get in their isolation, the certainty that no other hypothesis will explain the facts in question. For the more comprehensive the law and the wider the range of facts included in the system, the less likely is it that any other hypothesis will be found to account for them all ; and each instance of the law has the authority of the law behind it. And the confirmation that thus comes is mutual. Every fresh application of the law, every widening of its range to include and explain empirically established generalisations and connections, makes its own certainty greater.

These wider and more general laws are for the most part expressions, not of causal connections, but rather of the inter-relations of elements in systems according to definite and precise principles. This leads us to notice that we cannot get a full or final explanation in terms of causality, at least in the sense of the term in which causes are investigated by the inductive methods. Explanations in terms of causality are endless, not systematic. We find that B is the cause of A, and C the cause of B, and so on indefinitely. The series is endless, and so we never get completeness or satisfaction along that line. In a systematic whole like a gravitating system, the elements fit together in such a way that the system as a whole is more intelligible. The nature of such a system cannot be expressed in terms of causality. No doubt we get relations of causality within such systems, and we might even say that there is a cause for the elements of such systems cohering as they do. It is questionable, however, if cause is the right word to use in this connection. In the more comprehensive systems we cannot discover a reason why the elements cohere as they do, and, even if we could discover such a reason for the whole scheme of things, it would not be a cause in the sense of the term which we accepted above. In the case of the more comprehensive systems, all we can do is to try to discover in what way the elements are connected or what the

nature of the system is, not why it should have such a nature or relation of elements (*v.* p. 285). Once, however, we know the nature of a system, we can give that as a reason for the character of its elements.

To explain a fact or particular causal connection is to show it as an instance of a law, and to explain a more restricted law is to show it as an instance of a wider law. Accordingly, we may sometimes get the same fact or restricted law in either of two ways. (1) We may get it inductively, the fact by observation or experiment, the causal connection by the inductive methods, the restricted law by the method of hypothesis. In all of these observation is involved. (2) We may get it deductively from a wider law, or from the nature of the system which that law states. When we get a law inductively, it is said to be an empirical law. When we deduce a fact or more restricted law from a wider law, it is said to be explained.

That the orbits of the planets are elliptical was discovered inductively by Kepler. He started with particular observations, and put the law forward as a hypothesis to explain them. But the law might have been equally well deduced from the law of universal gravitation put forward by Newton. Of course not all the facts systematised by a law could, in the first instance, be deduced from it. Some of them have to be observed in order to suggest the law. And others have to be observed after they are deduced in order to confirm or verify the law. The Companion of Sirius was, as we have seen (p. 341), in the first place arrived at by deduction, but it might equally well have been observed first, and the irregularities of the movements of Sirius explained by it.

At the present time, different classes of physicists are attempting to account for nebulium, the unknown source of a particular kind of light observed in certain stars. The one class are working inductively, the other deductively. "The theoretical physicists are at work trying to find laws which will determine exactly the kind of light given

off by atoms in various stages of mutilation—so that it will be purely a matter of calculation to infer the atom from the light it emits. The experimental physicists are at work trying more and more powerful means of battering atoms, so that one day a terrestrial atom will be stimulated to give nebulium light." "It is a great race," adds Professor Eddington, "and I do not know which side to back." [1]

This shows how very closely connected are the methods of induction and deduction. It is often a matter of accident which of them is used to discover a particular fact or law. But we are not quite certain about it till it is given us by both. The restricted law got deductively from a wider law has to be confirmed by direct observation before we accept it as established, and if it is first arrived at by induction, we require deduction to explain it and make it intelligible. Of course, if it was itself the basis of suggesting the wider law which explains it, it has not been actually deduced, but it is in the same position as those which have.

In the method of hypothesis, whether it is used in ordinary life or in science, induction and deduction form different parts of one process of inference. They supplement and support one another. There is perhaps no concrete piece of thinking in which both are not employed. As we noticed before, and as becomes more obvious now, if induction is to be identified with the inferences employed in scientific procedure, it includes deduction as a part of it. Indeed the whole of the *reasoning* involved in it, as distinct from the observation, experiment, and suggestion of hypotheses, is deductive. The suggested hypotheses are tested, and confirmed or rejected, by deduction and observation. There is no process of inference involved except the deduction of consequences from the suggested law. The laws or hypotheses from which the consequences are deduced are themselves got from observation of facts. Thus, we begin and end with observation, observation to suggest the laws and observation to confirm or reject them; and the

[1] *Stars and Atoms*, p. 55.

process between these observations is deductive. The only step, other than the observation and deduction involved in establishing hypotheses or laws, is the suggestion of the latter. This step, however, is not a process of inference. It is direct insight. If we wish to retain the distinction between induction and deduction, we might say that a fact is explained deductively, if we already know the law by which it is explained before we ask for the explanation ; whereas it is inductively explained, if the law by which it is explained has itself to be suggested and established before the explanation is possible.

When a law is put forward as a tentative hypothesis and deductions are drawn from it, which are found to be confirmed by observation, it is said to be verified. But the process of verification does not prove the law beyond possibility of question. Nevertheless, every verification of it, every new successful prediction on the basis of it, gives it added probability. It is equally true, however, that what we call the facts, which the law is put forward to explain, are not themselves known to be correctly interpreted until we see them connected with others as elements in a system or instances of a law. The process of verification is thus two-sided. It is a verification both of the hypothesis and of our interpretation of the observed data which we call the facts.

This confirms what we noticed before, that observation is a very active process, and that the ordinary view which separates facts and theories by an impassable gulf is without any foundation. Was the Companion of Sirius, when deduced by calculations in 1844, a theory, and did it become a fact when it was observed in 1862 ? Was the passage of the blood from the arteries to the veins a theory, when Harvey deduced it from the circulation of the blood, and did it become a fact when Malpighi saw it through a microscope ? Fact is verified theory, and sometimes only familiar or unquestioned theory, like the ' fact ' that we see the sun move.

There is no use therefore in trying to get facts without theories or inferences. "No one," says Darwin,[1] "can be a good observer unless he is also an active theoriser." It is theory that enables us to select the facts or data to observe, amid the bewildering complexity of the concrete phenomena with which nature presents us. It often leads us to look for and discover facts whose existence we never suspected. It explains and connects and makes intelligible the facts that we do observe. And finally it confirms our observations if they are right, that is, it makes us sure that we have observed rightly, that we did not misinterpret the given data; and it corrects our observations when they are wrong, that is, it reinterprets the data, as when we think we observe that the sun is moving or the earth flat. In such cases theory, correct theory, denies 'facts,' which are nothing but familiar or accepted theories which are wrong.

Accordingly, when we are told, as we sometimes are, 'There are the facts. Do you mean to deny them?' we may answer, 'No, I do not deny the given data. But I do deny somebody's interpretation or rather misinterpretation of them.' *When the given data are organized* into a systematic whole according to a principle or law, so that no element can be removed or denied or differently interpreted without involving the collapse of the whole system and a new interpretation of all the facts, we feel some measure of certainty that we have got the facts. For we always assume that facts are consistent and intelligible, that they are connected together and throw light on one another, but we cannot see how this is so, until we see the law governing their relations. Accordingly, not only do we verify hypotheses, laws, principles, by an appeal to facts but the facts themselves are explained, better understood and made more certain, when they are organized into systems according to a law. And the comprehensiveness of the law is the measure of this intelligibility and certainty.

[1] *Life and Letters of Charles Darwin*, vol. i. p. 126. (Quoted by Hibben, *Logic Deductive and Inductive*, p. 292.)

The account which has been given of the method of hypothesis applies in all the sciences. The general principle is everywhere the same, and all hypotheses have to pass through the same main stages. But the degree of certainty attained by the method, the extent to which a hypothesis can be verified, and the details of the process of verification differ from science to science. The detailed methods of the different sciences must take account of the differences of their subject matter. The latter determine the extent to which, and the way in which, the general requirements of the method of hypothesis can be satisfied. Such detailed considerations must be left to expert knowledge in the sphere dealt with by a particular science. No purely formal account can be given of them. There are, however, some further general considerations which apply to all the sciences. These logic can indicate, leaving it to the different scientists to decide how they are to be satisfied in their respective departments.

One such general question is, What conditions must a hypothesis satisfy in order to be regarded as scientific? It follows from the nature of the method of hypothesis that only hypotheses on the basis of which deductions can be made are of value in science. If a hypothesis is to be of any use in science it must be capable of proof or disproof. Occasions do arise in which the facts that follow from a hypothesis are incapable of being observed, but a scientific hypothesis must be such that actual consequences would follow from it and that under certain conditions it could be confirmed or disproved. It would not be a scientific hypothesis if one were to suggest that a ghost moved the furniture in a room, opened drawers and stole some valuable articles, or that God causes the good weather. This is so, irrespective of the correctness or otherwise of belief in supernatural agency. For we cannot make any deductions from such a hypothesis. We do not know enough about such agencies to be able to calculate how the suggested cause would operate on other occasions. If it is suggested that the supernatural agency that causes

the weather acts according to the known laws of nature, it may be replied that the laws themselves are all that science needs or can use ; and they form a basis for deduction.

It is this characteristic which distinguishes a scientific from a speculative or metaphysical hypothesis. The latter is an attempt to interpret the facts so as to show their meaning. There may be several such hypotheses while the facts remain the same, but we cannot deduce from them what to expect under certain circumstances. Like the scientific, speculative hypotheses must be able to explain any facts that actually happen, or the facts must be capable of explanation consistently with them, but we cannot predict facts on the basis of them. We cannot predict any particular fact that will follow if an Idealistic view of the universe is true and not follow if Naturalism is the correct doctrine, or that would follow from Theism as distinct from Pantheism. As distinct from such speculative hypotheses, a scientific hypothesis must furnish a basis for deduction. Moreover a scientific hypothesis must not violate any of the principles on which our knowledge as a whole rests. It "must be thinkable consistently with the fundamental assumptions of the science which makes it"[1] and with the general principles of all our knowledge. "It would be an illegitimate hypothesis on the part of a bank clerk confronted with a small discrepancy in his books to suppose that on this occasion 2 + 2 make 3."[2]

Given a hypothesis which satisfies the above requirements, we may summarise as follows the main conditions which it must satisfy before it can be regarded as established : (1) It must be comprehensive enough to account for all the relevant known facts. (2) The relevant observed facts must be found to be in harmony with the deductions made from the hypothesis. (3) It should fit into the rest of our knowledge. Occasions arise, however, in which, in the conflict between a suggested hypothesis and the relevant

[1] Joseph, *Introduction to Logic*, p. 461. [2] *Ibid.* p. 464.

section of our present knowledge, it is the latter that has to give way. This happens when the fundamental principles of a science are called in question. But even in such cases the rest of our knowledge must be reinterpreted in the light of the new hypothesis, and until that is done the hypothesis cannot be regarded as fully established. (4) It must be the only hypothesis that will explain all the facts, observed and deduced. We can seldom be sure that any hypothesis satisfies this condition, but our certainty that it does so increases as it is seen to fit into the whole fabric of knowledge, and as our knowledge itself becomes more organized and systematic.

When we have two or more hypotheses which seem to satisfy the first three of the above requirements, we look for a fact which will enable us to decide between them. Such a fact is called a *crucial instance*. If it is got by experiment, the experiment is called a *crucial experiment*. We may take as an illustration of such an experiment the experiment of Sir Humphry Davy, already referred to (p. 354), which provided a means of deciding between the caloric and the motion theories of heat. As long as no such test is available, the simplest theory should be regarded as the most satisfactory.

Before illustrating the use of the method of hypothesis in science and practical life, let us review the different kinds of inference we have met, in order to see their relations to each other and their adequacy to express the ideal of scientific knowledge. The purpose of science is two-fold, to establish laws and to explain facts by means of them. It therefore needs both induction and deduction. We have met two main kinds of induction, the inductive methods and the method of hypothesis. We considered various kinds of law that can be established by these methods: causal connections between facts, and laws expressing the inter-relation of elements or facts within less or more comprehensive systems. As we go from particular connections between comparatively isolated facts to the laws that

systematise wider ranges of fact, the demands of thought for order and intelligibility are being progressively realized.

Now we can explain facts in terms of any of these laws, as being the effect of a particular cause or an element in a narrower or wider system. And the adequacy and satisfactoriness of the explanation increase with the comprehensiveness of the law in terms of which the explanation is given. Sometimes we are not interested in the more ultimate explanation. We may be quite content, in reply to an enquiry as to the cause of the frost withering the flowers, to be told, 'Frost always does.' In other cases the limits of present knowledge prevent a final explanation. But where it is forthcoming, the more adequate explanation is always in terms of the wider law.

Now deduction is involved both in establishing laws and in explaining facts by means of them. We met two kinds of deduction, syllogistic and systematic or non-syllogistic. We use both kinds of deduction to establish inductive laws and to explain facts by means of them. But in establishing laws of the interconnection of elements in a system and in explaining facts by means of them, we use non-syllogistic deduction mainly, if not entirely. In other words, the more ultimate and satisfactory explanations are arrived at by non-syllogistic deduction. Syllogism, as we have seen, cannot give us the individual or distinguishing characteristics of a fact. It can only give us the characters which it has in common with all members of the kind to which it belongs. In this respect syllogism is like the law expressing causal connection, which gives only what is true in all cases of the connection, and not what distinguishes one from another. Both syllogism and causal connections are concerned with comparatively isolated facts which recur. Neither of them can give us system in the full sense. They are both useful in science and ordinary life. They give us laws and explain facts. But the explanations are very incomplete, though we have often to be content with them. More complete explanations demand that the facts explained by them should be connected with the rest of our knowledge into a

systematic whole. And the deductions involved in the establishing of such wholes, and in explaining facts by means of them, are non-syllogistic.

We could not deduce syllogistically from the data possessed by Bessel the position and character of the Companion of Sirius. These were deduced from its place in the system of gravitating bodies. Without a body having the characteristics of the Companion of Sirius, there would be a gap in the system. The full explanation of a concrete fact always demands non-syllogistic deduction. For no fact is the expression of just one law. Several laws combine to give a fact its precise character, and the inference by which we deduce its character from their combination cannot be expressed as a syllogism. Consider, for example, the way in which the law of gravity, the resistance of the air, and its initial force, combine to explain the trajectory of a bullet or a golf ball. If all the forces acting on it were exactly known, the exact position of the bullet at a certain instant could be deduced from them in just the same way as the time of an eclipse can be calculated. In both cases we get a system of elements combining to give a precise result. Accordingly, if the ideal of scientific knowledge is to show that facts are so connected together into systems that you cannot deny or reinterpret one without denying the system itself, the method of hypothesis combined with non-syllogistic deduction is the most adequate expression of its nature. The progress of knowledge is always double-edged. It involves induction and deduction, establishing of laws and explanation of facts. The facts confirm the laws and the laws illumine the facts. In this process all forms of inference play a part. The inductive methods and the syllogism establish connections within narrow, recurring, and relatively isolated systems; while the method of hypothesis and non-syllogistic deduction reveal further connections, and show the facts in narrower systems as elements in more comprehensive systems.

We find this union of induction and deduction in all concrete thinking, in ordinary life as well as in science.

Which aspect of it we employ in a particular case depends on our purpose and the degree of our knowledge. The one aspect is used by the scientist in arriving at his principles in his laboratory or study, the other in expounding a subject to a class or in writing a textbook. The one is used by the detective following a criminal, the other by the counsel for the prosecution in opening the case in Court. The ordinary man uses the one in trying to make up his own mind, the other in trying to convince his neighbour. In all these different departments, the method of hypothesis combined with non-syllogistic deduction is the natural procedure in the more serious kinds of inference,[1] but these methods are confirmed and supported by the other inductive and deductive methods.

A few examples of the usc of the method of hypothesis will show how closely induction and deduction are related in concrete thinking and what an important part non-syllogistic deduction plays in it.

(1) *Newton's Law of Universal Gravitation.*[2] Whatever be the truth about the tradition of the falling apple, Newton seems to have been led to formulate the hypothesis of universal gravitation by observing the fact that all heavy bodies have a tendency to fall to the earth. Using the data provided by Kepler, he calculated that the attractive force of the sun on the planets varied in inverse proportion to the square of the distance between it and them. He next calculated the influence which the earth would exert on the moon according to the same law. But the calculation did not quite square with the facts, and as a result he discarded the theory for many years. It was later discovered that part of the data on which he based his calculation (the earth's magnitude) was inaccurate. When the necessary correction was made, it was found that the deductions from the theory were what

[1] Cf. Bosanquet, *Implication and Linear Inference*, ch. v.

[2] This example is taken from the *Encyclopædia Britannica*, ed. xi, vol. 19, pp. 586 ff.

the movements of the moon as observed demanded. During the next few years (1679-1685), observations on other heavenly bodies and calculations from the law were found to be in such agreement that Newton became quite convinced of the truth of the hypothesis; and many subsequent deductions and observations have further confirmed it.

In this illustration we note: (1) The theory was first suggested as a hypothesis. Deductions were made from it and were found true in fact. The theory explained the facts and *they* confirmed *it*. (2) When a scientist is asked for an explanation of a particular fact, *e.g.* the movements of the moon or the orbits of the planets, he replies, 'The system is so. Therefore the fact must be so.' (3) It does not matter whether a particular element in the system is first arrived at by inference or observation, by deduction or induction. (4) The deductions involved are non-syllogistic. They are based on the inter-relation of the elements of the system as stated in the law.

(2) *Harvey's Discovery of the Circulation of the Blood.*[1] A rough idea of the circulating system will help to make the illustration clearer. The heart consists of two main chambers divided by a wall, the septum. From the left chamber starts the main artery, which branches out to different parts of the body and gets smaller and smaller until it seems to get lost in the capillaries near the surface of the skin. A vein leaves the right chamber of the heart and branches in the same way as the artery. Now the blood leaves the heart by the arteries, its colour being light red, travels through the capillaries, where its colour changes to dark red, and returns by the veins. This is the systemic or larger circulation, but there is another smaller or pulmonary circulation. The blood which enters the right chamber of the heart by the veins leaves it by an artery which leads to the lungs. This artery branches in the same way as the other, and in passing through the capillaries in the lungs the blood changes colour again, and returns to

[1] Taken from Singer, *The Discovery of the Circulation of the Blood.*

the left chamber of the heart a light red. This description of the complete circulation omits many of the complications but it will serve our purpose.

According to the theory current for some fourteen centuries before the time of Harvey (1578-1657), the blood in the arteries and veins was different, and it moved backwards and forwards in both cases. But the theory required that there should be some connection between the two, and this was supposed to take place through pores in the septum. This theory accounted for all the facts known in ancient times. The one difficulty in its way was that no passage could be discovered in the septum (for the very good reason that there is none), but one was assumed in the interests of the theory.

Now Harvey, basing his work on that of his predecessors, some of whom saw difficulties in the accepted theory though they had nothing to suggest in its place, observed (1) the expansion and contraction of the heart which causes the pulse beats and which forces the blood into the arteries, (2) the fact that the blood comes in spurts when an artery is cut, (3) the fact that there are many valves in the arteries and some in the veins and others at the junctions of both to the heart, and that these open in one direction only.

How many of these facts Harvey observed before he formed the hypothesis of the circulation we do not know. But having formed the theory he proceeded to verify it further. He demonstrated by experiments, *e.g.* bandaging arteries and veins, that there is a flow of blood in both arteries and veins, and that in the arteries it is away from the heart and in the veins towards the heart. But there was one requirement of the theory that Harvey could not verify by observation, namely, the passage of the blood from the arteries to the veins. But from the nature of the system he argued that such a passage must exist. For if there is a forward movement there must be a circulation, otherwise the supply of blood could not be maintained; and if there is a circulation, the blood must get from the arteries to the veins. And this must happen both in the

systemic system and in the lungs, for there is no passage through the septum as the old theory held. As the result of a large number of observations and experiments Harvey concluded that the passage must be there, and that the circulation is confirmed " by argument and ocular demonstration." A further confirmation of the theory was found in the fact that it explained what seemed a strange anomaly to the ancient physiologists, that the blood in the arteries going to the lungs had the same colour as that in the veins, (other than veins from the lungs to the heart, which in this respect resembled the arteries.) The explanation was that the blood changed colour in passing through the capillaries.

The passage itself was not seen till four years after Harvey's death, when Malpighi saw it with the compound microscope in the lungs of a frog. Some years later (1688) Leuwenhoeck with a more powerful microscope " clearly saw not only the systemic capillary circulation but also the actual passage of the blood corpuscles through the minute capillaries." [1] With this the ocular demonstration of the truth of Harvey's theory was complete.

In this illustration we note not only the function of the hypothesis in directing observations and experiment and explaining facts, but also that it did not matter whether any particular part of the system was first observed or arrived at by deduction, and that the deductions involved are not syllogisms. The character of the deduced part is determined by the nature of the system into which it fits.

(3) *The procedure of the detective.* A detective comes to the scene of a crime and gathers some facts. These suggest to him an hypothesis, or it may be several, as to the identity of the criminal. If one hypothesis is correct, such and such facts are likely; but he finds that they are not so and the hypothesis is rejected. He tries another, and tests it in the same way, looking for facts which will help to prove or disprove it. In the end, one hypothesis is confirmed by all the facts, and seems to be the only one left standing. Then the counsel for the prosecution in opening

[1] Singer, *Op. cit.* p. 74.

the case in Court describes the situation which the facts in favour of the hypothesis constitute, and proceeds to produce evidence that the facts are so. Given that system of facts, he asks the Court to draw the inference that a particular person committed the crime. That is the only hypothesis that will make the facts intelligible, the only interpretation that the system will bear. His argument is, 'Either you must deny the system, or admit the guilt of the prisoner.' The business of the defence is to overthrow the system by showing either that there are facts which are inconsistent with it, or that the facts admit of another interpretation, or that some of the facts are not as represented.

Take the following opening of a case [1] as such a system. Here the defence was able to overthrow the system, as usually happens in detective stories, but nevertheless it illustrates the method of hypothesis, and the deduction that draws a conclusion from a system of facts, equally well.

"The case that is now before the Court involves a charge of wilful murder against the prisoner, Alfred Draper, and the facts, in so far as they are known, are briefly these: On the night of Monday the 27th Sept. the deceased, Charles Hearn, dined with some friends on board the yacht *Otter*. About midnight he came ashore, and proceeded to walk towards Sundersley along the beach. As he entered St. Bridget's Bay, a man who appears to have been lying in wait, and who came down Shepherd's Path, met him, and a deadly struggle seems to have taken place. The deceased received a wound of a kind calculated to cause almost instantaneous death, and apparently fell down dead.

"And now what was the motive of this terrible crime? It was not robbery, for nothing appears to have been taken from the corpse. . . . Nor clearly was it a case of a casual affray. We are consequently driven to the conclusion that the motive was a personal one, a motive of interest or revenge, and with this view the time, the place, and the evident deliberateness of the murder are in full agreement.

[1] From R. Austin Freeman, *Dr. Thorndyke's Case Book*, chap. i.

"So much for the motive. The next question is, Who was the perpetrator of this shocking crime? And the answer to that question is given in a very singular and dramatic circumstance. . . . The murderer was wearing a very remarkable pair of shoes, and those shoes left very remarkable footprints in the smooth sands . . . and those tell-tale shoes have been identified. . . .

"And who now is the owner of those very singular, those almost unique shoes? I have said that the motive of the crime must have been a personal one, and, behold! the owner of those shoes happens to be the one person in the whole district who could have had a motive for compassing the murdered man's death. Those shoes belong to . . . Alfred Draper . . . the only person in this neighbourhood who was acquainted with the deceased. . . . I shall prove to you, that the deceased was often an unwelcome visitor at the prisoner's house and that the prisoner appeared constantly to shun and avoid him.

"One more question and I have finished. Where was the prisoner on the night of the murder? . . . In a house little more than half a mile from the scene of the crime. And who was with him in that house? Who was there to observe and testify to his going forth and his coming home? No one. He was alone in the house. . . .

"Such are the facts of the case. . . . And I assert that taken together they are susceptible of only one explanation, which is that the prisoner, Alfred Draper, is the man who murdered the deceased, Charles Hearn."

In this case what were called the facts were, for the most part, mistaken interpretations of the data supplied by observation. This shows how difficult it is to distinguish between facts and theories, or to be certain that a particular hypothesis is the only possible explanation of the facts.

These illustrations show the functions of hypotheses in guiding investigation and explaining facts. They show how inseparable induction and deduction are in concrete

thought, and how facts and theories mutually support and confirm one another. These features we have considered already, but there are two other consequences which deserve to be mentioned.

(1) They explain why so much of what is obviously argument appears so like description, and why so few conclusions either in science or ordinary life are drawn from two premises, as syllogistic logic would lead us to expect. If we consider the procedure of counsel opening a case in Court, or a politician putting a case (*e.g.* the case for tariff reform or free trade) to a meeting, or a scientific lecturer leading up to a conclusion, we find him making statement after statement without drawing any inference. He is explaining a system or situation, and he cannot do so in two or three propositions. But once we understand the system and accept it, a certain conclusion is inevitable. We must either accept the conclusion which he wishes us to draw, or deny that he has accurately described the system. That conclusion rests on, or follows from, not two premises but a large number. It is required to make the situation which has been described intelligible, to make the facts a consistent whole.

We see the same principle operative in the kind of answer which a scientist is obliged to make if anyone denies or questions a statement which he makes on the strength of his scientific knowledge. If an astronomer says that "the velocity of the earth travelling round the sun is 20 miles a second",[1] or an economist that "an increase in the money supply of a country must raise prices",[2] and the statement is challenged, the astronomer or economist cannot answer the challenge except by unfolding the system from which the statement follows as a conclusion. The economist, *e.g.* must explain the nature of the monetary system of a country and its function in the economic life of the community. From the economic system thus revealed the conclusion follows as a simple deduction.

[1] Eddington, *Stars and Atoms*, p. 15.

[2] Taussig, *Principles of Economics*, vol. ii. p. 50.

This is the main difference between the unorganized knowledge of the average man and the systematised knowledge of the scientist in his own department. You can deny a particular statement that the ordinary man makes without his feeling anything more than that he has made a mistake. But if you deny a conclusion that the scientist draws from his scientific knowledge, you are denying not it only but the whole system to which it belongs. The scientist realizes the implications of your denial, how much must go if this fact goes. The fact has its roots in the system, and fact and system must stand or fall together. Such deductive arguments are not syllogisms. But they are common enough not only in science but also in ordinary life. It is common enough to hear, 'These are the facts and they force me to believe . . . they leave me no option but to conclude . . . they are capable of only one explanation.' Perhaps most of the conclusions that we really care to defend are drawn in this way and have more than two premises.

(2) The illustrations and the facts just urged indicate where we must look for the criterion or test of truth. Facts and law, elements and system, mutually imply and support one another. If a law is comprehensive enough to cover all the relevant facts, and explains them consistently with one another, we regard it as true. This two-edged test of comprehensiveness and consistency is called the test of coherence. It is our only ultimate guarantee of the certainty of either facts or theories. The degree of certainty of a fact depends on the degree to which we see that it is an element in a coherent system and that if we deny it many other facts have to be denied too. Our knowledge becomes more certain and more difficult to overthrow as it becomes more organized into coherent systems. The certainty that our interpretation of a particular fact is right, and that our explanation of it is adequate, increases with the consistency and comprehensiveness of the system in which it is an element. And in the same way, that law is most certain which systematises into a coherent whole

the widest range of facts. In both cases the degree of certainty is measured by the answer to the question How much must you deny if you deny this?

EXERCISE XXIII

1. Explain and illustrate the part played by hypotheses in scientific investigation.

2. What conditions must a hypothesis satisfy (*a*) in order to be regarded as scientific, (*b*) in order to be regarded as established?

3. 'No explanation in terms of causality is final.' Discuss this statement and consider the light which it throws on the limits of the Inductive Methods.

4. What is meant by scientific explanation? How are (*a*) observed phenomena, and (*b*) scientific laws, explained?

5. What are the logical steps involved in the establishment of a law of nature? Illustrate your answer by a concrete example.

6. Explain and exemplify the way in which induction and deduction are combined in a concrete piece of reasoning

7. Can an inductive generalization ever be logically justified?

8. Explain and illustrate the processes involved in inductive generalization, and discuss the statement that 'the whole of the reasoning involved in induction is deductive.'

9. Consider carefully the relation between fact and theory.

10. Explain and illustrate: verification, crucial experiment, negative instance, residual phenomenon.

11. What kind of inference do you consider most adequate to express the ideal of scientific knowledge, and why?

CHAPTER XXIV

FALLACIES

A FALLACY may be defined as a faulty or unsound inference. Some writers use the term more widely to cover all mistakes that lead to incorrect statements, but it is more usual to restrict it to mistaken inferences. Other writers again define it as a faulty inference that is apparently conclusive. But the addition of the words 'apparently conclusive' does not add much to the significance of the definition. Unless an inference has some appearance of conclusiveness we can scarcely call it an inference, even a faulty one. Moreover, the distinction between inferences which are apparently conclusive and inferences which are not is very difficult to draw and varies from one person to another.

An account of fallacies is included in a book on logic not because they directly illustrate the principles of inference, but rather because they help by contrast to put these principles in a clearer light, and because a knowledge of them enables us more easily to guard against them. Moreover, many fallacies fall under distinct types, and some of the types are of common occurrence. To these names have been given, and a knowledge of them will enable us to refer to them more easily.

It is impossible to classify fallacies exhaustively. They do not proceed on any principles and there are no limits to their kinds. "Truth may have its norms, but error is infinite in its aberrations." [1] One possible classification is an indirect one, by reference to the logical principles

[1] Joseph, *Introduction to Logic*, p. 569.

which the fallacies violate. But the same argument may violate several logical principles, and so be an instance of several types of fallacy. We have already illustrated many kinds of fallacy, in dealing with the different kinds of inference and the possible ways in which they may be violated, like undistributed middle, illicit process of the major or minor term, non-exhaustive disjunction, etc. ; but there are others to which specific names have been given but to which we have not given any special consideration. Of these, several classifications have been suggested, but from the nature of the case none of them is without defect. We shall follow the list and the terminology of Aristotle, which is in the main followed even by those writers who introduce other classifications. We shall indicate where modern writers with some unanimity use a term in a sense different from that in which it is used by Aristotle.

Aristotle divides fallacies, by dichotomy, into those which are, and those which are not, due to some ambiguity or looseness in the verbal expression.[1] His successors called the first class fallacies *in dictione*. Such fallacies are untranslatable from one language to another. Aristotle pays rather more attention to them than modern writers do, for in his treatment of fallacies he was concerned not so much with the discovery of truth as with the art of disputation, the refutation of an adversary. The second class, called fallacies *extra dictionem*, have no common principle except that they are not merely verbal. Aristotle enumerates six fallacies *in dictione*[2] and seven *extra dictionem*[3] as follows :

(*a*) *In dictione :*

1. *Equivocation* is an ambiguity in the terms used. In some instances the ambiguity is of quite a trifling character and would deceive no one, as in the argument :

> No human beings are made of paper,
> All pages are made of paper,
> Therefore no human beings are pages.

In other instances the ambiguity is more difficult to detect

[1] *Soph El* 165^b 23. [2] *Ibid* ch 4 [3] *Ibid* ch 5.

and is often a considerable source of error, as in the argument, 'Everyone ought to contribute to the support of the unfortunate; therefore there is no harm in a law which compels him to do so.' Here the ambiguity lies in confusing moral and legal obligation. This fallacy may be regarded as an instance of the fallacy of four terms.

2. *Amphiboly* is an ambiguity due to the structure of a sentence. Such ambiguities are of more frequent occurrence in the classical languages, but they are not unknown in English. We find an instance of it in the statement of the clergyman who said, 'I will wear no clothes to distinguish me from my christian brethren.' Or again, 'The Duke yet lives that Henry shall depose.'

3 and 4. *Composition* and *Division* are the converse one of the other and it is therefore better to take them together. According to Aristotle's usage, the fallacy of Composition is committed when words are taken together which ought to be taken separately, like 'Platinum and iron are rare and useful metals; therefore platinum is a rare and useful metal, and iron is a rare and useful metal'; while the fallacy of Division consists in taking separately words which ought to be taken together, like 'Jews and Gentiles are men and brethren; therefore Jews are men and Gentiles are brethren.'

Aristotle seems to have regarded these fallacies as merely verbal, but some of his examples can be interpreted as involving a confusion between a distributive and a collective term, as '3 and 2 are odd and even; 3 and 2 are 5; therefore 5 is odd and even.' This is the sense in which the terms are mainly used by modern writers. According to them, the fallacy of Composition consists in inferring from the distributive use of a term to its collective use, *e.g.* 'As every member of a jury is liable to be mistaken, we can place no confidence in trial by jury'; and Division consists in arguing from a term used collectively as if it had been used distributively, *e.g.* 'The people of America have a prejudice against Negroes; Mr. A. is an American; therefore he is prejudiced against Negroes.'

5. By the fallacy of *Accent*, Aristotle meant a mistaken inference due to the confusion of words that differ only in accent. But as such fallacies cannot be committed in modern languages, modern writers have used the term to describe the fallacy due to the alteration of meaning which results from emphasising a particular word or words in a sentence. Compare, *e.g.* the difference of meaning of the sentence, 'And he said to them: saddle me the ass; and they saddled him the ass,' with and without special emphasis on 'me' and 'him.' The term is sometimes extended to cover a more important fallacy, that due to the alteration of the meaning of phrases or sentences when they are divorced from their context, and especially when words not italicized in the original are italicized in a quotation.

6. The fallacy of *Figure of Speech* is committed when we regard the same grammatical form (*e.g.* the same inflection or prefix) as having the same force in every case in which it occurs, as would happen if one said that "*important* is a negative notion because *imperturbable* or *impenitent* is."[1]

(*b*) *Extra dictionem*:

1. The fallacy of *Accident* consists in arguing that what is true of the accidental qualities or relations of a thing is true of the thing itself. *E.g.* 'This dog is yours; this dog is a father; therefore this dog is your father.' This fallacy was not too clearly defined by Aristotle, and modern writers on logic tend to ignore it. They retain, however, the term 'fallacy of Accident,' and use it to describe one aspect of the fallacy which is usually called *Secundum Quid*. The other aspect they call the Converse fallacy of Accident.

2. The fallacy of *Secundum Quid*, as just noted, has two forms. Both forms consist in ignoring the conditions under which a particular statement is true. In the one form we infer that what is true normally, or as a general rule, is true absolutely and in every particular case. The other is the converse of this, and argues that what is true

[1] Joseph, *op. cit.* p. 584.

under particular circumstances or conditions is true normally. The following may serve as examples :

Staying in bed is good for a person's health, when he is unwell ; therefore it is normally so.

A man should be allowed to do what he likes with his own ; therefore the law should not interfere with the conditions under which he employs labour.

What is bought in the market is eaten ; raw meat is bought in the market ; therefore raw meat is eaten.

A walk is good for the health ; therefore a walk in bad weather is so.

3. *Ignoratio Elenchi* or *Irrelevant Conclusion.* This fallacy consists in proving the wrong point. The reasoning involved may be perfectly valid, but the conclusion arrived at is not exactly what was wanted in the circumstances. Aristotle applied the term to cases in which a disputant proved some point other than the logical contradictory of the thesis maintained by his opponent, *i.e.* something other than what was required to refute his opponent ; but modern usage applies it to all cases of proving something other than what is wanted. When a judge or jury is affected by the character of a prisoner rather than by the evidence of his having committed the crime of which he is accused, we have an instance of this fallacy.

The fallacy of *Ignoratio Elenchi* occurs so frequently, and can be committed in so many ways, that special names have been given to some of its more common forms. The form of irrelevant conclusion called *argumentum ad hominem* consists in proving something about the person who puts forward a doctrine or proposal rather than about the merits of the doctrine or proposal. This is a common form of political argument. Instead of proving that a particular reform or law is bad, its opponents often point out that its proposer is not a fit person to advocate it or that it is inconsistent with his former views. Another form, the *argumentum ad verecundiam*, consists in appealing to authority instead of establishing our contention on its merits. If one were to argue against the abolition of slavery on the

ground that Aristotle regarded it as justifiable, or that it was sanctioned by the Christian church, one would be guilty of this fallacy. The *argumentum ad populum* consists in appealing to people's passions and prejudices rather than to their intelligence. We get it in such an argument as the following: 'If you oppose this measure the government will be defeated and the Socialists will get into power. They will undermine the constitution of the country and increase the Income-tax, etc.' Another form of irrelevant conclusion is the *argumentum ad ignorantiam*. This consists in proving something like the point that we profess to establish, trusting that the ignorance of the people to whom the argument is addressed will prevent them from noticing the difference between the two points.

4. The fallacy of *Petitio Principii*, or *begging the question*, is committed when we either assume the proposition we are proposing to prove, or prove it by premises which can only be proved by means of the proposition itself. In this fallacy we have only an appearance of inference. What we are really doing is repeating or reasserting what we are supposed to be proving. Mill, as we have seen (Chap. xviii), holds that this is what is done in every first figure syllogism, and it does take place in every syllogism in which the major premise is an enumerative proposition. If we say 'Opium produces sleep because it has a soporific quality,' or 'You ought to give alms because it is a duty to be charitable,' the reason we give for the conclusion is obviously just a reassertion of the proposition to be proved. But there are other cases in which the fallacy is committed less openly and directly. In the course of a long argument the conclusion to be proved, or something which implies it and has not been independently established, is slipped in in a disguised form. We are then said to argue in a circle. The following instance is quoted by Mr. Joseph,[1] from Dr. McTaggart.[2] "There are certain people who look on all punishment as essentially degrading. They do not, in their saner moods, deny that there may be cases in which it

[1] *Op. cit.* p. 593. [2] *Studies in Hegelian Cosmology*, § 142.

is necessary. But they think, if anyone requires punishment, he proves himself to be uninfluenced by moral motives, and only to be governed by fear. . . . They look on all punishment as implying deep degradation in some one—if it is justified the offender must be little better than a brute; if it is not justified the brutality is in the person who inflicts it. The reasoning appears to travel in a circle. Punishment, they say, is degrading, therefore it can work no moral improvement. But this begs the question. For if punishment could work a moral improvement, it would not degrade but elevate. The humanitarian argument alternately proves that punishment can only intimidate, because it is brutalizing, and that it is brutalizing because it can only intimidate."

5. Fallacy of the *Consequent.* Aristotle uses this term to describe the fallacies which we noticed when dealing with hypothetical reasonings, the fallacies which result when condition and consequent are regarded as convertible. *E.g.* 'If the weather is foggy the train is late; the train is late, therefore the weather is foggy; or the weather is not foggy, therefore the train is not late.' Some modern writers, however, use the term to describe any kind of loose or inconsequent argument.

6. Fallacy of the *False Cause*, or *Non Causa pro Causa.* This fallacy is committed when we try to disprove a proposition on the ground that some obviously false consequence results from it, while in fact the false consequence follows not from it but from something else. *E.g.* "It is ridiculous to suppose that the world can be flat; for a flat world would be infinite and an infinite world could not be circumnavigated, as this has been. Here the supposition inconsistent with the fact of the circumnavigation of the world is not that the world is flat, but that it is infinite; it might be flat and yet circumnavigable, if it were finite; the thesis of the flatness is therefore unfairly discredited." [1] The term False Cause is, however, more commonly used to indicate a fallacy which is of much more frequent occur-

[1] Joseph, *op. cit.* p. 594.

rence than the one just mentioned, the fallacy of regarding as a cause that which is not really a cause. The most important example of it in this sense is that called *Post hoc, ergo propter hoc*, the fallacy which consists in arguing that because one thing follows another the first is the cause of the second. As an example, we might take the argument that the series of bad summers which followed the introduction of 'Summer Time' was the result of the interference with the natural order of things supposed to be involved in the innovation. Most superstitions are instances of the *post hoc* fallacy.

7. The fallacy of *Many Questions* consists in asking a question with an implication, the question being so framed that any answer to it must grant the truth of the implication like, 'Have you given up telling lies?—Yes or no.' A famous instance of it is contained in the problem propounded by Charles II to the members of the Royal Society, namely, Why does a live fish, when placed in a bowl full of water, not cause the water to overflow, whereas a dead fish does? The members of the society offered various explanations of the difference, but no such difference in fact exists. Misleading questions of this kind are often asked in all good faith about mental and social phenomena. The question 'What is the seat of the soul or mind?' implies that the mind is in space in the same way as material objects. If this implication is false, the question is unanswerable. Similarly, the question 'Is poverty the cause of crime or *vice-versa*?' implies that one of the two is the cause, whereas the relation between them seems to be reciprocal.

EXERCISE XXIV

Analyse the following arguments and point out and name any fallacies which they contain:

(1) The end of life is perfection; death is the end of life; therefore death is the perfection of life.

(2) If any business man takes a holiday he suffers from loss of custom. Therefore a general holiday is undesirable, as everyone would thereby lose custom.

(3) One can live without food, for one can live without bread or potatoes or porridge or any other kind of food

(4) In reply to the speaker's argument, it need only be said that two years ago he himself advocated the very principles he now opposes

(5) The great plea for increase of armaments is that any nation which makes itself strong makes itself respected and feared.

(6) You say that this book is inaccurate, but I saw an excellent review of it

(7) How can you hold that self-government is good and deny that India should govern itself?

(8) "The only proof capable of being given that an object is visible, is that people actually see it The only proof that a sound is audible, is that people hear it In like manner, the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it If the end which the utilitarian doctrine proposes to itself were not, in theory and in practice, acknowledged to be an end, nothing could ever convince any person that it was so No reason can be given why the general happiness is desirable, except that each person desires his own happiness This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons" (Mill, *Utilitarianism*, ch iv.)

MISCELLANEOUS EXERCISES AND TEST QUESTIONS

1. How far is logic necessarily a study of language? To what extent would the accusation be justified that logic is unduly occupied with verbal issues?

2. How would you distinguish (*a*) a term from a word, (*b*) a proposition from a sentence, (*c*) a syllogism from an inference?

3. What is the relation between the first and each of the following propositions? Supposing the first to be true, determine the truth or falsity of the remainder, indicating in each case the processes by which you reach your conclusion.

(*a*) Only the brave deserve the fair.
(*b*) Some who deserve the fair are cowards.
(*c*) Some who are brave are not deserving of the fair.
(*d*) Some who deserve the fair are brave.
(*e*) All cowards are undeserving of the fair.
(*f*) All who do not deserve the fair are cowards.

4. In what sense, if in any, can it be said that (*a*) the negative (*b*) the hypothetical proposition states a fact?

5. Write down the rules of definition, and apply them to the following definitions:

(*a*) Division is the exposition of the denotation of a term.
(*b*) A science is an organized body of knowledge.
(*c*) Coins are lumps of metal. (*O.*)

6. Explain and illustrate the distinction between Abstract and Concrete terms, General and Collective terms, Definition and Description, Real and Verbal propositions. (*O.*)

7. Point out the ambiguities in the terms, extension and intension. In what sense is it true that proper names do not, and that common names do, have intension? Do *man* and *Asquith* have extension in the same sense, and if not, what is the difference?

8. Put the following into logical form, and give the converse, the contradictory, and the obverse of each:

(*a*) Only the actions of the just
Smell sweet and blossom in the dust.
(*b*) Several creditable witnesses vouched for the fact.
(*c*) With the exception of Mr. Jones, no one escaped.
(*d*) A few men will not suffice to move a steam roller.

9. Examine the statements :

(*a*) A term may be non-connotative.

(*b*) Two particular propositions cannot prove a conclusion.

(*c*) If a negative premise occurs in a sorites, it must be the last premise. (*O.*)

10. Give concrete examples of the following : (*a*) a hypothetical syllogism in which the consequent is denied, (*b*) the *modus tollendo ponens*, (*c*) a complex constructive dilemma, (*d*) a Goclenian sorites containing a particular premise.

11. How are the following propositions logically related to each other ? (*a*) All *x* is *y*, (*b*) All *y* is *x*, (*c*) All not-*y* is not-*x*, (*d*) Some not-*y* is *x*.

12 Explain the nature of classification and the criteria of a good classification How far can a natural classification conform to the rules of logical division ?

13. Explain what exactly you mean by a Law of Thought. Is a Law of Thought one which is necessarily exemplified in every thought, or one which ought to be exemplified but need not be ?

14. What are the principal objections to the view that the primary import of propositions is to classify ?

15. State and examine the grounds on which it has been held that all deductive inferences are reducible to the form of syllogism.

16. What conclusion, if any, can be drawn from each pair of the following sentences, taken two and two together :

(*a*) Only gentlemen are members of the club.

(*b*) Some members of the club are not officers.

(*c*) All members of the club are invited to compete

(*d*) All officers are invited to compete.

17. Given that everything is either *q* or *r*, and that *r* is *q* unless it is not-*p* Prove that all *p* is *q*.

18. In how many ways can 'No S is P' be proved syllogistically ? Prove your answer from the general rules of the syllogism.

19 (*a*) Explain carefully the grounds of the syllogistic rule regarding the distribution of the middle term.

(*b*) All P is M and All S is M yield no conclusion, but from their equivalents All not-M is not-P and All not-M is not-S a conclusion does follow. Does this conflict with the above syllogistic rule ?

20. Examine the following arguments, and name the fallacy (if any) involved :

(*a*) It hoots : so it's only an owl.

(*b*) You say that he is dumb. But silence gives consent ; so he confesses his guilt.

(*c*) If the Snark be a Boojum, the man will vanish away. The man vanished away, therefore the Snark was a Boojum. (*O.*)

21. How far are universal judgments involved in all valid reasoning ? Discuss in the light of your answer the nature of the reasoning in syllogism. (*O.*)

22. 'The formulation of the best definition may be said to be the goal of scientific thought.' 'Thinking, if it is to be of any value, must start with clear definitions.' Examine these two statements.

23. Why does Mill deny that the syllogism is a valid form of inference? What function does he assign to it in reasoning?

24. If in a valid syllogism the middle term is distributed only in one premise, and the other premise is negative, determine its figure and mood.

25. Show that if two valid syllogisms have a common premise, while the other premises are contradictories, both the conclusions must be particular.

26. Prove that if the middle term of a valid syllogism be twice distributed the conclusion must be particular.

27. If p is always accompanied by q, and r is always accompanied by s, and q and s are never found together, what can you infer as to the relation between p and r?

28. If the major term of a valid syllogism is the predicate of the major premise, what do you know about the minor premise?

29. 'Induction may be defined as the operation of discovering and proving general propositions.' Is this definition satisfactory? (*O.*)

30. Consider how far any of the methods described by Mill could be used in solving any two of the following:

(*a*) The cause of a low rate of wages in rural districts.
(*b*) The reasons for the colouring of a particular insect's wings.
(*c*) The explanation of a by-election result. (*O.*)

31. Can the Joint Method of Agreement and Difference be so stated as to be logically satisfactory and practically useful?

32. What is the logical force of 'circumstantial evidence'? Discuss from the point of view of Logic the statements:

(*a*) One piece of circumstantial evidence is worthless, but the cumulative effect of many may be convincing.

(*b*) No chain of evidence is stronger than its weakest link. (*O.*)

33. What is meant by imperfect induction, plurality of causes, verifying a hypothesis, negative instance, simple enumeration?

34. What do you take to be the essence of induction? Can it ever yield more than probable conclusions?

35. Explain and exemplify the hypothetical method in induction, carefully explaining the logical processes used at each stage. Why are hypotheses and deduction the favourite methods of advanced sciences?

36. (*a*) What is meant by experiment? (*b*) When is it possible? (*c*) What are the advantages of using it? (*O.*)

37. Does causality necessarily involve succession?

38. What is meant by a plurality of causes? How far are the methods of the investigator determined by a plurality of causes?

39. Explain and exemplify *petitio principii*, rebutting dilemma, division by dichotomy, irrelevant conclusion, crucial instance.

40. Analyse the following arguments, explain their logical character and point out any fallacies which they contain

(1) A mouse is an animal, therefore a large mouse is a large animal

(2) If we can be sure that the inconsistent is unreal, we must logically be just as sure that the real is consistent

(3) Existent circles are all imperfect There must therefore be some thing other than existent circles which is perfect

(4) Only some men are liars, therefore there are some liars who are not men

(5) Only if you break the law are you imprisoned. That is why you are let free

(6) It is desirable to reduce the number of deaths from poisoning Prohibition of the sale of poisons would have this effect and is therefore desirable

(7) Barristers cannot afford to be sincere and no one who is insincere is trustworthy Therefore no trustworthy people will be found to be barristers

(8) The existence of capital punishment would be justifiable only if it led to a diminution of crime, but it certainly does not do this

(9) Everything must have a cause, for if anything wanted a cause it would produce itself, *i e* exist before it existed

(10) If I live for others neglecting myself, I shall make myself unhappy, and if I live for myself, neglecting others, others will make me unhappy But either I shall live for others or I shall live for myself, therefore I shall be unhappy

(11) It is a greater fault rigidly to censure than to commit a small oversight The one showeth himself man in mistaking, the other no man in not pardoning a light mistake

(12) Man must be governed by force or by reason But not by force, for they rebel against it, nor by reason, for few listen to it Therefore it is impossible to govern them

(13) When all is said, it remains true that there are but three ways of living possible in this world—by working, by robbing, or by begging To beg is infamous, to rob is criminal if a man will not work neither shall he eat

(14) Boswell "I asked if he (a certain traveller) was not a man of sense" Johnson "Why, sir, he is not a distinct relater, and I should say that he is neither abounding nor deficient in sense I did not perceive any distinct superiority of understanding" Boswell "But will you not allow him a certain nobleness of resolution in penetrating into distant regions" Johnson "That, sir, is not to the present purpose We are talking of his sense A fighting cock has a nobleness of resolution"

(15) If competitive examinations do not tend to select the best men, they should be abolished, and if they do so tend they should be applied to every appointment Therefore they ought to be abolished or they ought to be applied to every appointment

(16) If Newton had not lived, the law of gravitation would still have been discovered, and if Darwin had not lived, someone else would have discovered the principle of evolution Why then should we deplore so much the premature loss of this gifted man of science?

(17) Considering the immense sums of money that were raised during the war, it is absurd to say that we cannot afford expensive schemes of social reform.

(18) The proposed League of Nations is to be viewed with the greatest suspicion. In the first place, we cannot afford to trust our safety from aggression to a body that will be weak and divided. In the second place, the control of our national aspirations by an external body on which our representatives form a minority exposes us to oppression and endangers our prosperity. The whole idea is an astute move of President Wilson's for party advantage, and we may therefore neglect his arguments. Moreover, we shall not gain the favour of the Americans by supporting the League, for in this matter as everyone knows President Wilson is in a hopeless minority in his own country.

(19) "The reckoning, sir; and did you sponge on any man for a reckoning? Sir, no man should enter the door of a public-house without paying his lawing."

"I admit the general rule, sir," I replied, "But this was a parting cup between Darsie and me; and I should conceive it fell under the exception of *Doch an Dorroch*."

"You think yourself a wit," said my father, . . . "but I reckon you did not eat your dinner standing, like the Jews at the Passover? And it was decided in a case before the town-bailies of Cupar-Angus, when Luckie Simpson's cow had drunk up Luckie Jamieson's browst of ale, while it stood at the door to cool, that there was no damage to pay, because the crummie drank without sitting down; such being the very circumstance constituting *Doch an Dorroch*, which is a standing drink, for which no reckoning is paid. Ha, sir! What says your advocateship (fieri) to that? *Exceptio probat regulam.*"

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